Graph Search

EECS 214, Fall 2018

Questions we might ask about graphs

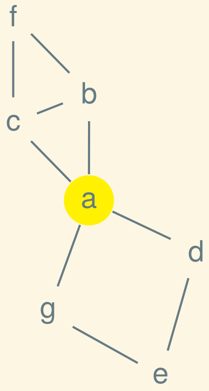
- Is there a path from v to u?
- What's the shortest path from v to u?
- Are there any cycles?

Graph search: basic idea

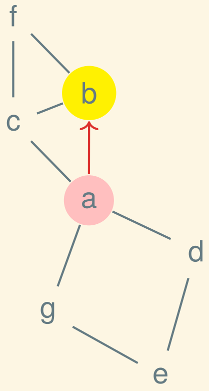
To answer whether there's a path (among other things), we can use:

- Depth-first search (DFS): go as far as you can along a path, then go back and try anything you haven't tried yet
- Breadth-first search (BFS): explore all the successors of a vertex before exploring their successors in turn

DFS example



DFS example



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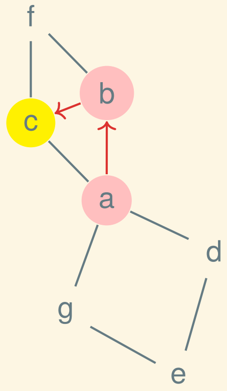
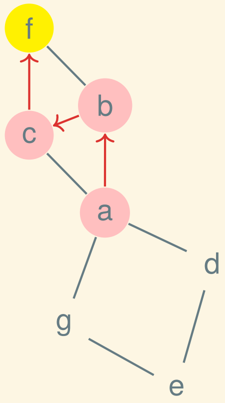
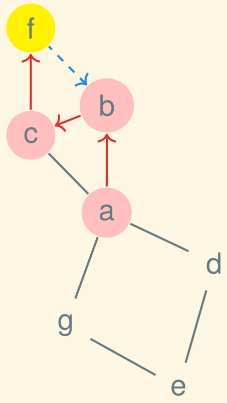


