Transformations II

Lecture Set 6

CS5600 ***Computer Graphics***
by
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What About Elementary Inverses?

- Scale
- Shear
- Rotation
- Translation

Scale Inverse

$$\begin{bmatrix} l & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/l & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} l & 0 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1/l & 0 \\ 0 & 1 \end{bmatrix}$$

Shear Inverse

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -b & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Shear Inverse

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -a \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ b & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ -b & 1 \end{bmatrix}$$

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Rotation Inverse

$$\begin{bmatrix} \cos q & -\sin q \\ \sin q & \cos q \end{bmatrix} \begin{bmatrix} \cos(-q) & -\sin(-q) \\ \sin(-q) & \cos(-q) \end{bmatrix}$$

$$= \begin{bmatrix} \cos q & -\sin q \\ \sin q & \cos q \end{bmatrix} \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix}$$

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Rotation Inverse

$$\begin{bmatrix} \cos q & -\sin q \\ \sin q & \cos q \end{bmatrix} \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix} =$$

$$= \begin{bmatrix} (\cos^2 q + \sin^2 q) & (\cos q \sin q - \cos q \sin q) \\ (\cos q \sin q - \cos q \sin q) & (\sin^2 q + \cos^2 q) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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Rotation Inverse

$$\begin{bmatrix} \cos q & -\sin q \\ \sin q & \cos q \end{bmatrix}^{-1} = \begin{bmatrix} \cos q & \sin q \\ -\sin q & \cos q \end{bmatrix}$$

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Translation Inverse

$$\begin{bmatrix}1 & 0 & d_x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -d_x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=$$

$$\begin{bmatrix}1 & 0 & (-d_x+d_x)\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

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Translation Inverse

$$\begin{bmatrix}1 & 0 & d_x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}^{-1}=\begin{bmatrix}1 & 0 & -d_x\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

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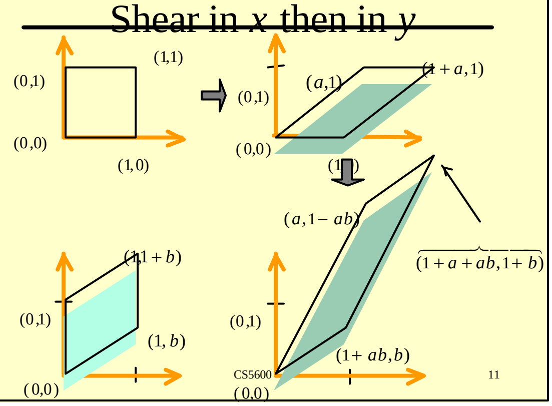
Shear in *x* then in *y*



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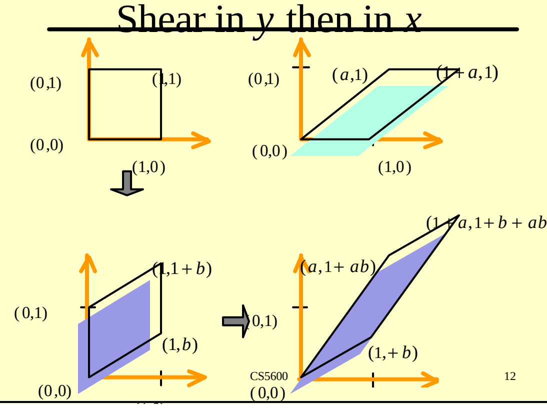
Shear in *y* then in *x*



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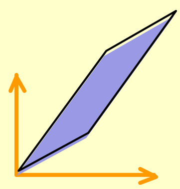
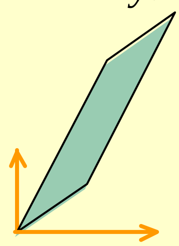
Results Are Different

