Inductive Learning and Decision Trees

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with slides from Pedro Domingos, Bryan Pardo

Outline

- Announcements
  - Homework #1 to be assigned soon
- Inductive learning
- Decision Trees

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  - Homework #1 to be assigned soon
- **Inductive learning**
- Decision Trees

Instances

- E.g. Four Days, in terms of weather:

Sky	Temp	Humid	Wind	Forecast
sunny	warm	normal	strong	same
sunny	warm	high	strong	same
rainy	cold	high	strong	change
sunny	warm	high	strong	change

Functions

- "Days on which Anne agrees to get lunch with me"

INPUT

OUTPUT

Sky	Temp	Humid	Wind	Forecast	f(x)
sunny	warm	normal	strong	same	1
sunny	warm	high	strong	same	1
rainy	cold	high	strong	change	0
sunny	warm	high	strong	change	1

Inductive Learning!

- **Predict** the output for a new instance (generalize!)

$\overbrace{\hspace{20em}}^{\text{INPUT}}$ OUTPUT

Sky	**Temp**	**Humid**	**Wind**	**Forecast**	**f(x)**
sunny	warm	normal	strong	same	1
sunny	warm	high	strong	same	1
rainy	cold	high	strong	change	0
sunny	warm	high	strong	change	1
rainy	**warm**	**high**	**strong**	**change**	**?**

General Inductive Learning Task

DEFINE:

- Set X of Instances (of n-tuples $\mathbf{x} = \langle x_1, ..., x_n \rangle$)
  - E.g., days decribed by *attributes* (or *features*):

  Sky, Temp, Humidity, Wind, Forecast
- Target function $f : X \rightarrow Y$, e.g.:
  - GoesToLunch $X \rightarrow Y = \{0,1\}$
  - ResponseToLunch $X \rightarrow Y = \{$"No," "Yes," "How about tomorrow?"$\}$
  - ProbabililityOfLunch $X \rightarrow Y = [0, 1]$

GIVEN:

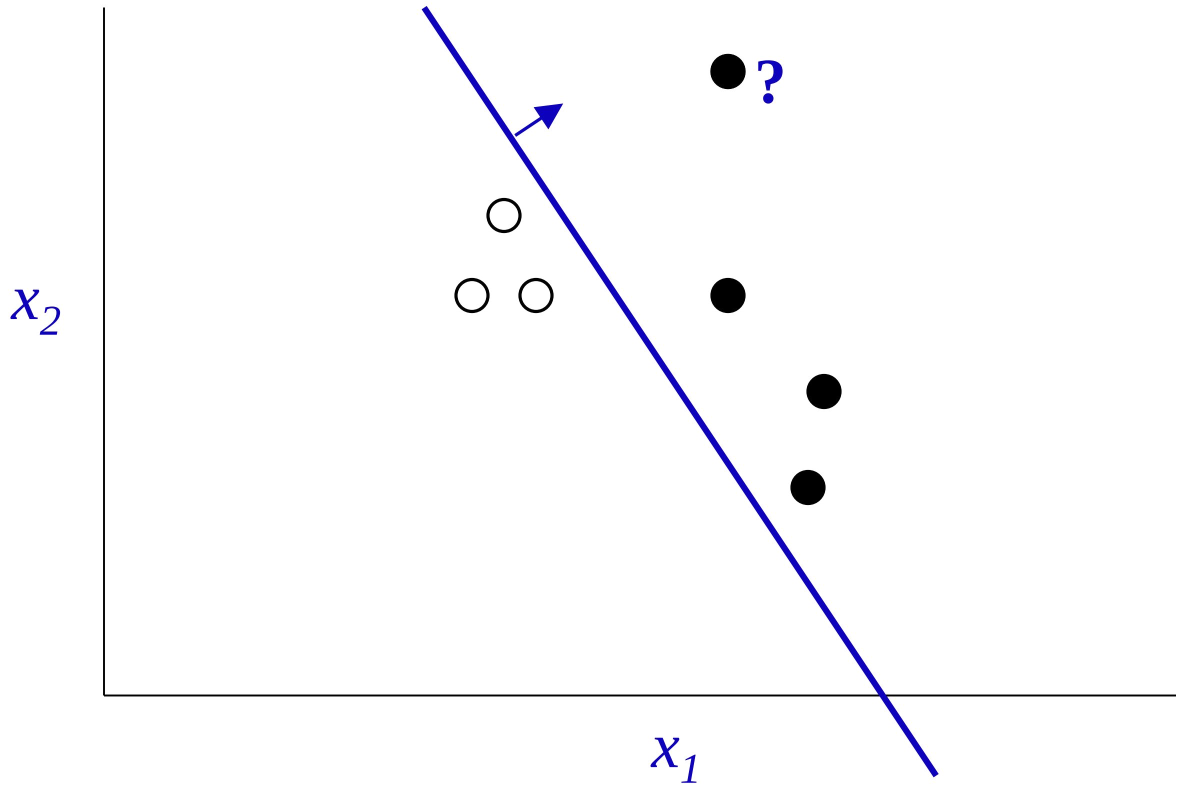
- *Training examples D*
  - examples of the target function: $\langle \mathbf{x}, f(\mathbf{x}) \rangle$

FIND:

- A hypothesis h such that $h(\mathbf{x})$ approximates $f(\mathbf{x})$.

Example w/ continuous attributes

Learn function from $\mathbf{x} = (x_1, \ldots, x_d)$ to $f(\mathbf{x}) \in \{0, 1\}$
given labeled examples $(\mathbf{x}, f(\mathbf{x}))$



Hypothesis Spaces

- Hypothesis space *H* is a **subset** of all $f : X \rightarrow Y$ e.g.:
  - Linear separators
  - Conjunctions of constraints on attributes (humidity must be low, and outlook != rain)
  - Etc.

- In machine learning, we restrict ourselves to *H*

Examples

- Credit Risk Analysis
  - ***X*:** Properties of customer and proposed purchase
  - $f(\mathbf{x})$**:** Approve (1) or Disapprove (0)
- Disease Diagnosis
  - ***X*:** Properties of patient (symptoms, lab tests)
  - $f(\mathbf{x})$**:** Disease (if any)
- Face Recognition
  - ***X*:** Bitmap image
  - $f(\mathbf{x})$**:**Name of person
- Automatic Steering
  - ***X*:** Bitmap picture of road surface in front of car
  - $f(\mathbf{x})$**:** Degrees to turn the steering wheel

Inductive Learning *tasks*

- Defined in terms of **inputs** and **outputs**:
  - Predicting outcomes of sporting events
    - Input: A game (two opponents, a date)
    - Output: which team will win (classification)
- On the other hand, these are *not* tasks:
  - "Studying the relationship between weather and sports game outcomes."
  - "Applying neural networks to natural language processing."