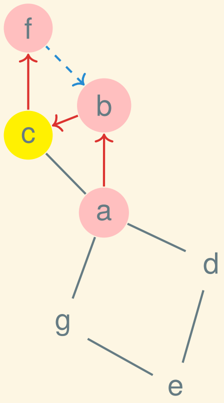
DFS example



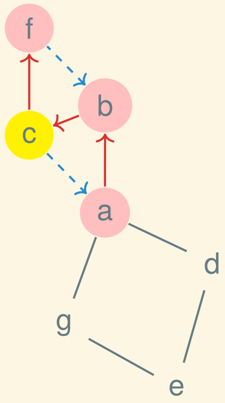
DFS example



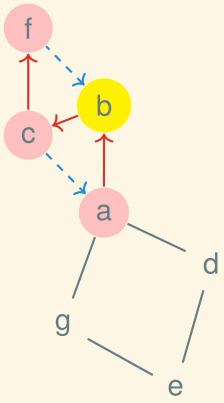
DFS example



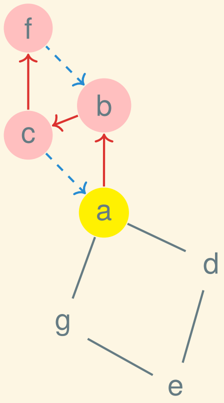
DFS example



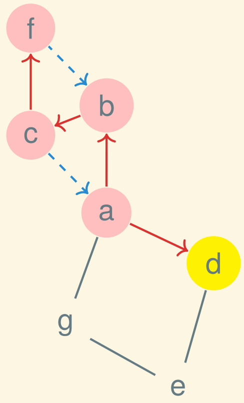
DFS example



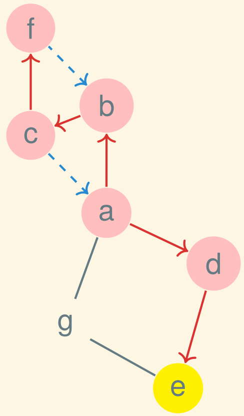
DFS example



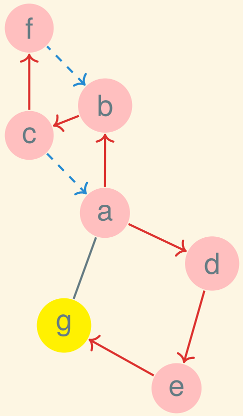
DFS example



DFS example



DFS example



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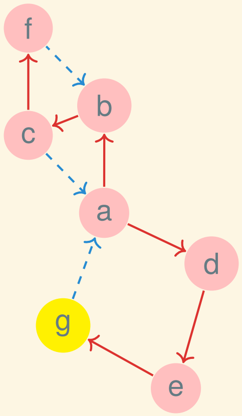
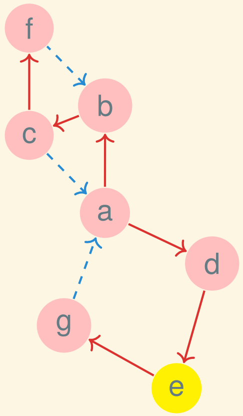


DFS example



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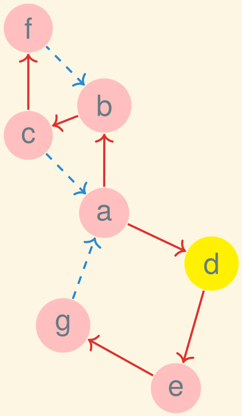

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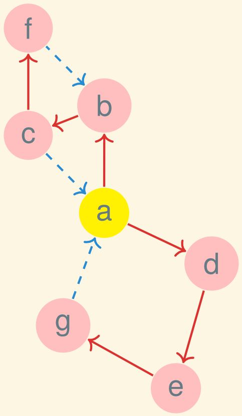


Recursive DFS algorithm (one source)

```
Procedure DFS(graph, start) is
    seen ← new array (same size as graph, filled with false);
    Procedure Visit(v) is
        if not seen[v] then
            seen[v] ← true;
            for u in Successors(graph, v) do
                Visit(u)
            end
        end
    end
    Visit(start);
    return seen
end
```

Recursive DFS algorithm (one source, lifted)

```
Procedure Visit(graph, seen, v) is
    if not seen[v] then
        seen[v] ← true;
        for u in Successors(graph, v) do
            Visit(graph, seen, u)
        end
    end
end

Procedure DFS(graph, start) is
    seen ← new array (same size as graph, filled with false);
    Visit(graph, seen, start);
    return seen
end
```

Recursive DFS algorithm (1 src., builds tree)

```
Procedure DFS(graph, start) is
    preds ← new array (same size as graph, filled with false);
    Procedure Visit(pred, v) is
        if not preds[v] then
            preds[v] ← pred;
            for u in Successors(graph, v) do
                Visit(v, u)
            end
        end
    end
    Visit(true, start);
    return preds
end
```

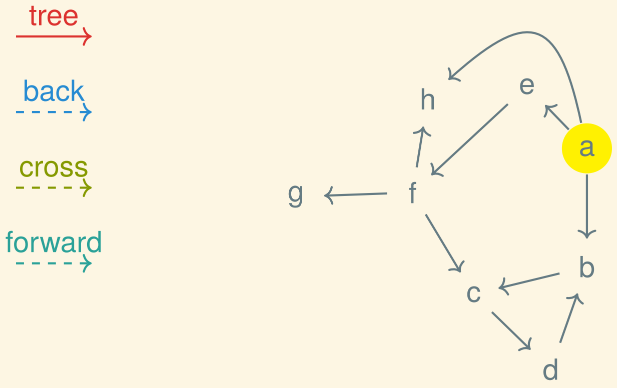
Recursive DFS algorithm (full)

```
Procedure DFS(graph) is
    preds ← new array (same size as graph, filled with false);
    Procedure Visit(pred, v) is
        if not preds[v] then
            preds[v] ← pred;
            for u in Successors(graph, v) do
                Visit(v, u)
            end
        end
    end
    for v in Vertices(graph) do
        Visit(true, v)
    end
    return preds
end
```