y then *x:*



x then *y:*

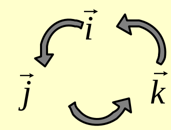


Want the RHR to Work

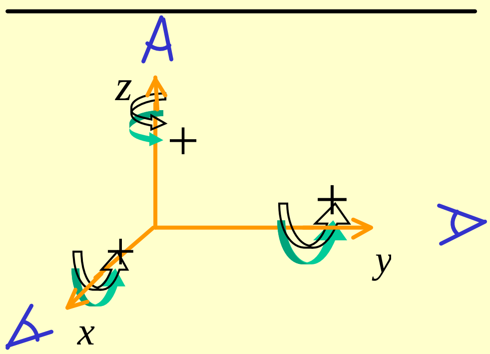
$\vec{i} \times \vec{j} = \vec{k}$

$\vec{j} \times \vec{k} = \vec{i}$

$\vec{k} \times \vec{i} = \vec{j}$



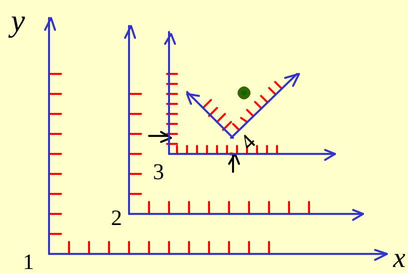
3D Positive Rotations



Transformations as a Change in Coordinate System

- Useful in many situations
- Use most natural coordination system locally
- Tie things together in a global system

Example



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Example

$M_{i \leftarrow j}$ is the transformation that takes a point $p^{(j)}$ in coordinate system j and converts it to a point $p^{(i)}$ in coordinate system i

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Example

- $p^{(i)} = M_{i \leftarrow j} \; p^{(j)}$
- $p^{(j)} = M_{j \leftarrow k} \; p^{(k)}$
- $M_{i \leftarrow k} = M_{i \leftarrow j} M_{j \leftarrow k}$

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Example

- $M_{2 \leftarrow 1} = T(4, 2)$
- $M_{2 \leftarrow 3} = S(2,2) \bullet T(2,3)$
- $M_{3 \leftarrow 4} = R(-45^{\circ}) \bullet T(6.7, 1.8)$

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Recall the Following

$$(AB)^{-1} = B^{-1} A^{-1}$$

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Since $M_{i \leftarrow j}^{-1} = M_{j \leftarrow i}$

- $M_{1 \leftarrow 2} = T(-4, -2)$
- $M_{3 \leftarrow 2} = T(-2, -3) \bullet S(\frac{1}{2}, \frac{1}{2})$
- $M_{4 \leftarrow 3} = T(-6.7, -1.8) \bullet R(+45^{\circ})$

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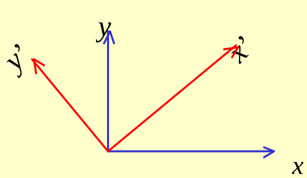
Change of Coordinate System

- Describe the *old* coordinate system in terms of the *new* one.



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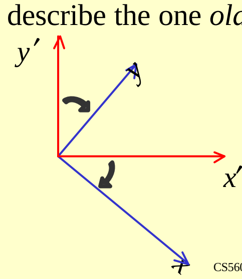
Change of Coordinate System

Move to the *new* coordinate system and describe the one *old*.



Old is a negative rotation of the *new*.

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What is "Perspective?"

- A mechanism for portraying 3D in 2D
- "True Perspective" corresponds to projection onto a plane
- "True Perspective" corresponds to an ideal camera image

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Many Kinds of Perspective Used

- Mechanical Engineering
- Cartography
- Art

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Perspective in Art

- Naïve (wrong)
- Egyptian
- Cubist (unrealistic)
- Esher
- Miro
- Matisse

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Egyptian Frontalism

- Head profile
- Body front
- Eyes full
- Rigid style

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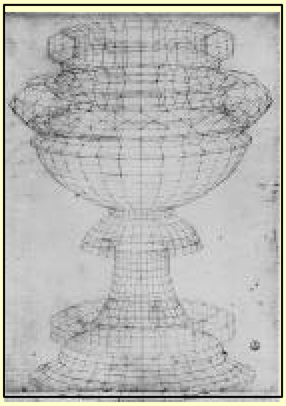

Uccello's (1392-1475) hand drawing was the first extant complex geometrical form rendered according to the laws of linear perspective

Perspective Study of a Chalice, Drawing, Gabinetto dei Disegni, Uffizi, Florence, ca 1430)

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Perspective in Cubism

Braque, Georges

Woman with a Guitar

Sorgues, autumn 1913

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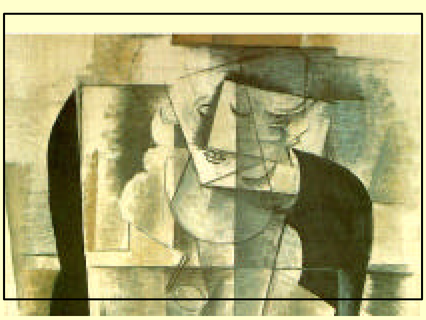
Perspective in Cubism



