First-Order Logic

Doug Downey
Northwestern EECS 348 – Intro to AI
Based on slides by Stuart Russell

Pros and Cons of Propositional Logic

- ☺ Declarative: pieces of syntax correspond to facts
- ☺ Allows partial/disjunctive/negated info
  - Unlike most data structures and DBs
- ☺ Compositional:
  - Meaning of $A \vee B$ derived from meanings of A, B
- ☺ Meaning is context-independent
- ☹ Very limited expressiveness
  - Can’t say “pits cause breezes in adjacent squares” except by writing a sentence for each square

First-order Logic

- Objects
  - Greg, Sheridan Road, 25, blue, Moby Dick (the novel), …
- Relations
  - Tall, four-lane, perfect square, has color, has read, …
- Functions
  - Height, father of, square root, …

FOL: Syntax

Constants	$KingJohn,\ 2,\ UCB, \ldots$
Predicates	$Brother,\ >, \ldots$
Functions	$Sqrt,\ LeftLegOf, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Connectives	$\wedge \quad \vee \quad \neg \quad \Rightarrow \quad \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall\ \exists$

Atomic Sentences

Atomic sentence $=$ $predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term $=$ $function(term_1, \ldots, term_n)$
or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

Complex Sentences

- Formed from simple sentences using connectives:

$$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$$

- E.g.,
  - MayorOf(Daley, Chicago) => LivesIn(Daley, Chicago)
  - Eat(Doug, Nachos) $\vee$ Eat(Doug, Grapes)

Truth in FOL

- A *model* in FOL is a set of objects, and full definitions for relations and functions
- Model enumeration totally infeasible
  – For each relation (of arity *k*)
    - For each distinct subset of all combinations of *k* objects…
- Technically, objects/relations in model are mapped to particular KB symbols through an *interpretation*
  – This provides yet more complexity

Universal Quantification

- ∀ <variables> <sentence>
- Everyone at Northwestern is smart:

  $\forall x$ At(x, Northwestern) => Smart(x)
- $\forall x\ P$ is true in a model *m* if *P* is true with **any** object from *m* substituted for *x* in *P*

Common Mistake to Avoid

- Typically => is the main connective with $\forall$

- Don't use $\wedge$ by mistake, e.g.:

  $\forall x$ At(x, Northwestern) $\wedge$ Smart(x)

Existential Quantification

- $\exists$ <variables> <sentence>

- Someone at Northwestern is rich:
  $\exists x$ At(x, Northwestern) $\wedge$ Rich(x)

- $\exists x\ P$ is true in a model m if P is true with **some** object from m substituted for x in P

Common Mistake to Avoid

- Typically $\wedge$ is the main connective with $\exists$

- Don't use => by mistake, e.g.:

  $\exists x$ At(x, Northwestern) => Rich(x)

- True if anyone is not at Northwestern!

Properties of Quantifiers

- $\forall x\ \forall y$ is the same as $\forall y\ \forall x$
- $\exists x\ \exists y$ is the same as $\exists y\ \exists x$
- $\exists x\ \forall y$ is **not** the same as $\forall y\ \exists x$

- Related through negation:
  - $\forall x$ Likes(x, IceCream) $\Leftrightarrow \neg\exists x \neg$ Likes(x, IceCream)
  - $\exists x$ Likes(x, Broccoli) $\Leftrightarrow \neg \forall x \neg$ Likes(x, Broccoli)