When to use?

- Inductive Learning is appropriate for building a face recognizer
- It is not appropriate for building a calculator
  - You'd just write a calculator program
- Question:
  What general characteristics make a problem suitable for inductive learning?

Think/Pair/Share

What general characteristics make a problem suitable for inductive learning?

Think

Start

End

Think/Pair/Share

What general characteristics make a problem suitable for inductive learning?

Pair

Start

End

Think/Pair/Share

What general characteristics make a problem suitable for inductive learning?

Share

Appropriate applications

- Situations in which:
  - There is no human expert
  - Humans can perform the task but can’t describe how
  - The desired function changes frequently
  - Each user needs a customized f

Outline

- Announcements
  - Homework #1
- Inductive learning
- **Decision Trees**

Why Decision Trees?

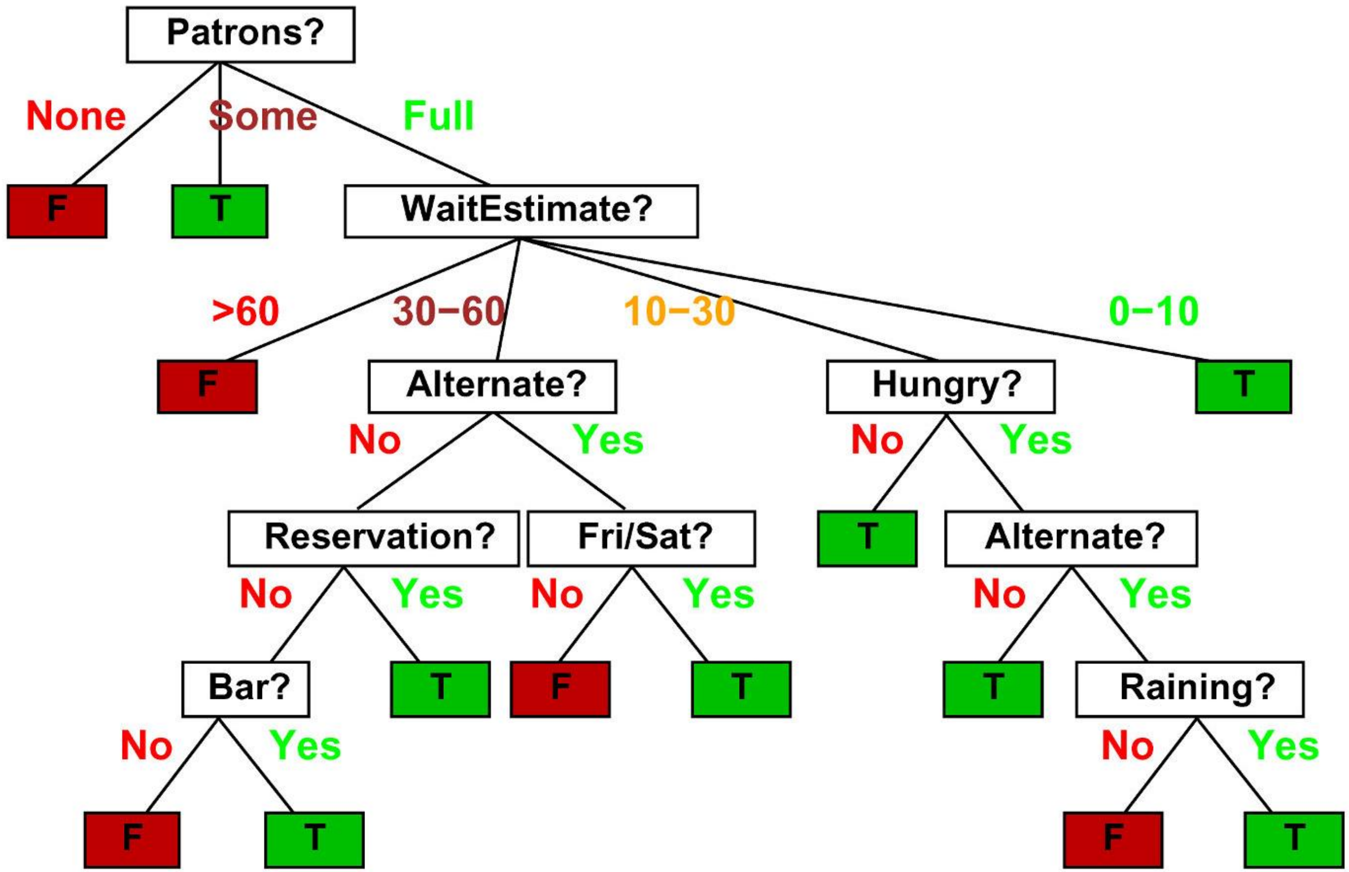
- Simple inductive learning approach
  - Training procedure is easy to understand
  - Models are easy to understand
- Popular
  - The most popular learning method, according to surveys [Domingos, 2016]

Task: Will I wait for a table?

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*T*	*F*	*F*	*T*	*Some*	*$$$*	*F*	*T*	*French*	*0–10*	*T*
X_2	*T*	*F*	*F*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*30–60*	*F*
X_3	*F*	*T*	*F*	*F*	*Some*	*$*	*F*	*F*	*Burger*	*0–10*	*T*
X_4	*T*	*F*	*T*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*10–30*	*T*
X_5	*T*	*F*	*T*	*F*	*Full*	*$$$*	*F*	*T*	*French*	*>60*	*F*
X_6	*F*	*T*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Italian*	*0–10*	*T*
X_7	*F*	*T*	*F*	*F*	*None*	*$*	*T*	*F*	*Burger*	*0–10*	*F*
X_8	*F*	*F*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Thai*	*0–10*	*T*
X_9	*F*	*T*	*T*	*F*	*Full*	*$*	*T*	*F*	*Burger*	*>60*	*F*
X_{10}	*T*	*T*	*T*	*T*	*Full*	*$$$*	*F*	*T*	*Italian*	*10–30*	*F*
X_{11}	*F*	*F*	*F*	*F*	*None*	*$*	*F*	*F*	*Thai*	*0–10*	*F*
X_{12}	*T*	*T*	*T*	*T*	*Full*	*$*	*F*	*F*	*Burger*	*30–60*	*T*