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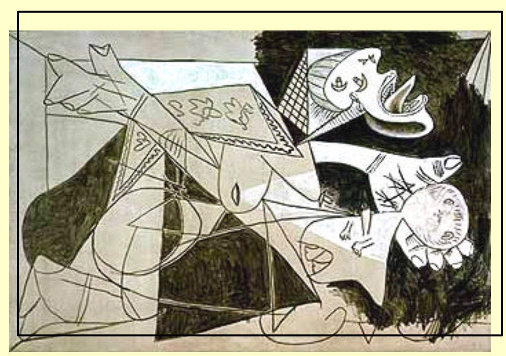

Pablo Picaso, Madre con niño muerto (1937)

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Pablo Picaso
Cabeza de mujer llorando con pañuelo

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Perspective (Mural) Games

M C Esher, *Another World II* (1947)

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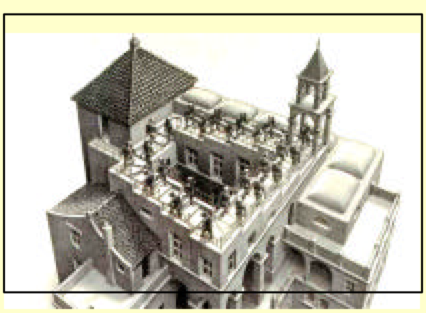
Perspective



M.C. Escher, Ascending and Descending (1960)

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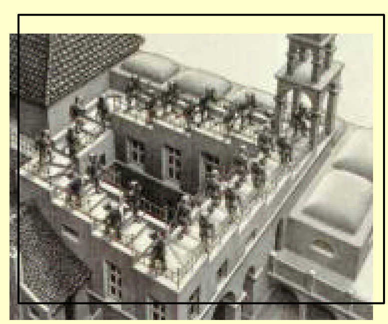
M. C. Esher



M.C. Escher, Ascending and Descending (1960)

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M. C. Esher

- Perspective is "local"
- Perspective consistency is not "transitive"
- Nonplanar (hyperbolic) projection

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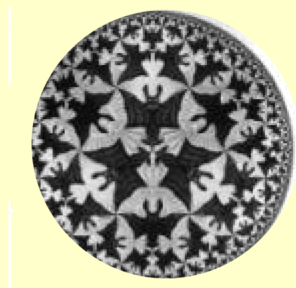
Nonplanar (Hyperbolic) Projection



M C Esher,
Heaven and Hell

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Nonplanar (Hyperbolic) Projection

M C Esher,
Heaven and
Hell

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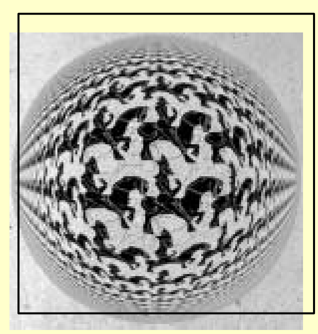
David McAllister



The March of Progress, (1995)

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Joan Miro: Flat Perspective

The Tilled Field

What cues are missing?

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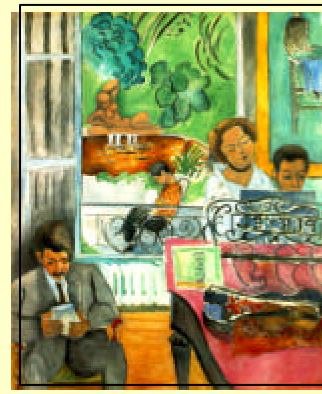
Flat Perspective: What cues are missing?



Henri Matisse, *La Lecon de Musique*

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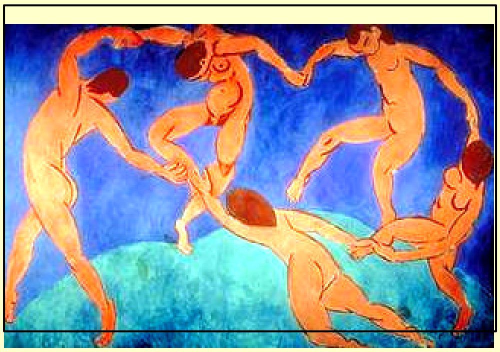

Henri Matisse, Danse II (1910)

