

# Higher-Order Functions (Part II)

# FAE with Deferred Substitution

`(interp {with {y 10} {fun {x} {+ y x}}})`

⇒

`(interp {fun {x} {+ y x}})`

`(interp {{fun {y} {fun {x} {+ y x}}} 10})`

⇒

`(interp {fun {x} {+ y x}})`

# FAE with Deferred Substitution

```
(interp {{with {y 10} {fun {x} {+ y x}}}}  
        {with {y 7} y}})
```

Argument expression:

```
(interp {with {y 7} y})
```

⇒

```
(interp y = 7  
        y) ⇒ 7
```

Function expression:

```
(interp {{with {y 10} {fun {x} {+ y x}}}})
```

⇒

```
(interp y = 10  
        {fun {x} {+ y x}}) ⇒ ?
```

# F AE Values

A function value needs to keep its deferred substitution

```
(define-type FAE-Value
  [numV (n number?)]
  [closureV (param-name symbol?)
            (body FAE?)
            (ds DefSub?)])
```

```
(define-type DefSub
  [mtSub]
  [aSub (name symbol?)
        (value FAE-Value?)
        (rest DefSub?)])
```

```
(test (interp {with {y 10} {fun {x} {+ y x}}})
      (closureV 'x {+ y x}
                (aSub 'y (num 10) (mtSub))))
```

# Continuing Evaluation

Function: `{fun {x} {+ y x}}` y = 10

Argument: 7

To apply, interpret the function body with the given argument:

`(interp {+ y x})` x = 7                      y = 10

# FAE Interpreter with Deferred Substitution

```
; interp : FAE? DefSub? -> FAE-Value?
(define (interp a-fae ds)
  (type-case FAE a-fae
    [num (n) (numV n)]
    [add (l r) (num+ (interp l ds) (interp r ds))]
    [sub (l r) (num- (interp l ds) (interp r ds))]
    [id (name) (lookup name ds)]
    [fun (param-name body)
         (closureV param-name body ds)]
    [app (fun-expr arg-expr)
         (local [(define fun-val
                   (interp fun-expr ds))
                  (define arg-val
                   (interp arg-expr ds))]
                 (interp (closureV-body fun-val)
                         (aSub (closureV-param-name fun-val)
                               arg-val
                               (closureV-ds fun-val)))))]))
```