Classification of examples is positive (T) or negative (F)

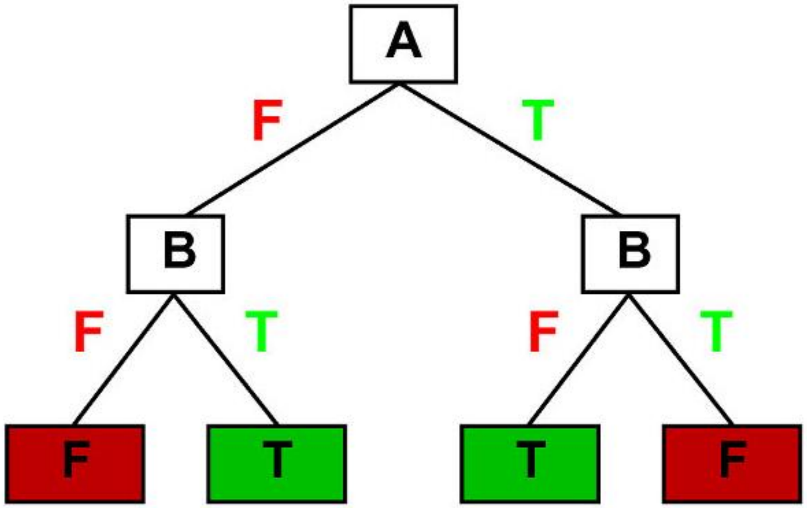
A Decision Tree for “Will I Wait”



Expressiveness of D-Trees

- Decision Trees can represent *any* Boolean function
  - E.g., for two binary attributes {A,B}, the tree for binary function *f*(A, B) = A xor B:

A	B	A xor B
F	F	F
F	T	T
T	F	T
T	T	F



Inductive Learning with Decision Trees

- In inductive learning, our goal is to *learn* a decision tree from a data set, such that it can *generalize* to new examples.
- What tree might you learn from the following **three** examples?

A	B	$f(A, B)$
F	F	F
F	T	T
T	F	T

Think/Pair/Share

What tree might you learn from the following three examples?

A	B	$f(A, B)$
F	F	F
F	T	T
T	F	T

Think

Start

End

Think/Pair/Share

What tree might you learn from the following three examples?

A	B	*f*(A, B)
F	F	F
F	T	T
T	F	T

Pair

Start End

Think/Pair/Share

What tree might you learn from the following three examples?

A	B	$f(A, B)$
F	F	F
F	T	T
T	F	T

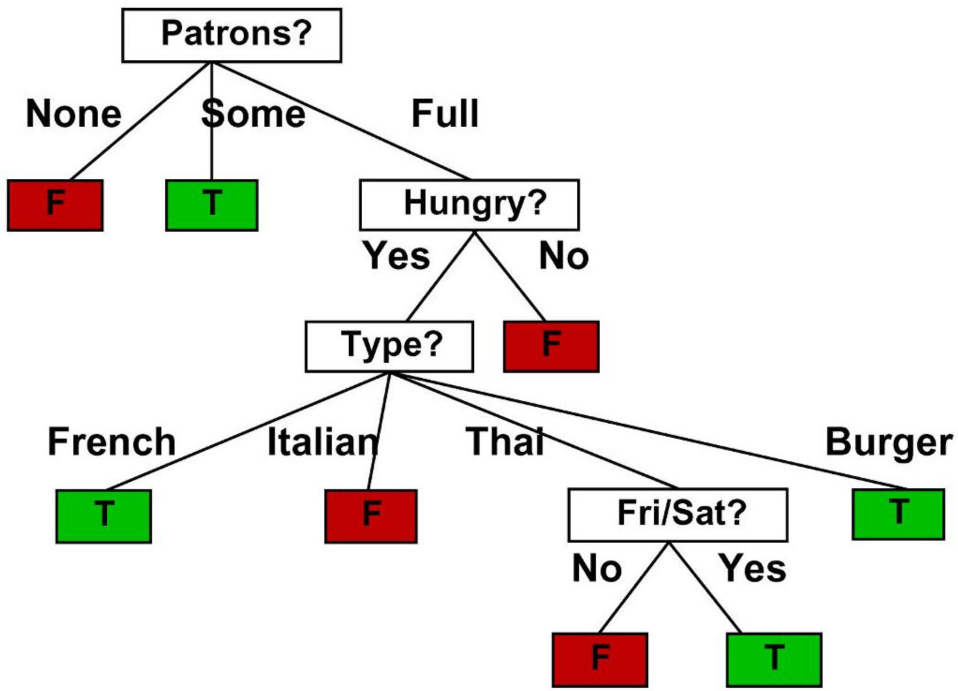
Share

Inductive Bias

- To learn, we **must** prefer some functions to others
  - **Selection bias**
    - use a **restricted** hypothesis space, e.g.:
      - linear separators
      - depth-2 decision trees
  - **Preference bias**
    - use the whole function space, but state a **preference** over functions, e.g.:
      - *Lowest-degree* polynomial that separates the data
      - *shortest* decision tree that fits the data

A learned decision tree

Decision tree learned from the 12 examples:



Substantially simpler than "true" tree—a more complex hypothesis isn't justified by small amount of data

Summary

- Inductive Learning
  - Given **examples** of a **target function** *f*
    - **example** = **instance** (a vector of **attributes**) and its corresponding target function value
  - Learn a **hypothesis** that approximates the function
- Decision Trees
  - One way of *representing* a hypothesis
  - Can represent any Boolean function
- Inductive Bias
  - Bias in favor of some functions over others
  - Necessary for learning

Outline

- Decision Tree Learning (ID3)

Decision Tree Learning (ID3*)

Goal: Find a (small) tree consistent with *examples*

Function ID3(*examples*, *default*) **returns** a tree

if *examples* is empty

return tree(*default*)

else if all *examples* have same classification **or** no non-trivial splits are possible:

return tree(MODE(*examples*)))

Returns most frequent class label in examples

else:

best ← CHOOSE-ATTRIBUTE(*examples*)

t ← new tree with root test *best*

for each $value_i$ of best:

$examples_i$ ← {elements of *examples* with *best* = $value_i$}

subtree ← ID3(*examplesi*, MODE(*examples*)}

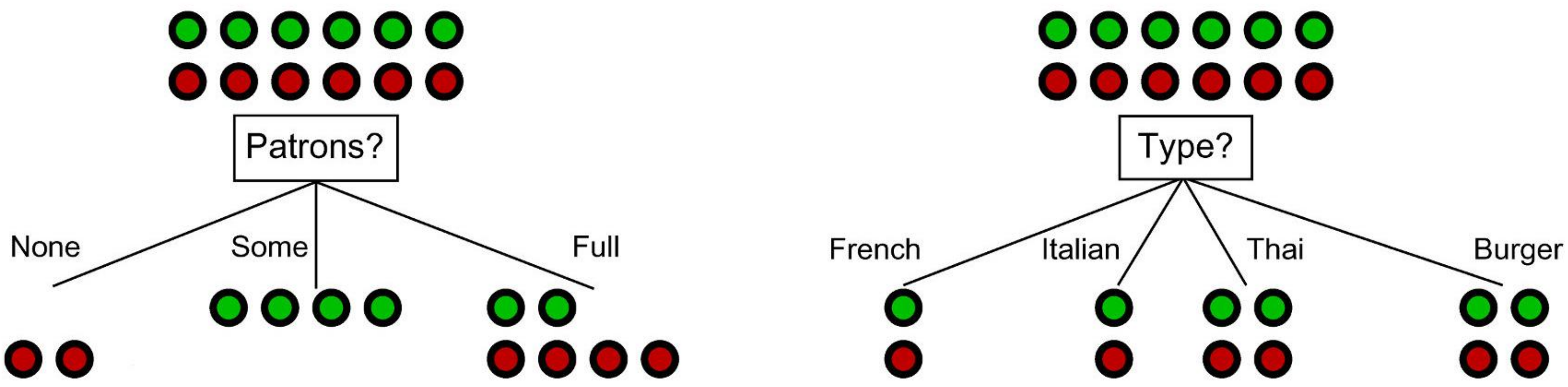
add branch to *t* with label $value_i$ and subtree *subtree*

return *t*

* Our algorithm's termination conditions differ in small ways from the original published ID3

Choosing an attribute

Idea: a good attribute splits the examples into subsets that are (ideally) "all positive" or "all negative"



Patrons? is a better choice—gives **information** about the classification

Think/Pair/Share

How should we choose which attribute to split on next?

Think

Start

End

Think/Pair/Share

How should we choose which attribute to split on next?

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Start

End

Think/Pair/Share

How should we choose which attribute to split on next?

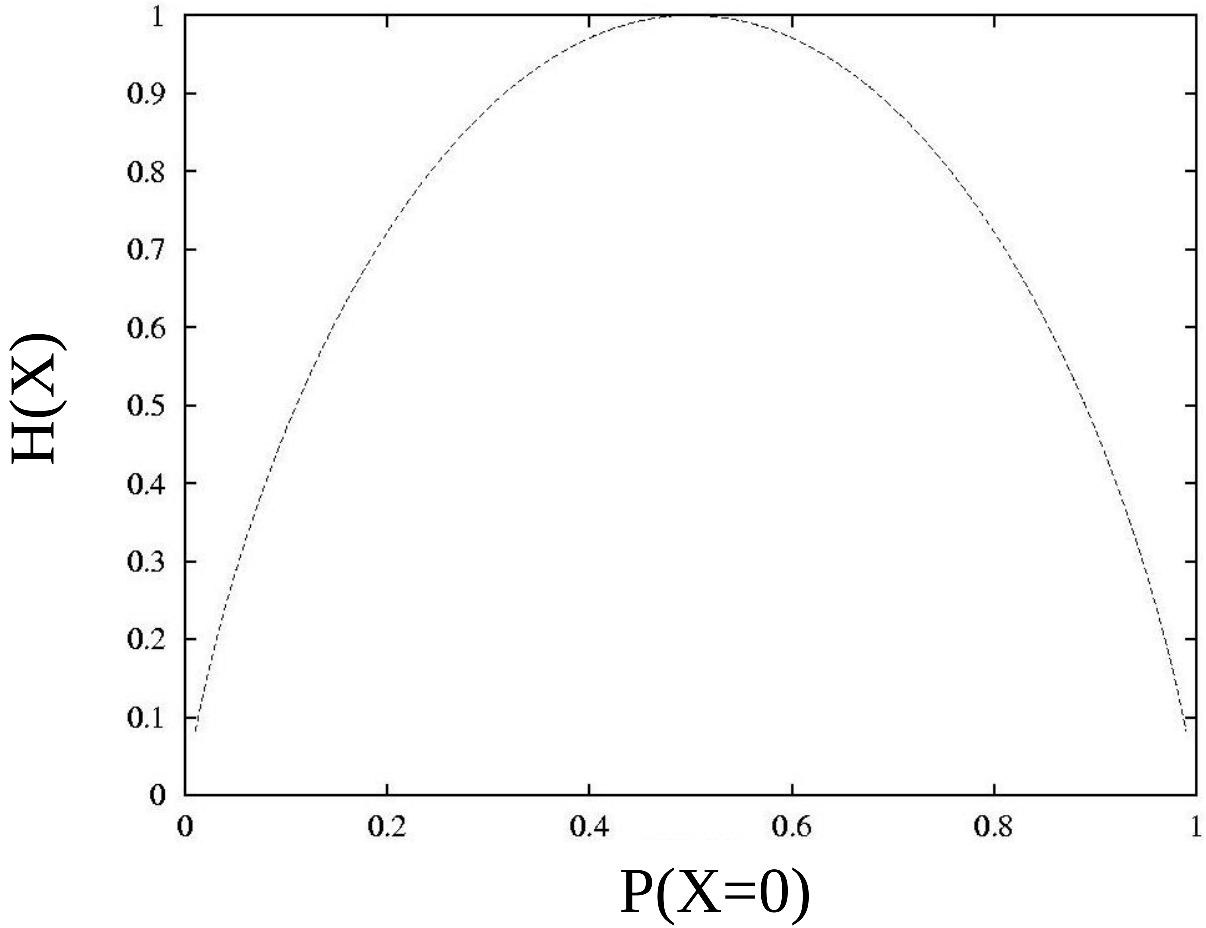
Share

Information

- Brief sojourn into information theory
  - (on board)