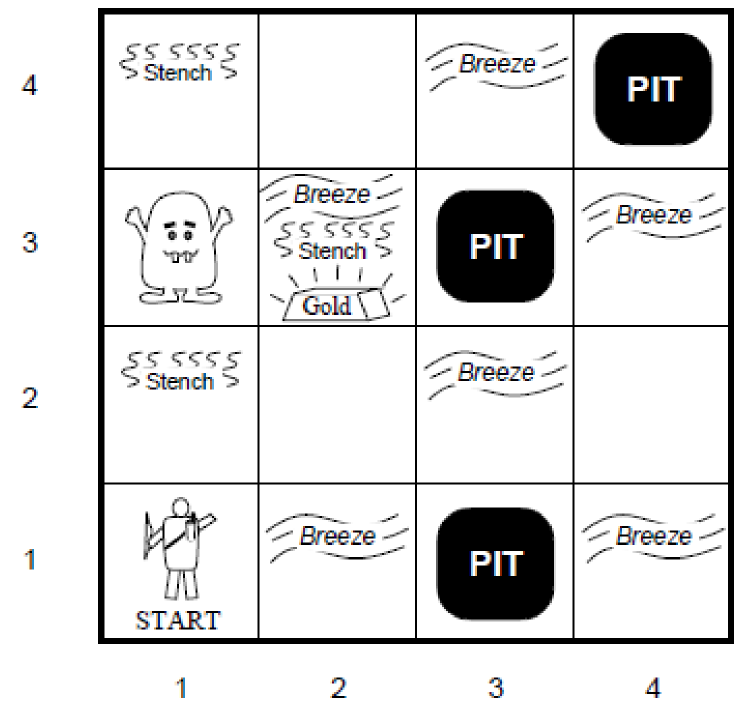
Examples!

- Brothers are siblings
  - $\forall x\ \forall y$ Brother(x , y) => Sibling(x , y)
- Sibling is symmetric:
  - $\forall x\ \forall y$ Sibling(x , y) => Sibling(y , x)
- One's mother is one's female parent
  - $\forall x\ \forall y$ Mother(x, y) <=> (Female(x) $\wedge$ Parent(x, y))
- A first cousin is a child of a parent's sibling…

Payoff in the Wumpus World

- FOL is concise
  - Can define Adjacent(x, y) for squares x, y
  - Then:

$$\forall y \text{ Breezy}(y) \Leftrightarrow [\exists x \text{ Pit}(x) \wedge \text{Adjacent}(x, y)]$$



Interlude: Recap and RoadMap

- So far – classical AI
  - Overview and Philosophy of AI (e.g., Turing Test)
  - General search methods and techniques
    - A*, Local Search, CSPs, etc.
  - Logic
    - Logical Agents, Propositional Logic, FOL (inference: today)
- To Come – "modern AI"
  - Challenges for classical AI
  - Leveraging data (machine learning)
  - The "big questions" (Larry Birnbaum guest lecture June 2)

Interacting with FOL KBs

- Say a wumpus-world agent perceives a smell and a breeze, but no glitter, at *t* = 5:

$$Tell(KB, Percept([Smell, Breeze, None], 5))$$
$$Ask(KB, \exists a \;\; Action(a, 5))$$

- I.e., does the KB entail any actions at *t* = 5?
  - Answer: Yes, {*a* / Shoot} <= **substitution**
- Ask(KB, S) returns the substitutions of S that KB entails

Inference in first-order knowledge bases

- We just said:
  - Ask(KB, S) returns the substitutions of S that KB entails
- How?
  - Propositionalize
    - "Instantiation"
  - Forward/Backward Chaining
  - *Resolution*

Key Idea: **Unification**

Resolution use in practice

- Theorem provers
  - Provided major mathematical results
    - “Otter” proved a conjecture (the Robbins algebra) which had been unsolved for 60 years
  - Verification
    - Hardware (adders, CPUs)
    - Algorithms
      - RSA encryption
    - Software
      - Spacecraft control

Logic as a Foundation for AI

- Logic: extremely expressive, powerful
  - Theorem provers: useful in practice
- But:
  - Writing down needed knowledge is hard
    - So-called Frame, qualification, ramification problems
    - => Knowledge acquisition bottleneck
  - Logic systems are "incomplete"
  - Logic systems are brittle

