n-way Mutual Exclusion

EECS 395 “Rust”

Jan. 21, 2016

Filter algorithm for *n* threads

```
template <int N>
class FilterLock : public GuardedLockBase {
  int level_[N] = {0};
  int waiting_[N];

  bool exists_competition(int level)
  {
    for (auto k : thread::all_ids())
      if (k != i() && level_[k] >= level)
        return true;
    return false;
  }
  ⋮
};
```

```
template <int N>
class FilterLock : public GuardedLockBase {
    ⋮
public:
    virtual void lock() override
        for (int level = 1; level < N; ++level) {
            level_[i()]        = level;
            waiting_[level] = i();
            while (exists_competition(level) && waiting_[level] == i())
            { }
        }
    }

    virtual void unlock() override
        level_[i()] = 0;
    }
};
```

Filter lock properties

- Mutual exclusion
  - By induction, one thread gets stuck in each level…
- Deadlock freedom
  - Like Peterson—only one thread can wait per level
- Starvation freedom
  - Like Peterson—every thread advances if any does

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- **Fairness?**
  - No—threads can overtake others

Bounded waiting

Idea: If thread *A* "starts before" *B*, then *A* enters CS before *B*.

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But what is “starts before”?

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r-Bounded Waiting Guarantee: If $D_A^k \rightarrow D_B^j$ then $CS_A^k \rightarrow CS_B^{j+r}$.

"If A enters the doorway for the kth time before B enters it for the j time, then A's kth critical section happens before B's $(j + r)$th critical section."

r-Bounded waiting

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r-Bounded waiting

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- Filter algorithm (for n) has $r = \infty$
- Bakery algorithm (for n) has $r = 0$ (first-come-first-served)

Helper class: lexigraphically-ordered pairs

```
template <typename A, typename B>
struct LexPair {
   A x;
   B y;
};

template <typename A, typename B>
bool operator>(const LexPair<A,B>& p,
               const LexPair<A,B>& q)
{
   return p.x > q.x || (p.x == q.x && p.y > q.y)
}
```

Bakery algorithm for *n* threads

```
template <int N>
class BakeryLock : public GuardedLockBase {
    bool flag_[N]    = {false};
    int label_[N]    = {0};
    int max_label_ = 0;

    bool someone_is_ahead()
    {
        for (auto k : thread::all_ids())
            if (flag_[k] && LexPair{label_[i()], i()} > LexPair{label_[k], k})
                return true;
        return false;
    }
    ⋮
};
```

```
template <int N>
class BakeryLock : public GuardedLockBase {
  ⋮
public:
  virtual void lock() override
  {
    flag_[i()]  = true;
    label_[i()] = ++max_label_;
    while (someone_is_ahead()) {}
  }

  virtual void unlock() override
  {
    flag_[i()] = false;
  }
};
```

Bakery $Y2^{32}K$ bug

Does overflow matter?

Bits	**Does it?**
16	quite
32	maybe
64	no

Bakery lock properties

- Mutual exclusion
  - Between any two (label[k], k) pairs, one will defer to the other…
- Deadlock freedom
  - Must be some least (label[k], k) pair
- Starvation freedom
  - No thread takes the same number twice
- First-come-first served (0-bounded waiting)
  - First through the door has lower label, goes first

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- **Practical?**
  - Have to read *n* variables to lock

“Registers” (shared memory locations)

Flavors:

- Multi-reader/single-writer (flag[])
- Multi-reader/multi-writer (waiting)
- (Not that interesting: SRMW and SRSW)

Theorem

At least *n* MRSW (multi-reader/single-writer) registers are needed to solve deadlock-free mutual exclusion.

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Proof sketch. For $n = 2$, one register is insufficient because neither thread necessarily sees the other's write. Then by induction, the record of the first thread to enter always gets obliterated by the rest.

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For deadlock-free mutual exclusion of n threads:

- Best known algorithm uses $2n$ MRSW registers.
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$O(n)$ reads is too inefficient—we need something better, and we'll get it from the hardware