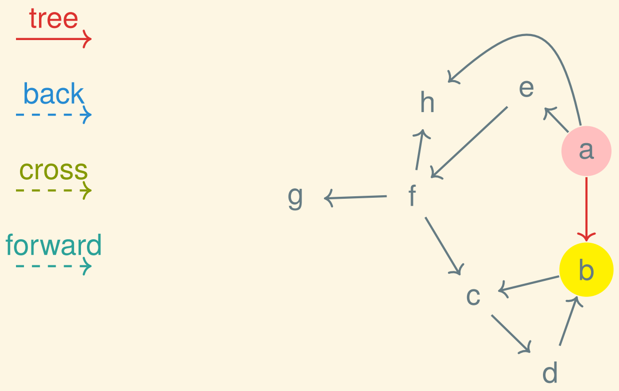
Iterative DFS algorithm

```
Procedure DFS($graph$, $start$) is
    $preds$ ← new array (same size as graph, filled with false);
    $todo$ ← new stack;

    $preds[start]$ ← true;
    Push($todo$, $start$);

    while $todo$ is not empty do
        $v$ ← Pop($todo$);
        for $u$ in Successors($graph$, $v$) do
            if not $preds[u]$ then
                $preds[u]$ ← $v$;
                Push($todo$, $u$)
            end
        end
    end
    return $preds$
end
```

Running DFS on a digraph



Running DFS on a digraph



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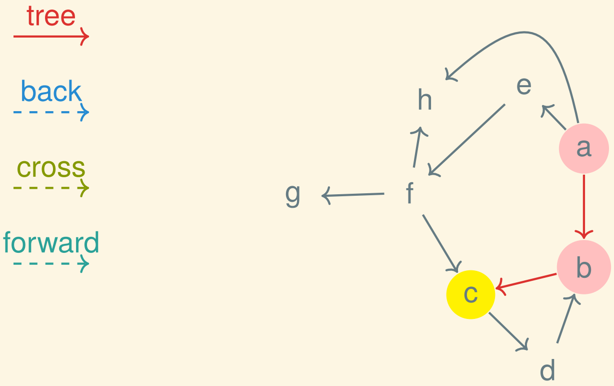

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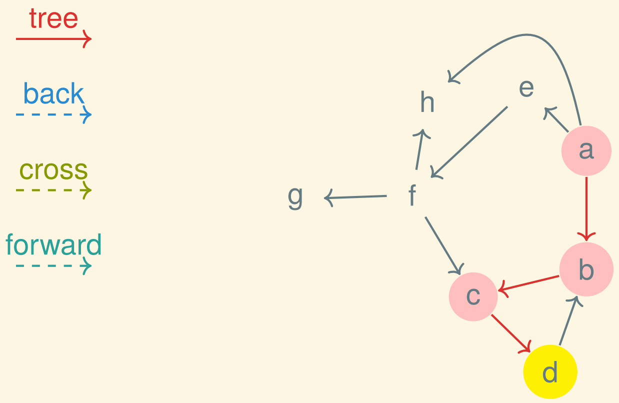

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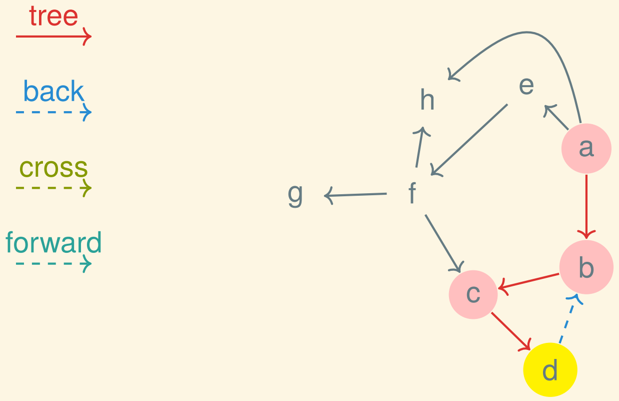

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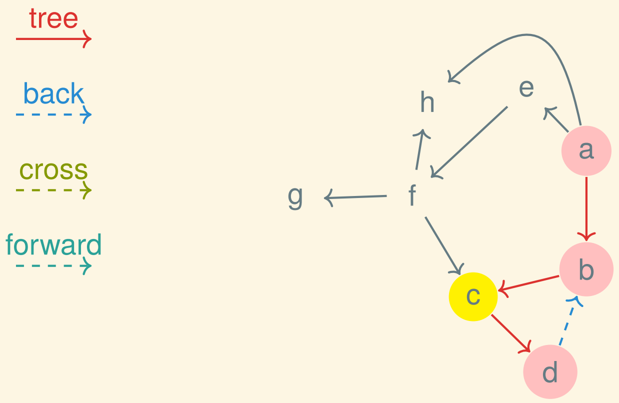