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Atlas Projection

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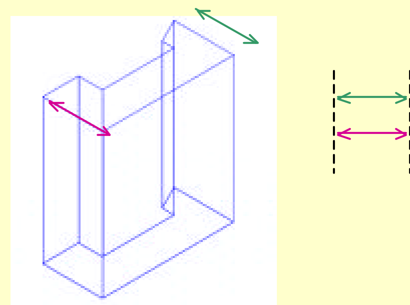
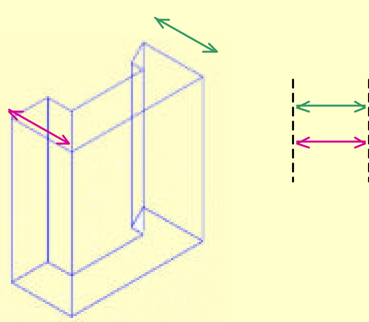
Norway is at High Latitude

There is considerable size distortion

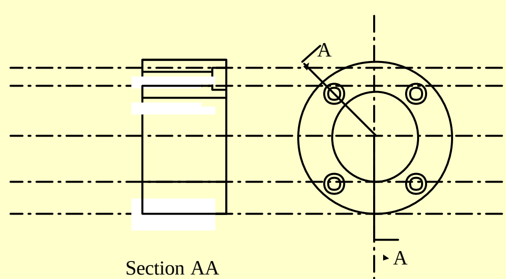
Isometric View



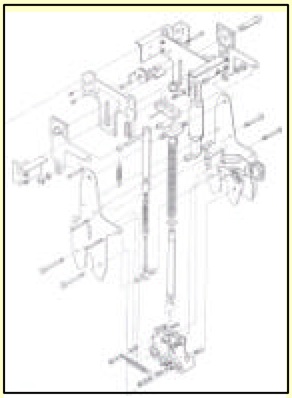
Isometric View



Engineering Drawing

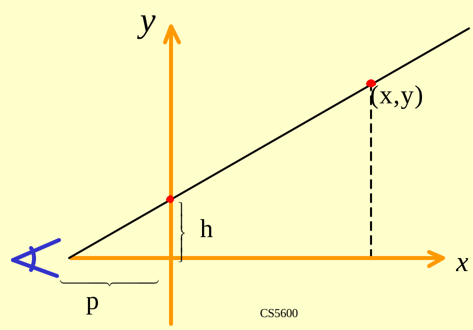


Engineering Drawing: Exploded View



Understanding 3D Assembly in a 2D Medium

“True” Perspective in 2D



“True” Perspective in 2D

$$\frac{h}{p} = \frac{y}{x+p}$$

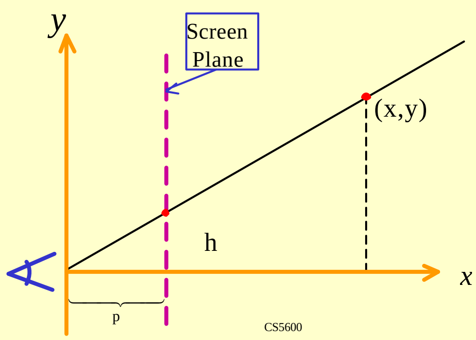
$$h = \frac{py}{x+p}$$

“True” Perspective in 2D

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/p & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ x/p + 1 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \\ \frac{x+p}{p} \end{bmatrix} = \begin{bmatrix} \frac{px}{x+p} \\ \frac{py}{x+p} \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} \frac{px}{x+p} \\ \frac{py}{x+p} \end{bmatrix}$$

Geometry is Same for Eye at Origin



What Happens to Special Points?

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1/p & 0 & 1\end{bmatrix}\begin{bmatrix}-p\\ 0\\ 1\end{bmatrix} = \begin{bmatrix}-p\\ 0\\ 0\end{bmatrix}$$

What is this point??

Let's Look at a Limit

Observe,

$$\lim_{n\to\infty}\begin{bmatrix}1\\ 0\\ \left(\frac{1}{n}\right)\end{bmatrix} = \begin{bmatrix}n\\ 0\\ 1\end{bmatrix} \Leftrightarrow \begin{bmatrix}n\\ 0\end{bmatrix}$$

We see that $\begin{bmatrix}n\\ 0\end{bmatrix} \Leftrightarrow +\infty$ on x-axis

Where does Eye Point Go?

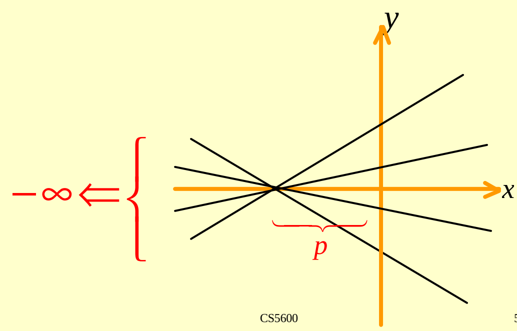
- It gets sent to $-\infty$ on x-axis
- Where does $+\infty$ on x-axis go?