Entropy

The entropy H(X) of a Boolean random variable X as the probability of X = 0 varies from 0 to 1



Using Information

- Say we have *n* attributes $A_1, A_2, \ldots A_n$
- The key question: how much information, on average, will I gain about the class *y* = *f*(***x***) by doing the split?
  - Choose attribute A_i that maximizes this expected value
- $InfoGain(A_i) = H_{prior} - \sum_v P(A_i = v) H(y|A_i = v)$
- Since H_{prior} is constant w.r.t. A_i, we can just choose attribute with minimum $\sum_v P(A_i = v) H(y|A_i = v)$

Evaluating Decision Trees

- *Accuracy* of a tree
  - Fraction of examples where tree output matches the output in the data set
- What is the accuracy of a tree on the examples used to train it?
  - Assuming the "noiseless" case where the same attribute vector $\mathbf{x}$ always maps to the same output $f(\mathbf{x})$.
  - …100%
- If I deployed a tree and used it to classify new examples, would I expect it to be 100% accurate?
  - No.
- How to estimate accuracy of tree on new examples?

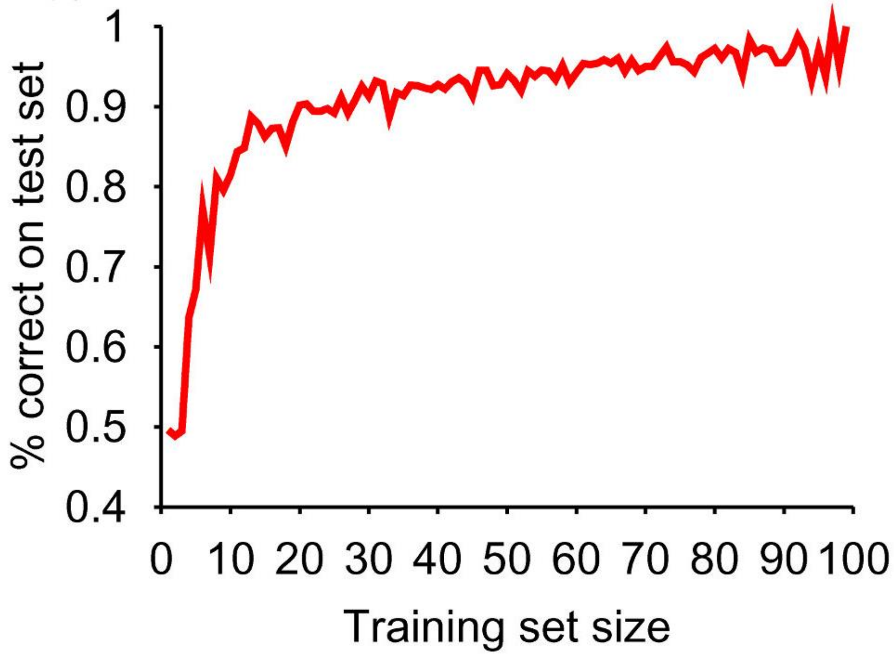
Measuring Performance

How do we know that $h \approx f$? (Hume's **Problem of Induction**)

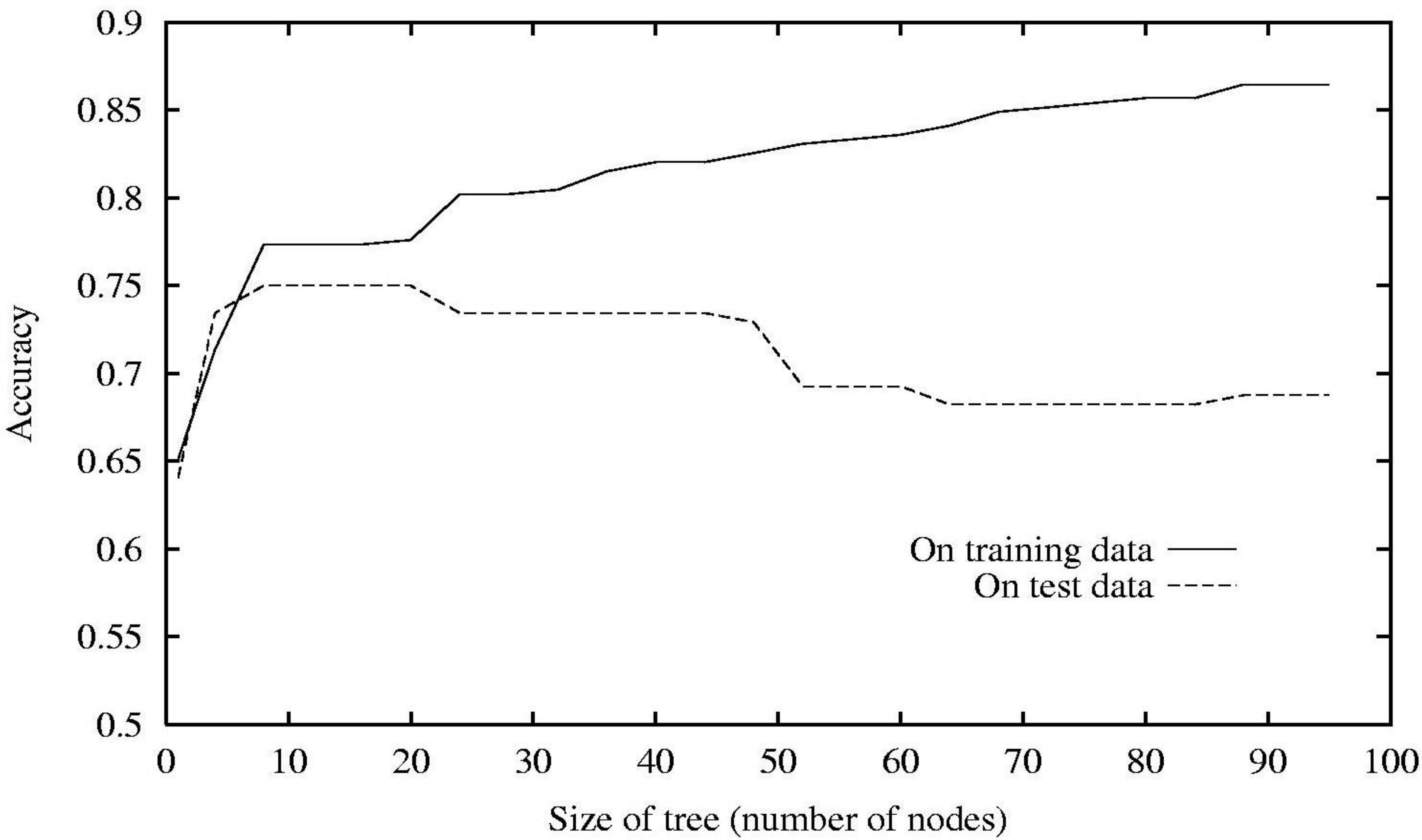
1) Use theorems of computational/statistical learning theory