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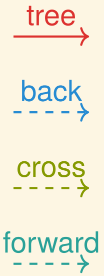

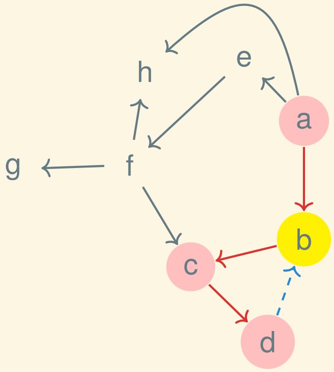

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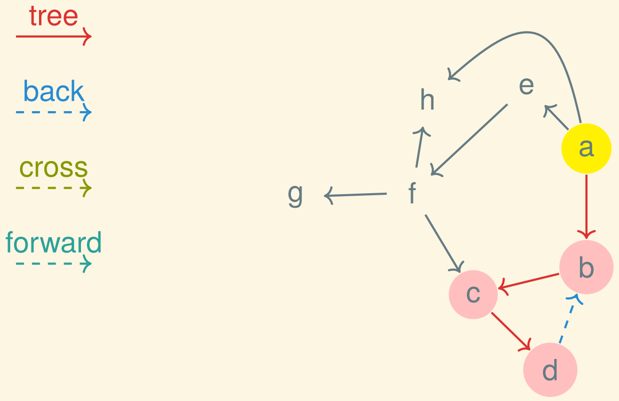

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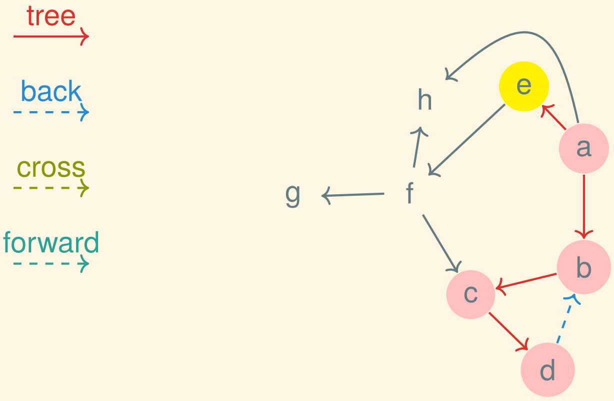

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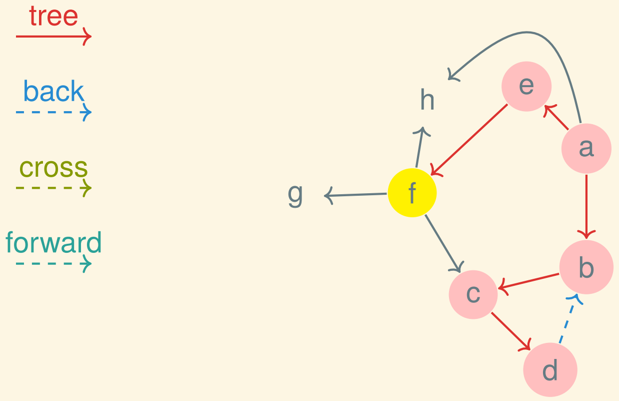

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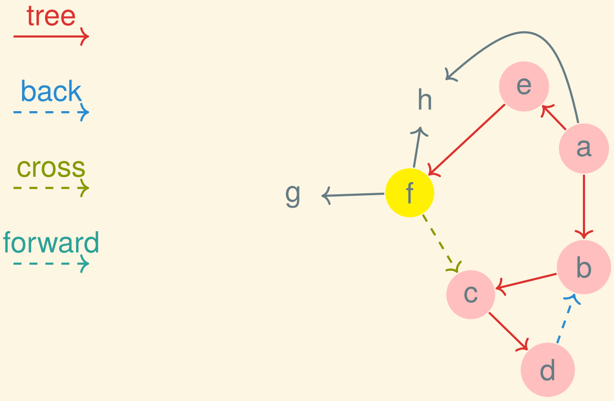

