# Bayes Net Learning and Logistic Regression

EECS 349 Spring 2015

# Learning in Bayes Nets – the upshot

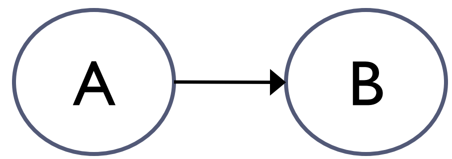
- Where does the structure come from?
  - Write it down (BNs most useful in this case), or
  - Learn it automatically from data
  - (take 395/495 PGMs course to learn more)

# Learning parameters in Bayes Nets

- Just statistical estimation for each CPT

Training Data

| A | B |
|---|---|
| 1 | 1 |
| 1 | 0 |
| 1 | 0 |
| 0 | 1 |
| 1 | 1 |
| 0 | 1 |
| 1 | 1 |




$P_{ML}(A) = 0.714$

$P_{ML}(B \mid A=1) = 0.6$

# Discriminative vs. Generative training

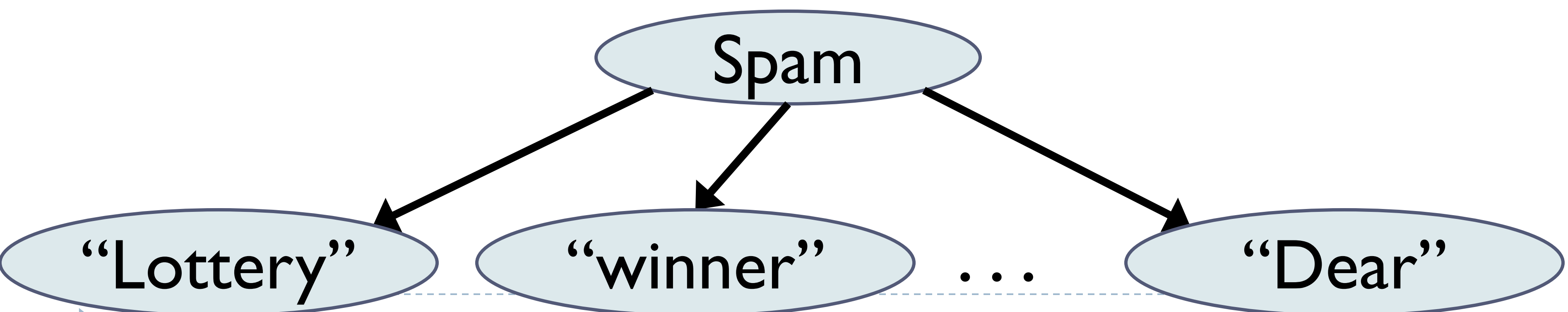
- Say our graph *G* has variables ***X*** , ***Y***
- Standard BN learning learns P(***X*** , ***Y*** )
- But often, the only inferences we care about are of form P(***Y*** | ***X***)
  - P(*Disease* | *Symptoms* = **e**)
  - P(*StockMarketCrash* | *RecentPriceActivity* = **e**)

# Discriminative vs. Generative training

- Learning P(***X*** , ***Y*** ): **generative** training
  - Learned model can "generate" the full data ***X, Y***
- Learning only P(***Y*** | ***X***): **discriminative** training
  - Model can't assign probs. to ***X*** – only ***Y*** given ***X***
- Idea: Only model what we care about
  - Don't "waste data" on params irrelevant to task
  - Side-step false independence assumptions in training (example to follow)

# Generative Model Example

- Naïve Bayes model
  - $Y$ binary {1=spam, 0=not spam}
    $\boldsymbol{X}$ an $n$-vector: message has word (1) or not (0)
  - Re-write P(Y | $\boldsymbol{X}$) using Bayes Rule, apply Naïve Bayes assumption
  - $2n + 1$ parameters, for $n$ observed variables