2) Try h on a new test set of examples
   (use **same distribution over example space** as training set)

Learning curve $=$ % correct on test set as a function of training set size



Overfitting



Overfitting is due to “noise”

- Sources of noise:
  - Erroneous training data
    - concept variable incorrect (annotator error)
    - Attributes mis-measured
  - More significant:
    - Irrelevant attributes
    - Target function not realizable in attributes

Irrelevant attributes

- If many attributes are noisy, information gains can be spurious, e.g.:
  - 20 noisy attributes
  - 10 training examples
  - Expected # of different depth-3 trees that split the training data perfectly using *only* noisy attributes: **13.4**

Not realizable

- In general:
  - We can rarely measure well enough for **perfect** prediction
  - => Target function is not uniquely determined by attribute values
  - Target outputs appear to be "noisy"
    - Same attribute vector may yield distinct output values

Not realizable: Example

Humidity	f(x)
0.90	0
0.87	1
0.80	0
0.75	0
0.70	1
0.69	1
0.65	1
0.63	1

Decent hypothesis:

Humidity > 0.70 → No

Otherwise → Yes

Overfit hypothesis:

Humidity > 0.89 → No

Humidity > 0.80

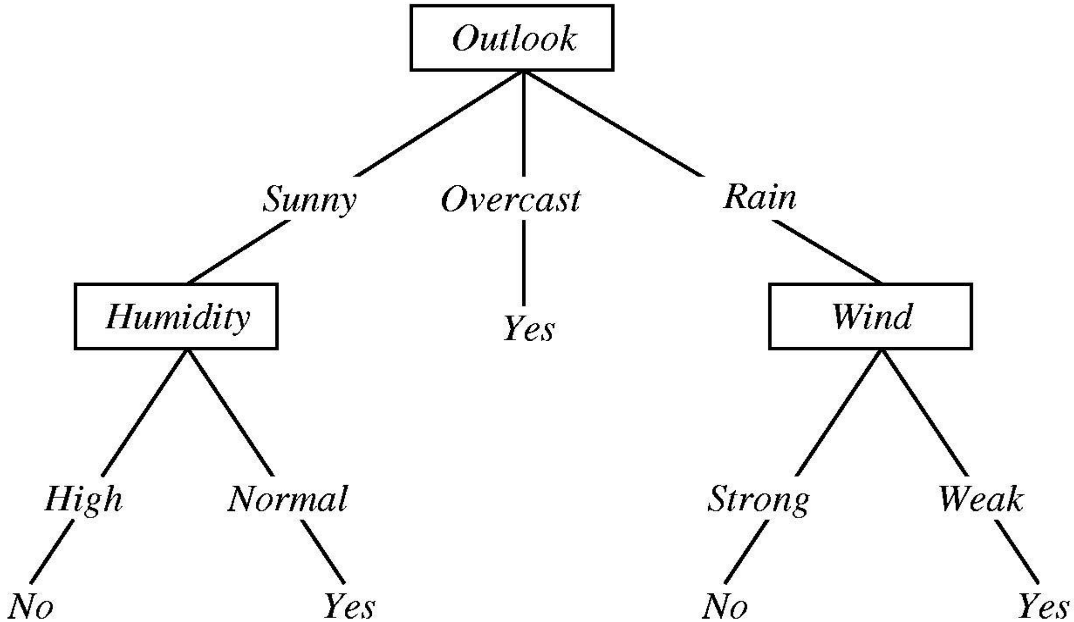
^ Humidity <= 0.89 → Yes

Humidity > 0.70

^ Humidity <= 0.80 → No

Humidity <= 0.70 → Yes

Overfitting in Decision Trees



Consider adding a noisy training example:

Sunny, Hot, Normal, Strong, PlayTennis=No

What effect on tree?

Avoiding Overfitting

- Approaches
  - Stop splitting when information gain is low or when split is not statistically significant.
  - Grow full tree and then **prune** it when done