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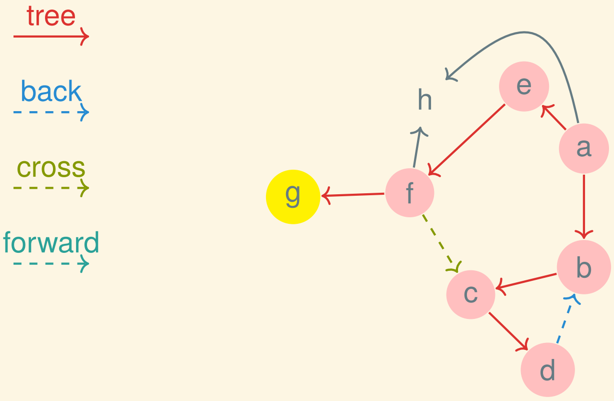

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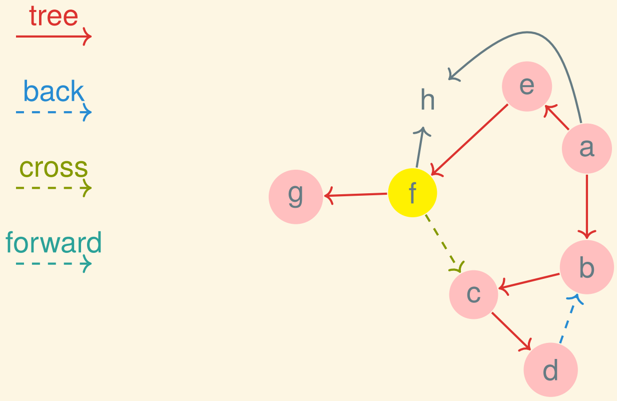

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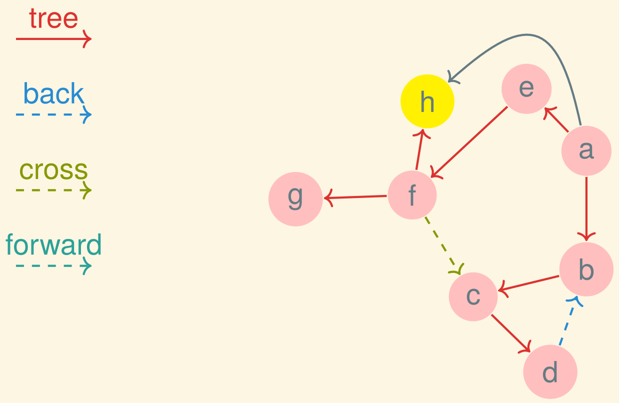

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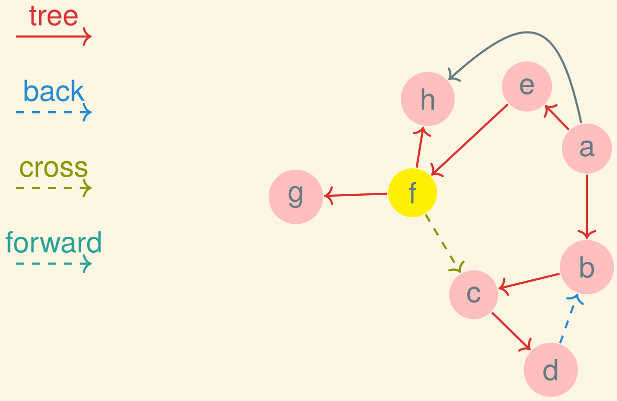

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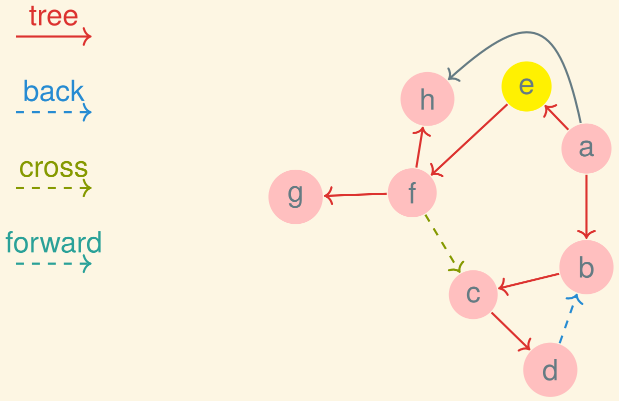