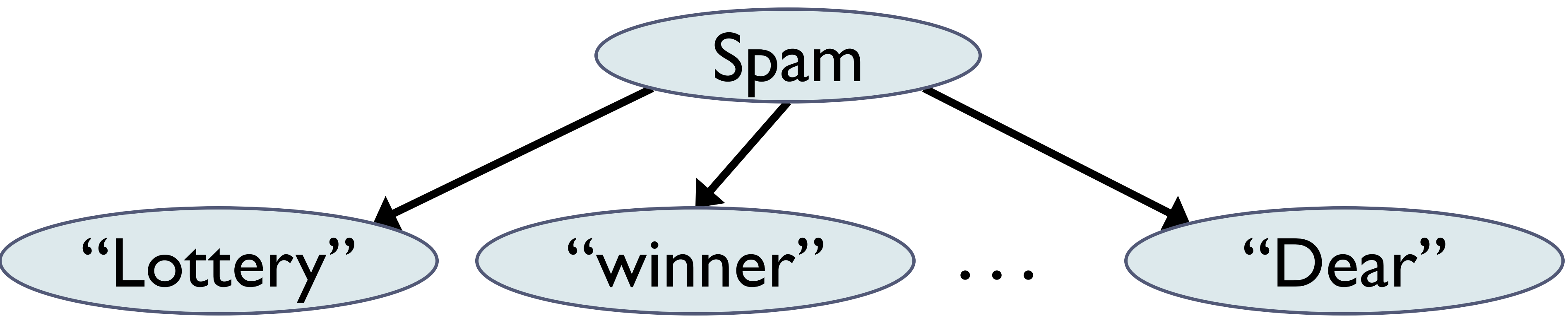


# Generative => Discriminative (1 of 3)

- But $P(Y \mid \boldsymbol{X})$ can be written more compactly

$$P(Y \mid \boldsymbol{X}) = \frac{1}{1 + \exp(w_0 + w_1 x_1 + \ldots + w_n x_n)}$$

- Total of $n$ + 1 parameters $w_i$



# Generative => Discriminative (2 of 3)

- One way to do conversion (vars binary):

$$\exp(w_0) = \frac{P(Y=0)\,P(X_1=0|Y=0)\,P(X_2=0|Y=0)\ldots}{P(Y=1)\,P(X_1=0|Y=1)\,P(X_2=0|Y=1)\ldots}$$

for $i > 0$:

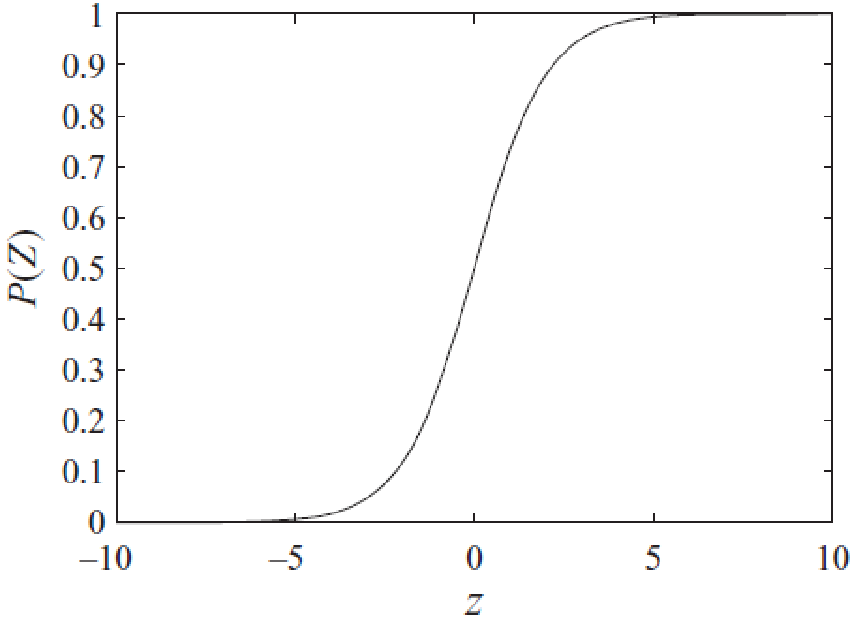
$$\exp(w_i) = \frac{P(X_i=0|Y=1)\,P(X_i=1|Y=0)}{P(X_i=0|Y=0)\,P(X_i=1|Y=1)}$$