The real world: Sensing and Acting

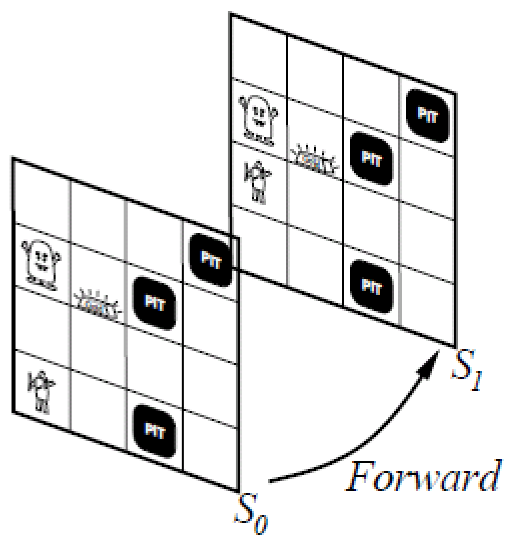
- Perception
  - three binary inputs [smell, breeze, glitter] at each time t
  - $\forall s, b, t$ Percept([s, b, Glitter] , t] => AtGold(t)

- $\forall t$ AtGold(t) => Action(Grab, t) ?
  - Infinite Loop!
- $\forall t$ AtGold(t) $\wedge$ $\neg$**Holding(Gold, t)** =>

  Action(Grab, t)

Keeping track of Change

- Facts hold in particular situations
  - E.g., Holding(Gold, *t*) may be False, Holding(Gold, *t+8*) true
- Agent must keep track of change



Frame Problem

- Effect axioms
  - $\forall t$ Standing((i, j), t) $\wedge$ Facing(Up, t) $\wedge$ Action(Forward, t)

    => Standing((i,j+1), t + 1)

- But…HaveArrow(t + 1) ?
- "Frame" axioms keep track of what *doesn't* change
  - Action(Forward, t) => (HaveArrow(t) $\wedge$ HaveArrow(t + 1))
  - Etc. etc. etc.

Representational Frame Problem

- Historically thought to be extremely tricky
- Can be solved by writing axioms about fluents rather than actions

Holding(Gold, t)

<=>

$\neg$ Holding(Gold, t-1) and action at t-1 made it true

or

Holding(Gold, t-1) and no action at t-1 made it false

Qualification Problem

- Action's preconditions can be complex

- Action(Grab, t) => Holding(t)

  ....unless gold is slippery or nailed down or too heavy or our hands are full or...

Ramification Problem

- Actions can have many consequences
  - $\forall t$ Standing((i, j), t) $\wedge$ Facing(Up, t) $\wedge$ Action(Forward, t)

    => Standing((i,j+1), t + 1)
  - But also

    => In(Basketball, (i, j+1), t + 1)

    if I'm holding a basketball
  - Writing all these down -- difficult

Knowledge Acquisition

- Remember the Colonel West story
  - We converted text to logic
  - In practice…who does this?
- Qualification, Ramification problems tell us we need *tons* of “common-sense” knowledge
- The infamous “knowledge acquisition bottleneck”

Knowledge Acquisition: Options

- Type it all in yourself
  - Cyc
- Get Web citizens to type it all in
  - Open Mind
- Extract it from the Web
  - KnowItAll, TextRunner

Logic as a Foundation for AI

- Logic: extremely expressive, powerful
  – Theorem provers: useful in practice
- But:
  – Writing down needed knowledge is hard
    - So-called Frame, qualification, ramification problems
    - => Knowledge acquisition bottleneck
  – **Logic systems are “incomplete”**
  – Logic systems are brittle

Gödel's Incompleteness Theorem

- Completeness Theorem: All valid statements have proofs in FOL

- Incompleteness Theorem: For any FOL KB enhanced to allow mathematical induction, there are true statements that *can't* be proved.

Gödel's Theorem: Sketch (1)

- Idea:

  This statement is false.

- More specifically:

  This statement has no proof.

