Binary trees

```
; a database is
; - false
; - (make-db-node person database database)
(define-struct db-node (person left right))
```

Lookup for binary trees

```
; lookup : number database → person or false
; to find the person with `ssn' in `bt' if possible
(define (lookup ssn bt)
  (cond
    [(boolean? bt) false]
    [else
      (cond
        [(ssn-matches? ssn (db-node-person bt))
         (db-node-person bt)]
        [else
         (pick (lookup ssn
                         (db-node-left bt))
                (lookup ssn
                         (db-node-right bt)))]))
```

Helper function for binary trees

```
; pick : person-or-false person-or-false → person-or-false
; to pick whichever, p1 or p2, that is a person,
; or false if none are
(define (pick p1 p2)
  (cond
    [(and (boolean? p1) (boolean? p2))
     false]
    [(and (person? p1) (boolean? p2))
     p1]
    [(and (boolean? p1) (person? p2))
     p2]
    [(and (person? p1) (person? p2))
     p1]))
; ssn-matches? : ssn person → boolean
(define (ssn-matches? ssn person)
  (= ssn (person-ssn person)))
```

```
(lookup
                                   =>
 5
  (make-bst
    (make-person 4 'tim)
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
      (make-bst
        (make-person 3 'ellen)
        false
        false))
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
```

(lookup	=> (pick
5	(lookup
(make-bst	5
(make-person 4 'tim)	(make-bst
(make-bst	(make-person 2 'jon)
(make-person 2 'jon)	(make-bst
(make-bst	(make-person 1 'anna)
(make-person 1 'anna)	false
false	false)
false)	(make-bst
(make-bst	(make-person 3 'ellen)
(make-person 3 'ellen)	false
false	false)))
false))	(lookup
(make-bst	5
(make-person 6 'jane)	(make-bst
(make-bst	(make-person 6 'jane)
(make-person 5 'george)	(make-bst
false	(make-person 5 'george)
false)	false
(make-bst	false)
<pre>(make-person 7 'emily)</pre>	(make-bst
false	<pre>(make-person 7 'emily)</pre>
false))))	false
	false))))

```
<= (pick
     (lookup
        (make-bst
          (make-person 2 'jon)
          (make-bst
            (make-person 1 'anna)
           false
           false)
          (make-bst
            (make-person 3 'ellen)
           false
           false)))
     (lookup
       5
        (make-bst
          (make-person 6 'jane)
          (make-bst
            (make-person 5 'george)
           false
           false)
          (make-bst
            (make-person 7 'emily)
           false
           false))))
```

pick	<=	(pick	
(pick		(lookup	
(lookup		5	
5		(make-bst	
(make-bst		(make-person 2	'jon)
(make-person 1 'anna)		(make-bst	
false		(make-person	1 'anna)
false))		false	
(lookup		false)	
5		(make-bst	
(make-bst		(make-person	3 'ellen)
(make-person 3 'ellen)		false	
false		false)))	
false)))		(lookup	
(lookup		5	
5		(make-bst	
(make-bst		(make-person 6	'jane)
(make-person 6 'jane)		(make-bst	
(make-bst		(make-person	5 'george)
(make-person 5 'george)		false	
false		false)	
false)		(make-bst	
(make-bst		(make-person	7 'emily)
<pre>(make-person 7 'emily)</pre>		false	
false		false))))	
false))))			

```
(pick
                                   =>
 (pick
  (lookup
    5
    (make-bst
      (make-person 1 'anna)
      false
      false))
  (lookup
    5
    (make-bst
      (make-person 3 'ellen)
      false
      false)))
 (lookup
  5
   (make-bst
     (make-person 6 'jane)
     (make-bst
       (make-person 5 'george)
       false
       false)
     (make-bst
       (make-person 7 'emily)
       false
       false))))
```