What happens to $+\infty$?

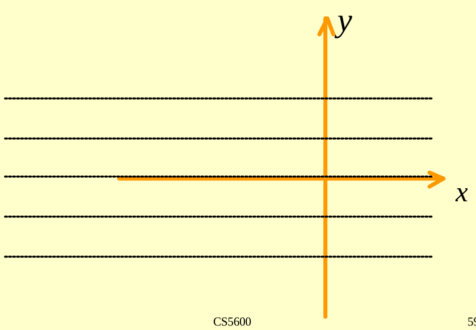
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{p} & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \frac{1}{p} \end{bmatrix} = \begin{bmatrix} p \\ 0 \\ 1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} p \\ 0 \end{bmatrix}$$

It comes back to virtual eye point!

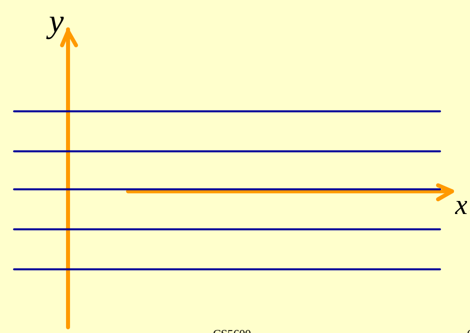
What Does This Mean?



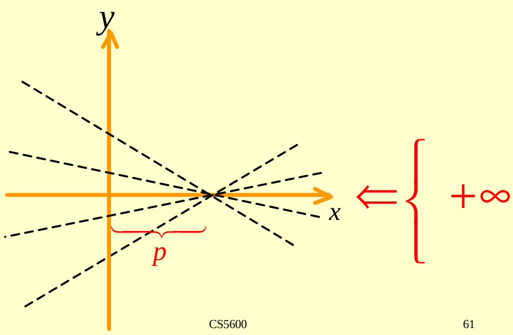
The "Pencil of Lines" Becomes Parallel



Parallel Lines Become a "Pencil of Lines" !



What Does This Mean?



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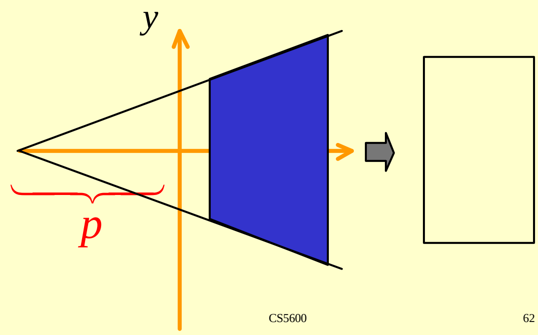
“True” Perspective in 2D



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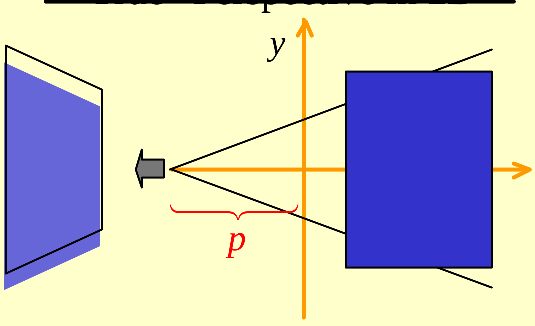
“True” Perspective in 2D



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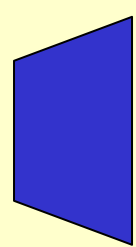
Viewing *Frustum*



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What happens for large p?”

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \frac{1}{p} & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\lim_{p \to \infty} \frac{1}{p} = 0$$

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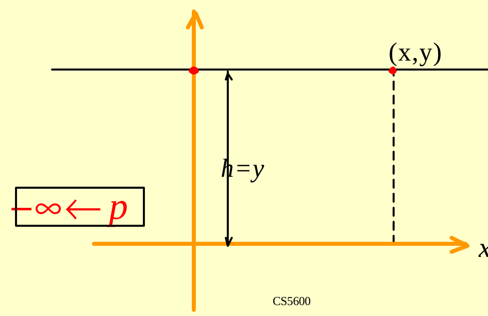
Projection Becomes Orthogonal: “Right Thing Happens”



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The End
of
Transformations II

Lecture Set 6

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