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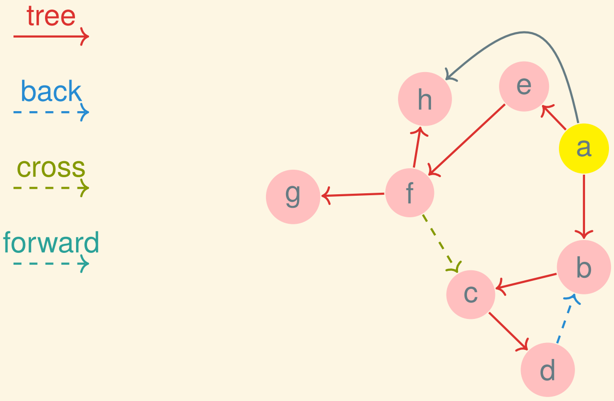

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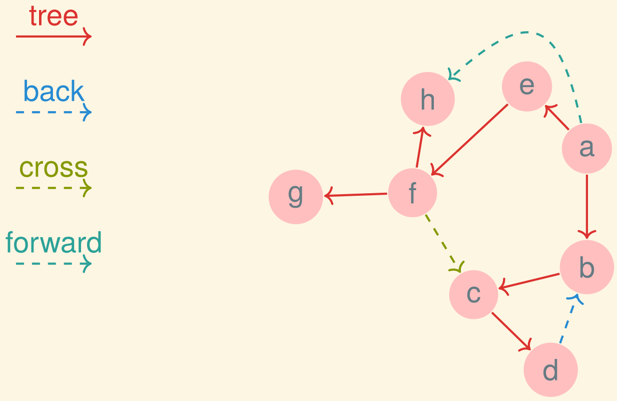
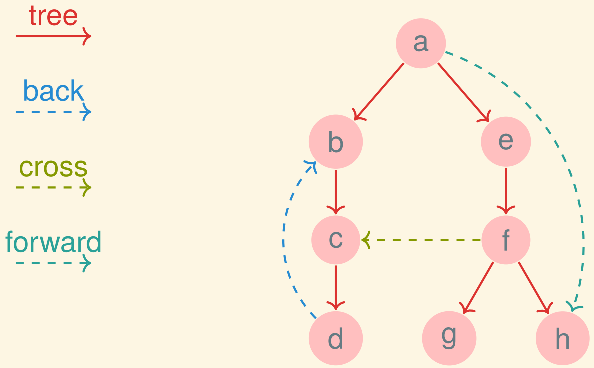


A DFS tree



DFS for cycle detection

```
Procedure FindCycle(graph) is
    started ← new array (same size as graph, filled with false);
    finished ← new array (same size as graph, filled with false);

    Procedure Visit(v) is
        if not finished[v] then
            if started[v] then
                we found a cycle!
            end
            started[v] ← true;
            for u in Successors(graph, v) do
                Visit(u)
            end
            finished[v] ← true;
        end
    end

    for v in Vertices(graph) do
        Visit(v)
    end
end
```

Breadth-first search

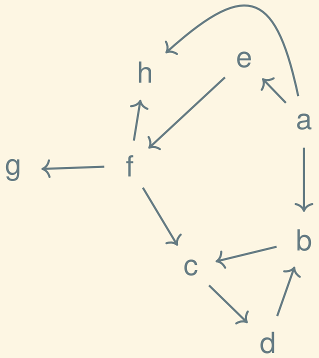
```
Procedure BFS(graph, start) is
    preds ← new array (same size as graph, filled with false);
    todo ← new queue;

    preds[start] ← true;
    Enqueue(todo, start);

    while todo is not empty do
        v ← Dequeue(todo);
        for u in Successors(graph, v) do
            if not preds[u] then
                preds[u] ← v;
                Enqueue(todo, u)
            end
        end
    end

    return preds
end
```

Running BFS on a digraph



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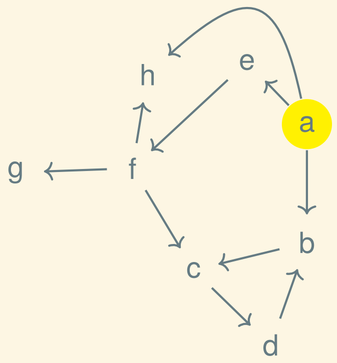
Running BFS on a digraph