pick	=> (pick
(pick	(pick
(lookup	(pick
5	(lookup 5 false)
(make-bst	(lookup 5 false))
(make-person 1 'anna)	(lookup
false	5
false))	(make-bst
(lookup	(make-person 3 'ellen)
5	false
(make-bst	false)))
<pre>(make-person 3 'ellen)</pre>	(lookup
false	5
false)))	(make-bst
(lookup	(make-person 6 'jane)
5	(make-bst
(make-bst	(make-person 5 'george
(make-person 6 'jane)	false
(make-bst	false)
<pre>(make-person 5 'george)</pre>	(make-bst
false	<pre>(make-person 7 'emily)</pre>
false)	false
(make-bst	false))))
<pre>(make-person 7 'emily)</pre>	
false	
false))))	

```
<= (pick
     (pick
      (pick
       (lookup 5 false)
        (lookup 5 false))
      (lookup
        5
         (make-bst
           (make-person 3 'ellen)
          false
          false)))
     (lookup
       5
        (make-bst
          (make-person 6 'jane)
          (make-bst
            (make-person 5 'george)
           false
            false)
          (make-bst
            (make-person 7 'emily)
           false
            false))))
```

```
<= (pick
(pick
 (pick
                                         (pick
  (pick false (lookup 5 false))
                                          (pick
  (lookup
                                           (lookup 5 false)
    5
                                           (lookup 5 false))
    (make-bst
                                          (lookup
      (make-person 3 'ellen)
                                            5
      false
                                            (make-bst
                                              (make-person 3 'ellen)
      false)))
                                              false
 (lookup
  5
                                              false)))
   (make-bst
                                         (lookup
     (make-person 6 'jane)
     (make-bst
                                           (make-bst
       (make-person 5 'george)
                                             (make-person 6 'jane)
       false
                                             (make-bst
       false)
                                               (make-person 5 'george)
     (make-bst
                                               false
       (make-person 7 'emily)
                                               false)
       false
                                             (make-bst
       false))))
                                               (make-person 7 'emily)
                                               false
                                               false))))
```

```
(pick
                                   =>
 (pick
  (pick false (lookup 5 false))
  (lookup
    5
    (make-bst
      (make-person 3 'ellen)
      false
      false)))
 (lookup
  5
   (make-bst
     (make-person 6 'jane)
     (make-bst
       (make-person 5 'george)
       false
       false)
     (make-bst
       (make-person 7 'emily)
       false
       false))))
```

```
(pick
                                   => (pick
 (pick
                                         (pick
  (pick false (lookup 5 false))
                                          (pick false false)
  (lookup
                                          (lookup
    5
                                            5
    (make-bst
                                            (make-bst
      (make-person 3 'ellen)
                                              (make-person 3 'ellen)
      false
                                              false
      false)))
                                              false)))
 (lookup
                                         (lookup
                                           5
  5
   (make-bst
                                           (make-bst
     (make-person 6 'jane)
                                             (make-person 6 'jane)
     (make-bst
                                             (make-bst
                                               (make-person 5 'george)
       (make-person 5 'george)
       false
                                               false
       false)
                                               false)
     (make-bst
                                             (make-bst
                                               (make-person 7 'emily)
       (make-person 7 'emily)
       false
                                               false
       false))))
                                               false))))
```

```
<= (pick
     (pick
      (pick false false)
      (lookup
        5
         (make-bst
           (make-person 3 'ellen)
          false
          false)))
     (lookup
       5
        (make-bst
          (make-person 6 'jane)
          (make-bst
            (make-person 5 'george)
           false
           false)
          (make-bst
            (make-person 7 'emily)
           false
           false))))
```

```
<= (pick
(pick
 (pick
                                         (pick
 false
                                          (pick false false)
  (lookup
                                          (lookup
                                            5
    5
    (make-bst
                                            (make-bst
      (make-person 3 'ellen)
                                               (make-person 3 'ellen)
      false
                                              false
      false)))
                                              false)))
 (lookup
                                         (lookup
                                           5
  5
   (make-bst
                                           (make-bst
     (make-person 6 'jane)
                                             (make-person 6 'jane)
     (make-bst
                                              (make-bst
       (make-person 5 'george)
                                                (make-person 5 'george)
       false
                                               false
       false)
                                               false)
     (make-bst
                                             (make-bst
       (make-person 7 'emily)
                                                (make-person 7 'emily)
       false
                                               false
       false))))
                                               false))))
```

```
(pick
                                   =>
 (pick
 false
  (lookup
    5
    (make-bst
      (make-person 3 'ellen)
      false
      false)))
 (lookup
  5
   (make-bst
     (make-person 6 'jane)
     (make-bst
       (make-person 5 'george)
       false
       false)
     (make-bst
       (make-person 7 'emily)
       false
       false))))
```

```
(pick
                                   => (pick
 (pick
                                         (pick
 false
                                          false
  (lookup
                                          (pick
    5
                                           (lookup 5 false)
    (make-bst
                                           (lookup 5 false)))
      (make-person 3 'ellen)
                                         (lookup
      false
                                           5
      false)))
                                           (make-bst
                                              (make-person 6 'jane)
 (lookup
  5
                                             (make-bst
   (make-bst
                                               (make-person 5 'george)
     (make-person 6 'jane)
                                               false
     (make-bst
                                               false)
       (make-person 5 'george)
                                             (make-bst
       false
                                                (make-person 7 'emily)