Gödel's Theorem: Sketch (2)

- Assign numbers to sentences, proofs
  - E.g. by sorting by length, then alphabetically
- Consider the sentence $\alpha(j, A)$
  - For all numbers *i*, statement #*i* is not a proof for statement #*j* from the axioms *A*
- Let σ be the sentence $\alpha(\#\sigma, A)$
  - σ false? But it has a proof!
  - σ true? It's unprovable!

Gödel's Theorem: Ramifications

- Argument: Computers are limited by Gödel's theorem, whereas humans aren’t.
- Thus, AI is doomed

Three counter-arguments

- Gödel's theorem applies to math induction systems, e.g. Turing Machines
  – Computers aren’t *really* Turing machines

- “Steve cannot say this sentence is true.”
  – But Steve might be able to do other cool stuff

- Are humans really immune to the theorem?

Logic as a Foundation for AI

- Logic: extremely expressive, powerful
  - Theorem provers: useful in practice
- But:
  - Writing down needed knowledge is hard
    - So-called Frame, qualification, ramification problems
    - => Knowledge acquisition bottleneck
  - Logic systems are "incomplete"
  - **Logic systems are brittle**

Brittleness of Logic Systems

- Consider a KB with just one contradiction
- That KB entails everything
- This is a problem because much of the world is uncertain
  - Perception, action, incomplete information, controversies, etc.

Toward "Modern" AI

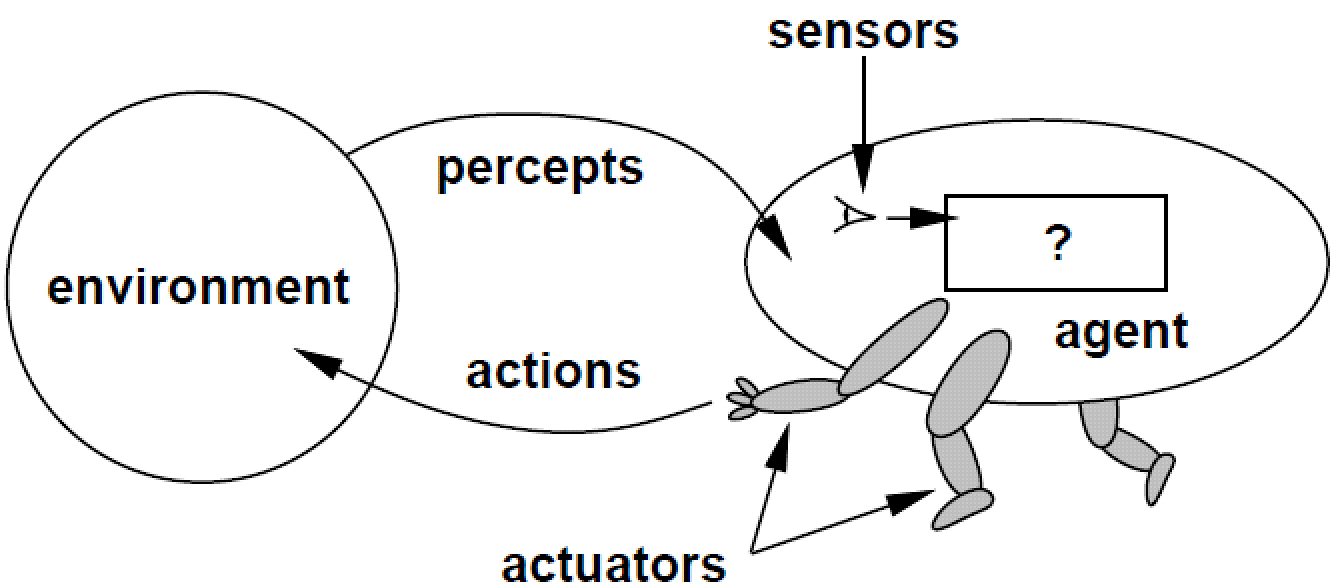
- Limitations:
  - Knowledge Acquisition Bottleneck, Brittleness