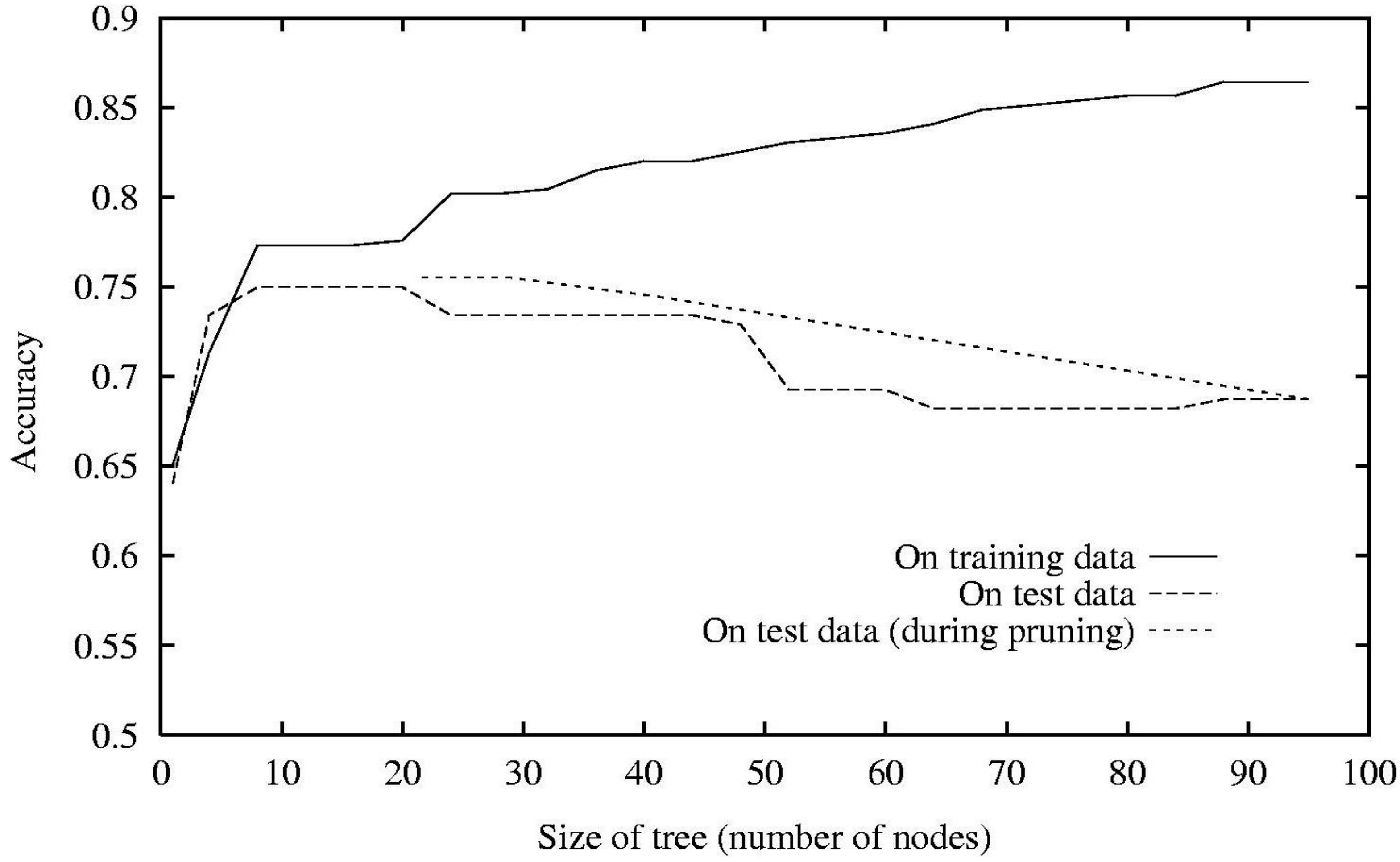
Reduced-Error Pruning

Split data into *training* and *validation* set

Do until further pruning is harmful:

1. Evaluate impact on *validation* set of pruning each possible node (plus those below it)
2. Greedily remove the one that most improves *validation* set accuracy

Effect of Reduced Error Pruning



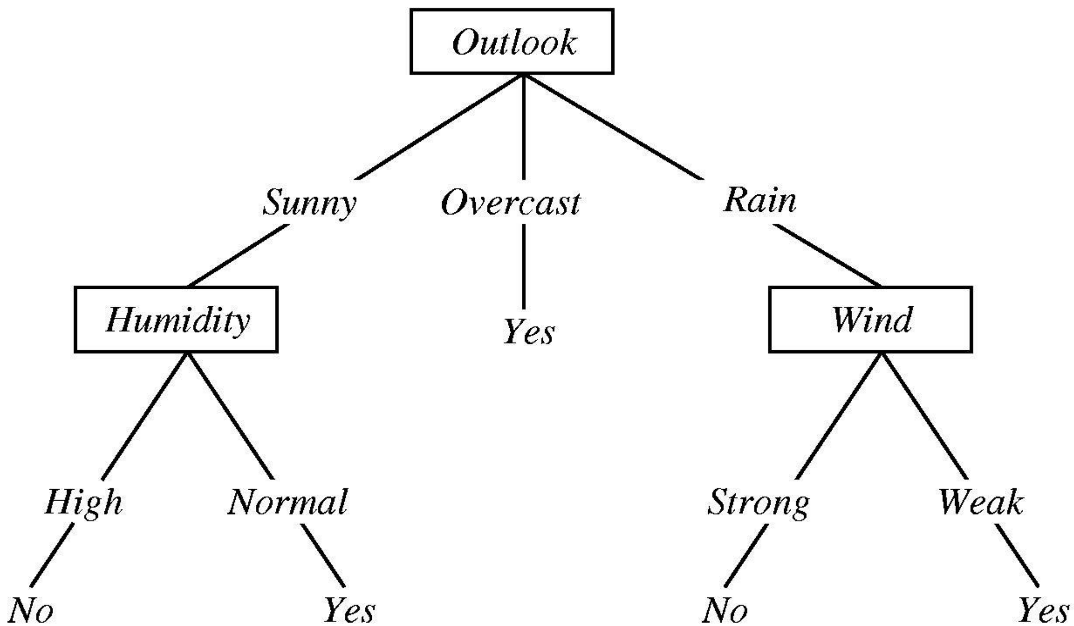
C4.5 Algorithm

- Builds a decision tree from labeled training data
- Generalizes simple "ID3" tree by
  - Prunes tree after building to improve generality
  - Allows missing attributes in examples
  - Allowing continuous-valued attributes

Rule post pruning

- Used in C4.5
- Steps
  1. Build the decision tree
  2. Convert it to a set of logical rules
  3. Prune each rule independently
  4. Sort rules into desired sequence for use

Converting A Tree to Rules



IF $(Outlook = Sunny)\ AND\ (Humidity = High)$
THEN $PlayTennis = No$

IF $(Outlook = Sunny)\ AND\ (Humidity = Normal)$
THEN $PlayTennis = Yes$

...

Other Odds and Ends

- Unknown Attribute Values?

Unknown Attribute Values

What if some examples are missing values of A?

