DUE Friday, May 17 at 11:59PM. Late assignments penalized 10% per day.

Propositional Logic

1. **(6 x 1/2 points)** Prove or find a counterexample for all statements below (1/2pt each)
   a. $P \wedge \neg P$ is valid
   b. $(A \rightarrow B) \vDash B$
   c. $if\ (A \vDash B)\ and\ (B \vDash C)\ then\ ((A\ or\ B) \vDash C)$
   d. $A \vee \neg(B \wedge \neg C) \vDash (A \vee C)$
   e. $\neg(A \wedge B) \wedge (A \rightarrow B)$ is satisfiable
   f. There are the same number of models for the statement $(A \wedge (B \vee C))$ as there are for the statement $\big((A \rightarrow \neg B) \wedge A\big)$

First-order Logic

2. **(2 x 1/2 points)** Translate the following first order logic sentences to English. Predicates and functions carry the obvious meanings.
   a. mascot(Northwestern, Willie Wildcat)
   b. $\forall x \forall y \forall z\ (odd(sum(x,y)) \wedge odd(z)) \rightarrow \neg odd(sum(\ sum(x,y),z))$

3. **(2 x 1/2 points)** Translate the following English sentences to first order logic. Name predicates and functions such that their meanings are obvious.
   a. Greg and John are students, and Greg plays tennis.
   b. For any two integers x and y, the product of x and y is an integer.

4. **(6 points total)** Security personnel are working to keep us safe by tracking the whereabouts of professors, and deducing advisor-advisee relationships. They enlist first-order-logic AI technology in order to better perform their job. The KB consists of the following statements:

   a. Each professor advises at least one student
   b. Each student has an advisor, who is a professor
   c. Each advisor meets with all of his or her advisees
   d. Advising meetings take place on campus
   e. Juan is a student
   f. Sarah is a professor

   (i) **(2 points)** Translate each of the eight English sentences above and below (a-h) into first-order logic. The relations should be Meet(X, Y), OnCampus(X), Advises(X, Y), Professor(X), and Student(X). (Here X and Y refer to people, and there are no functions in the KB.) Feel free to abbreviate the relations as M, O, A, P, and S. (1/4 of a point per translation)

(ii) **(3 points)** Using the above knowledge base only, i.e. not assuming any other real world facts, prove **by resolution** the following security bombshell:

g. Sarah has been on campus

You should include a drawing showing how the resolution proof proceeds, similar to the resolution proofs constructed in class and in the lecture notes posted online.

HINTS: When you are done converting your KB to CNF, you should have a total of 9 clauses. Two of these will be "ground" clauses, involving no variables. You will also have two Skolem functions, and each Skolem function will take a single variable. You should have a total of six free (universally quantified) variables. NOTE: some free variables may appear in multiple clauses. When you substitute for a variable v when unifying clauses, you must remember to repeat the same substitution later whenever you use another clause containing v.

Your resolution proof has led to increased security. Good work.

(iii) **(1 point)** Now consider the following statement **h** which *cannot* be proved using resolution. Your resolution proof from part (ii) contains a derived clause that is very similar to the FOL statement **h** (both involve Sarah and the Advises predicate). However, the derived clause cannot be unified with **h** (if it could, we would be able to prove **h** from the KB). What is the derived clause, and why can't it be unified with **h**?

h. Sarah advises Juan.