- "Modern" directions:
  - Situatedness, embodiment
  - Probability
  - Learning from data

Alternatives: Focus on Behavior

- Argument: we can’t even build systems that do what ants do
- In the timeline of evolution, simple cells->ants took much longer than ants->humans
- Let’s start by building ants
  - Environment, body can make tasks **easier**
  - Incrementally solve **real** problems end-to-end

Intelligent Agents



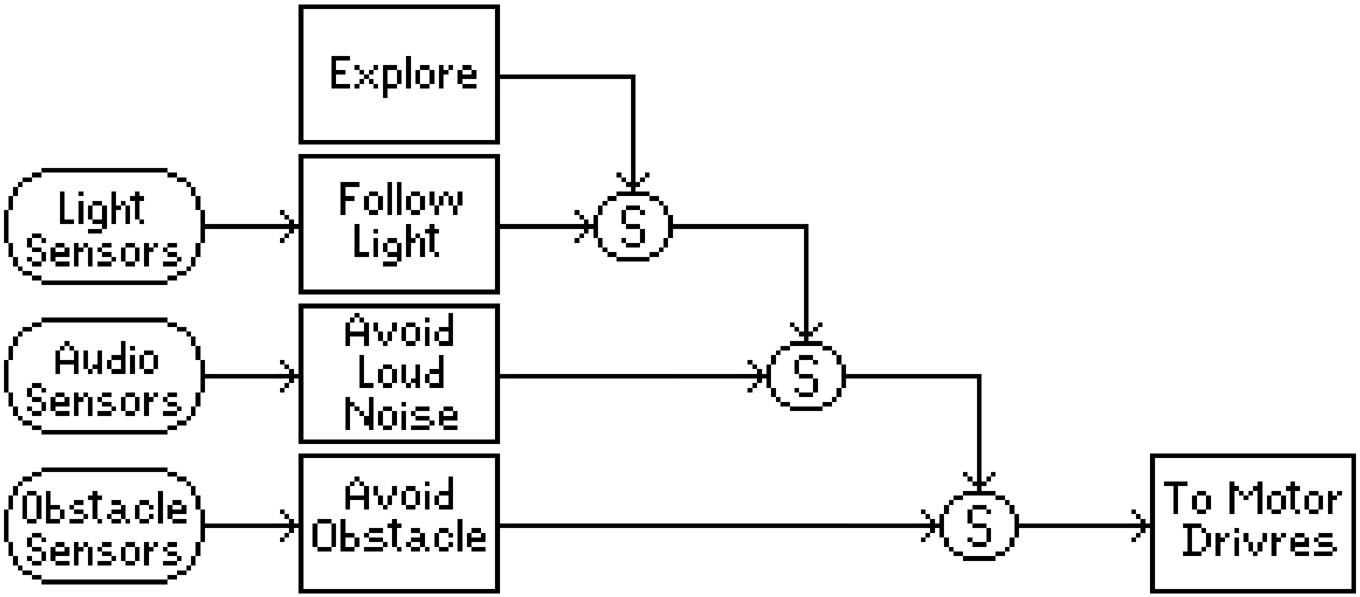
- Sensory/motor aspect
  - more **important**, more **coupled**, more **integrated** with rest of intelligence than originally thought

Behavior-based robots as a foundation for AI

- Common-sense knowledge arises from our interaction in the world
- Thus, the road to AI is paved with real-world interaction
  - We must build robots
- Another possibility: softbots

Subsumption Architecture

- Behavior-based robotics



Beam-wiki.org

Other "modern" trends

- Biological inspiration, e.g.:
  - Neural networks
  - Hexapod robots drawing on insect nervous systems followed subsumption architecture
- Probability theory
  - Handles uncertainty, overcomes brittleness
- Data

Learning from Data

- Quantities of data are exploding -- let’s learn from it

- “Machine learning”