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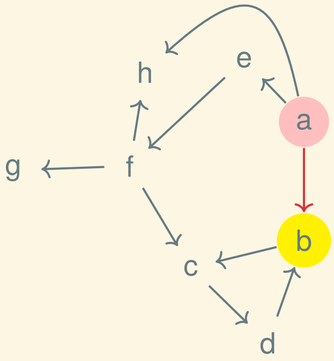
Running BFS on a digraph



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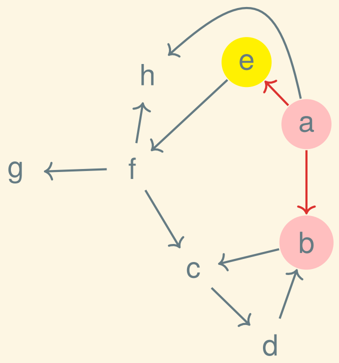
Running BFS on a digraph



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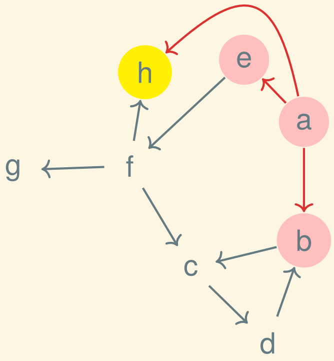
Running BFS on a digraph



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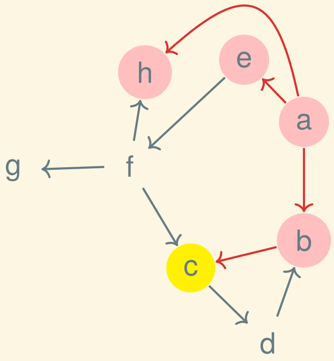
Running BFS on a digraph



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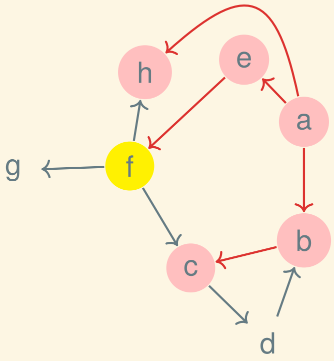
Running BFS on a digraph



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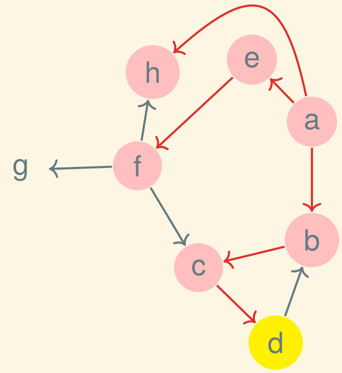
Running BFS on a digraph



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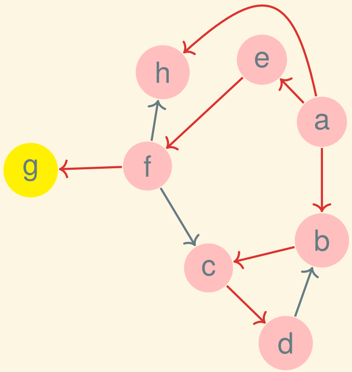
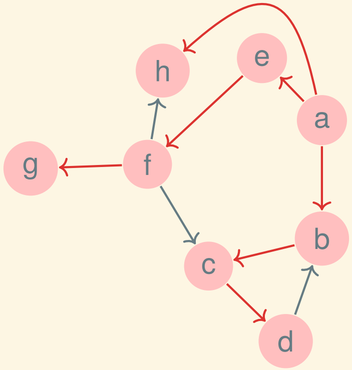
Running BFS on a digraph



Running BFS on a digraph



Generic graph search

If *todo* is a stack we get DFS; if *todo* is a queue we get BFS:

```
Procedure Search(graph, start) is
    preds ← new array (same size as graph, filled with false);
    todo ← new collection;

    preds[start] ← true;
    Add(todo, start);

    while todo is not empty do
        v ← Remove(todo);
        for u in Successors(graph, v) do
            if not preds[u] then
                preds[u] ← v;
                Add(todo, u)
            end
        end
    end

    return preds
end
```

Next time: shortest paths