Use training example anyway, sort through tree

- If node n tests A, assign most common value of A among other examples sorted to node n
- Assign most common value of A among other examples with same target value
- Assign probability p_i to each possible value v_i of A
  Assign fraction p_i of example to each descendant in tree

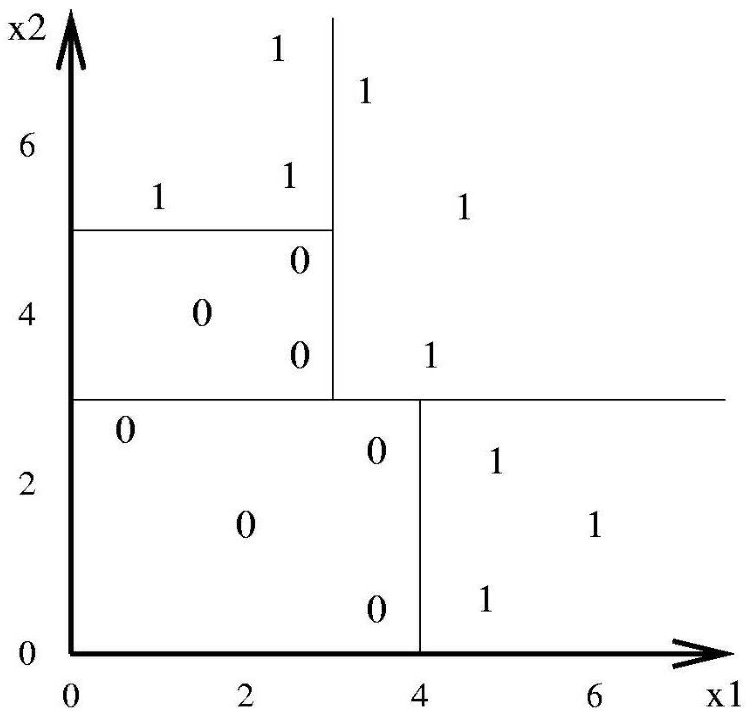
Classify new examples in same fashion

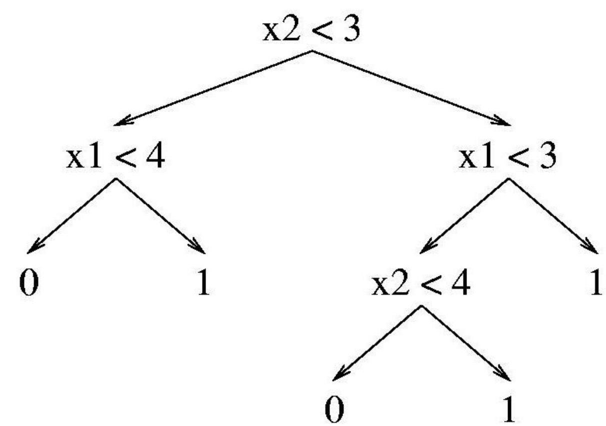
Odds and Ends

- Unknown Attribute Values?
- Continuous Attributes?

Decision Tree Boundaries

Decision trees divide the feature space into axis-parallel rectangles, and label each rectangle with one of the K classes.

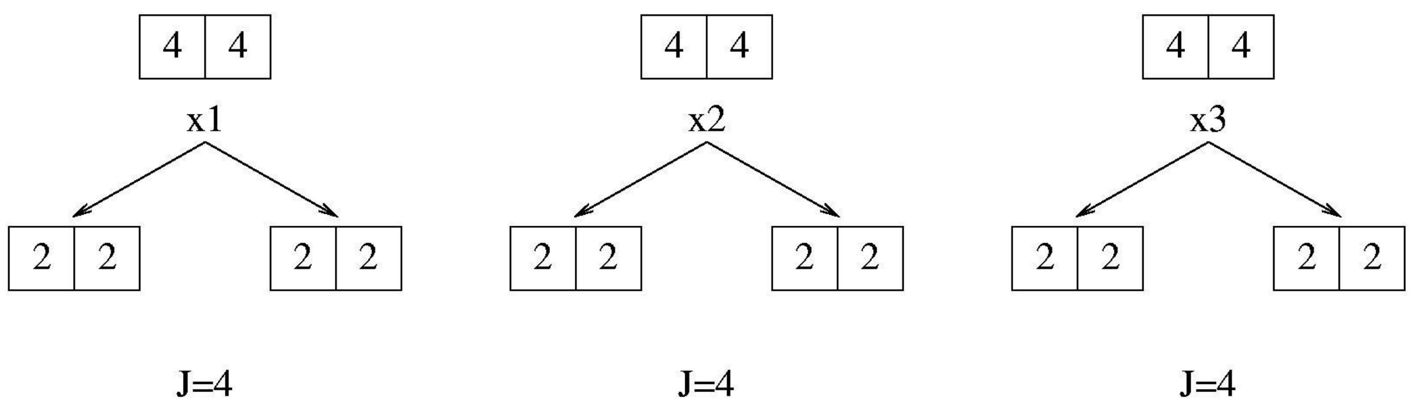




Learning Parity with Noise

When learning exclusive-or (2-bit parity), all splits look equally good. If extra random boolean features are included, they also look equally good. Hence, decision tree algorithms cannot distinguish random noisy features from parity features.

x_1	x_2	x_3	y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0



Decision Trees Bias

- How to solve 2-bit parity:
  - Split on *pairs* of attributes at once

- For *k*-bit parity, why not split on *k* attribute values at once?

=>*Parity functions are among the "victims" of the decision tree's inductive bias.*

Now we have choices

- Re-split continuous attributes?
- Handling unknown variables?
- Prune or not?
- Stopping criteria?
- Split selection criteria?
- Use look-ahead?

- In homework #1: one choice for each
- In practice, how to decide? An instance of *Model Selection*
  - In general, we could also select an H other than decision trees

Think/Pair/Share

We can do model selection using a 70% train, 30% validation split of our data. But can we do better?

Think

Start

End

Think/Pair/Share

We can do model selection using a 70% train, 30% validation split of our data. But can we do better?

Pair

Start

End

Think/Pair/Share

We can do model selection using a 70% train, 30% validation split of our data. But can we do better?

Share

10-fold Cross-Validation

- On board

Take away about decision trees

- Used as classifiers
- Supervised learning algorithms (ID3, C4.5)
- Good for situations where
  - Inputs, outputs are discrete
  - Interpretability is important
  - "We think the true function is a small tree"

Readings

- Decision Trees:
  - Induction of decision trees, Ross Quinlan (1986) (covers ID3)
    - https://link.springer.com/article/10.1007%2FBF00116251
      (may need to be on campus to access)
  - C4.5: Programs for Machine Learning (2014) (covers C4.5)
    https://books.google.com/books?hl=en&lr=&id=b3ujBQAAQBAJ&oi=fnd&pg=PP1&dq=c4.5&ots=sPanSTEtC4&sig=c2Np0fBu37b-le-dVUyhulPJsv4#v=onepage&q=c4.5&f=false
- Overfitting in Decision Trees
  - http://cse-wiki.unl.edu/wiki/index.php/Decision_Trees,_Overfitting,_and_Occam's_Razor
- Cross-Validation
  - https://en.wikipedia.org/wiki/Cross-validation_(statistics)