

n-way Mutual Exclusion

EECS 395 “Rust”

Jan. 21, 2016

Filter algorithm for n threads

```
template <int N>
class FilterLock : public GuardedLockBase {
    int level_[N] = {0};
    int waiting_[N];

    bool exists_competition(int level)
    {
        for (auto k : thread::all_ids())
            if (k != i() && level_[k] >= level)
                return true;
        return false;
    }
    :
};
```

```

template <int N>
class FilterLock : public GuardedLockBase {
    :
public:
    virtual void lock() override
        for (int level = 1; level < N; ++level) {
            level_[i()] = level;
            waiting_[level] = i();
            while (exists_competition(level) && waiting_[level] == i())
                { }
        }
    }

    virtual void unlock() override
        level_[i()] = 0;
    }
};

```

Filter lock properties

- Mutual exclusion
 - ▶ By induction, one thread gets stuck in each level...
- Deadlock freedom
 - ▶ Like Peterson—only one thread can wait per level
- Starvation freedom
 - ▶ Like Peterson—every thread advances if any does

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- **Fairness?**
 - ▶ No—threads can overtake others

Bounded waiting

Idea: If thread A “starts before” B , then A enters CS before B .

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But what is “starts before”?

Bounded waiting

Divide *lock()* operation into two intervals:

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“If A enters the doorway for the k th time before B enters it for the j time, then A ’s k th critical section happens before B ’s $(j + r)$ th critical section.”

r -Bounded waiting

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r -Bounded waiting

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- Filter algorithm (for n) has $r = \infty$
- Bakery algorithm (for n) has $r = 0$ (first-come-first-served)

Helper class: lexicographically-ordered pairs

```
template <typename A, typename B>
struct LexPair {
    A x;
    B y;
};
```

```
template <typename A, typename B>
bool operator>(const LexPair<A,B>& p,
               const LexPair<A,B>& q)
{
    return p.x > q.x || (p.x == q.x && p.y > q.y)
}
```

Bakery algorithm for n threads

```
template <int N>
class BakeryLock : public GuardedLockBase {
    bool flag_[N] = {false};
    int label_[N] = {0};
    int max_label_ = 0;

    bool someone_is_ahead()
    {
        for (auto k : thread::all_ids())
            if (flag_[k] && LexPair{label_[i()], i()} > LexPair{label_[k], k})
                return true;
        return false;
    }
    :
};
```

```
template <int N>
class BakeryLock : public GuardedLockBase {
    :
public:
    virtual void lock() override
    {
        flag_[i()] = true;
        label_[i()] = ++max_label_;
        while (someone_is_ahead()) {}
    }

    virtual void unlock() override
    {
        flag_[i()] = false;
    }
};
```

Bakery Y2³²K bug

Does overflow matter?

Bits	Does it?
16	quite
32	maybe
64	no

Bakery lock properties

- Mutual exclusion
 - ▶ Between any two $(\text{label}[k], k)$ pairs, one will defer to the other...
- Deadlock freedom
 - ▶ Must be some least $(\text{label}[k], k)$ pair
- Starvation freedom
 - ▶ No thread takes the same number twice
- First-come-first served (0-bounded waiting)
 - ▶ First through the door has lower label, goes first

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 - ▶ First through the door has lower label, goes first
- **Practical?**
 - ▶ Have to read n variables to lock

“Registers” (shared memory locations)

Flavors:

- Multi-reader/single-writer (flag[])
- Multi-reader/multi-writer (waiting)
- (Not that interesting: SRMW and SRSW)

Theorem

At least n MRSW (multi-reader/single-writer) registers are needed to solve deadlock-free mutual exclusion.

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Proof sketch. For $n = 2$, one register is insufficient because neither thread necessarily sees the other's write. Then by induction, the record of the first thread to enter always gets obliterated by the rest.

Summary

For deadlock-free mutual exclusion of n threads:

- Best known algorithm uses $2n$ MRSW registers.
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$O(n)$ reads is too inefficient—we need something better, and we'll get it from the hardware

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