                                               false
       false)
     (make-bst
                                               false))))
       (make-person 7 'emily)
       false
       false))))
```

```
<= (pick
     (pick
      false
      (pick
       (lookup 5 false)
       (lookup 5 false)))
     (lookup
       5
        (make-bst
          (make-person 6 'jane)
          (make-bst
            (make-person 5 'george)
           false
           false)
          (make-bst
            (make-person 7 'emily)
           false
           false))))
```

```
<= (pick
(pick
                                         (pick
 (pick
 false
                                          false
  (pick false (lookup 5 false)))
                                          (pick
                                           (lookup 5 false)
 (lookup
  5
                                           (lookup 5 false)))
   (make-bst
                                         (lookup
     (make-person 6 'jane)
                                           5
     (make-bst
                                           (make-bst
       (make-person 5 'george)
                                             (make-person 6 'jane)
       false
                                             (make-bst
       false)
                                               (make-person 5 'george)
     (make-bst
                                               false
       (make-person 7 'emily)
                                               false)
       false
                                             (make-bst
       false))))
                                               (make-person 7 'emily)
                                               false
                                               false))))
```

```
(pick
                                   =>
 (pick
 false
  (pick false (lookup 5 false)))
 (lookup
  5
   (make-bst
     (make-person 6 'jane)
     (make-bst
       (make-person 5 'george)
       false
       false)
     (make-bst
       (make-person 7 'emily)
       false
       false))))
```

```
(pick
                                   => (pick
 (pick
                                         (pick false (pick false false))
 false
                                         (lookup
                                          5
  (pick false (lookup 5 false)))
 (lookup
                                           (make-bst
  5
                                             (make-person 6 'jane)
   (make-bst
                                             (make-bst
     (make-person 6 'jane)
                                               (make-person 5 'george)
     (make-bst
                                               false
       (make-person 5 'george)
                                               false)
       false
                                             (make-bst
       false)
                                               (make-person 7 'emily)
     (make-bst
                                               false
       (make-person 7 'emily)
                                               false))))
       false
       false))))
```

```
<= (pick
(pick
 (pick false false)
                                         (pick false (pick false false))
 (lookup
                                         (lookup
  5
                                          5
   (make-bst
                                           (make-bst
     (make-person 6 'jane)
                                             (make-person 6 'jane)
     (make-bst
                                             (make-bst
                                               (make-person 5 'george)
       (make-person 5 'george)
       false
                                               false
       false)
                                               false)
     (make-bst
                                             (make-bst
       (make-person 7 'emily)
                                               (make-person 7 'emily)
       false
                                               false
       false))))
                                               false))))
```

```
(pick
  (pick false false)
  (lookup
    5
    (make-bst
          (make-person 6 'jane)
          (make-bst
           (make-person 5 'george)
           false
           false)
           (make-bst
                (make-person 7 'emily)
                false
                 false
                 false))))
```

```
=> (pick
(pick
                                        false
 (pick false false)
 (lookup
                                         (lookup
  5
                                           5
   (make-bst
                                           (make-bst
     (make-person 6 'jane)
                                             (make-person 6 'jane)
     (make-bst
                                             (make-bst
       (make-person 5 'george)
                                               (make-person 5 'george)
       false
                                               false
       false)
                                               false)
     (make-bst
                                             (make-bst
       (make-person 7 'emily)
                                               (make-person 7 'emily)
       false
                                               false
       false))))
                                               false))))
```

```
<= (pick
(pick
false
                                        false
 (pick
                                         (lookup
                                           5
  (lookup
    5
                                           (make-bst
    (make-bst
                                             (make-person 6 'jane)
      (make-person 5 'george)
                                             (make-bst
      false
                                               (make-person 5 'george)
      false))
                                               false
                                               false)
  (lookup
                                             (make-bst
    5
                                               (make-person 7 'emily)
    (make-bst
      (make-person 7 'emily)
                                               false
      false
                                               false))))
      false))))
```

```
(pick
                                   =>
false
 (pick
  (lookup
   5
    (make-bst
      (make-person 5 'george)
      false
      false))
  (lookup
   5
    (make-bst
      (make-person 7 'emily)
      false
      false))))
```

```
=> (pick
(pick
false
                                        false
                                         (pick
 (pick
  (lookup
                                          (make-person 5 'george)
    5
                                          (lookup
                                            5
    (make-bst
      (make-person 5 'george)
                                            (make-bst
                                              (make-person 7 'emily)
      false
      false))
                                              false
  (lookup
                                              false))))
    5
    (make-bst
      (make-person 7 'emily)
      false
      false))))
```

```
(pick
  false
  (pick
    (make-person 5 'george)
    (pick
        (lookup 5 false)
        (lookup 5 false))))
```

```
(pick
  false
  (pick
    (make-person 5 'george)
    (pick
       (lookup 5 false)
       (lookup 5 false))))
```

```
(pick
  false
  (pick
     (make-person 5 'george)
     (pick false false)))
```

```
(pick
    false
    (make-person 5 'george))
```

(that took forever!)

Binary search trees

```
; a database is
; - false
; - (make-db-node person database[left] database[right])
; where all people in left have smaller ssns than person, and
; all people in right have larger ssns than person.
(define-struct db-node (person left right))
```

Lookup for binary search trees

```
; lookup : number database → person or false
; to find the person with `ssn' in `bt' if possible
(define (lookup ssn bt)
  (cond
    [(boolean? bt) false]
    [else
     (cond
        [(ssn-matches? ssn (db-node-person bt))
         (db-node-person bt)]
        [(ssn-less? ssn (db-node-person bt))
         (lookup ssn (db-node-left bt))]
        [(ssn-more? ssn (db-node-person bt))
         (lookup ssn (db-node-right bt))]))
```

Helper functions for binary search trees

```
; ssn-less? : ssn person → boolean
  (define (ssn-less? ssn person)
        (< ssn (person-ssn person)))
; ssn-more? : ssn person → boolean
  (define (ssn-more? ssn person)
        (> ssn (person-ssn person)))
```

```
(lookup
                                  =>
  (make-bst
    (make-person 4 'tim)
    (make-bst
      (make-person 2 'jon)
      (make-bst
        (make-person 1 'anna)
        false
        false)
      (make-bst
        (make-person 3 'ellen)
        false
        false))
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
```

```
(lookup
                                  => (lookup
  (make-bst
                                         (make-bst
    (make-person 4 'tim)
                                           (make-person 6 'jane)
    (make-bst
                                           (make-bst
      (make-person 2 'jon)
                                             (make-person 5 'george)
      (make-bst
                                             false
        (make-person 1 'anna)
                                             false)
        false
                                           (make-bst
        false)
                                             (make-person 7 'emily)
      (make-bst
                                             false
        (make-person 3 'ellen)
                                             false)))
        false
        false))
    (make-bst
      (make-person 6 'jane)
      (make-bst
        (make-person 5 'george)
        false
        false)
      (make-bst
        (make-person 7 'emily)
        false
        false))))
```

```
<= (lookup
(lookup
  (make-bst
                                        (make-bst
    (make-person 5 'george)
                                          (make-person 6 'jane)
                                          (make-bst
   false
   false))
                                            (make-person 5 'george)
                                            false
                                            false)
                                          (make-bst
                                            (make-person 7 'emily)
                                            false
                                            false)))
```

```
(lookup
5
  (make-bst
        (make-person 5 'george)
        false
        false))
```

(much quicker that time, no?)