# Generative => Discriminative (3 of 3)

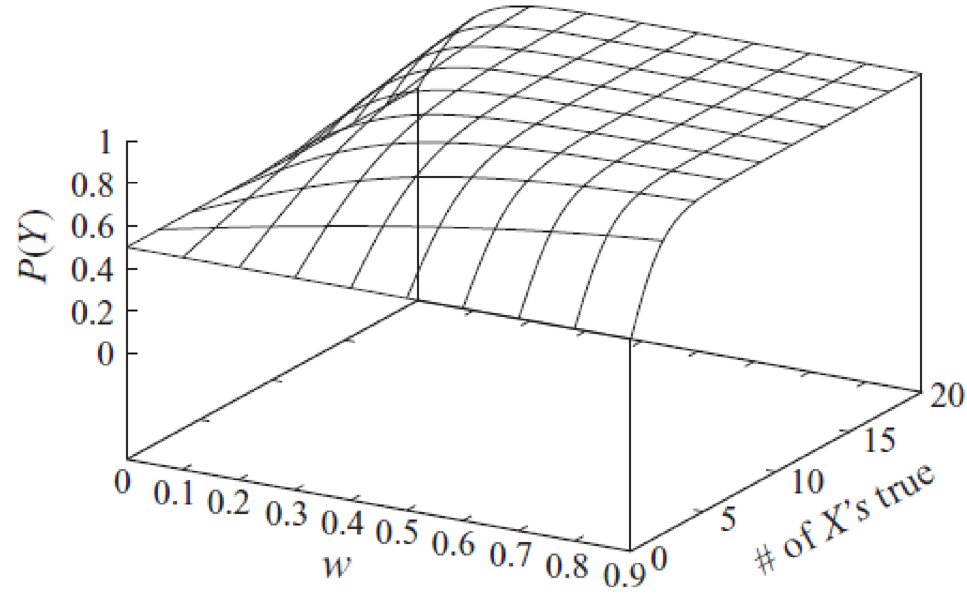
- We reduced $2n + 1$ parameters to $n + 1$
  - This must be *better*, right?
- Not exactly. If we construct $P(Y \mid \boldsymbol{X})$ to be equivalent to Naïve Bayes (as before)
  - then it's…equivalent to Naïve Bayes
- Idea: optimize the $n + 1$ parameters directly, using training data

# Discriminative Training

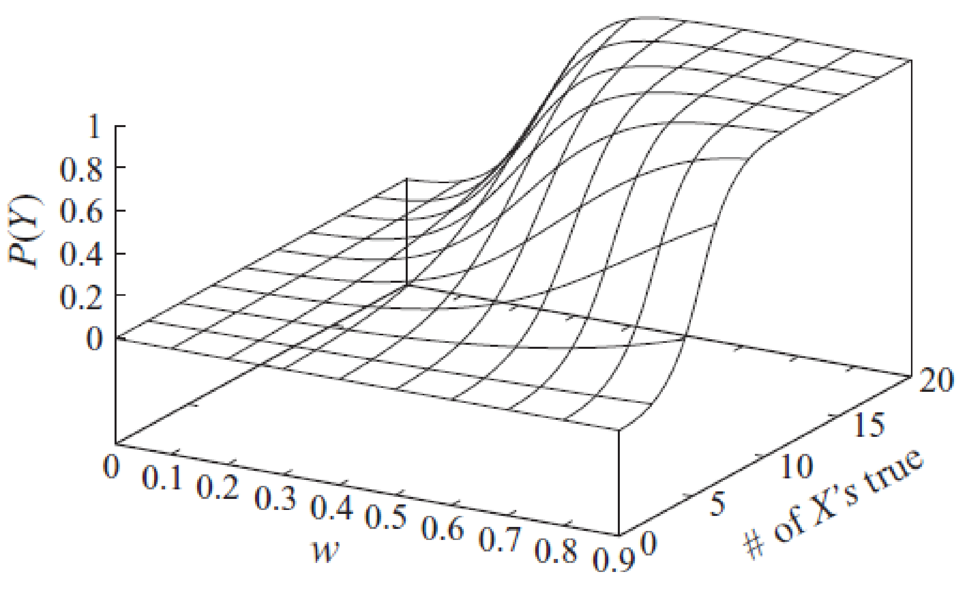
- In our example:
  $$P(Y \mid \boldsymbol{X}) = \frac{1}{1 + \exp(w_0 + w_1 x_1 + \ldots + w_n x_n)}$$
- Goal: find $w_i$ that maximize likelihood of training data *Y*s given training data ***X***s
  - Known as "logistic regression"
  - Solved with gradient ascent techniques
  - A convex optimization problem

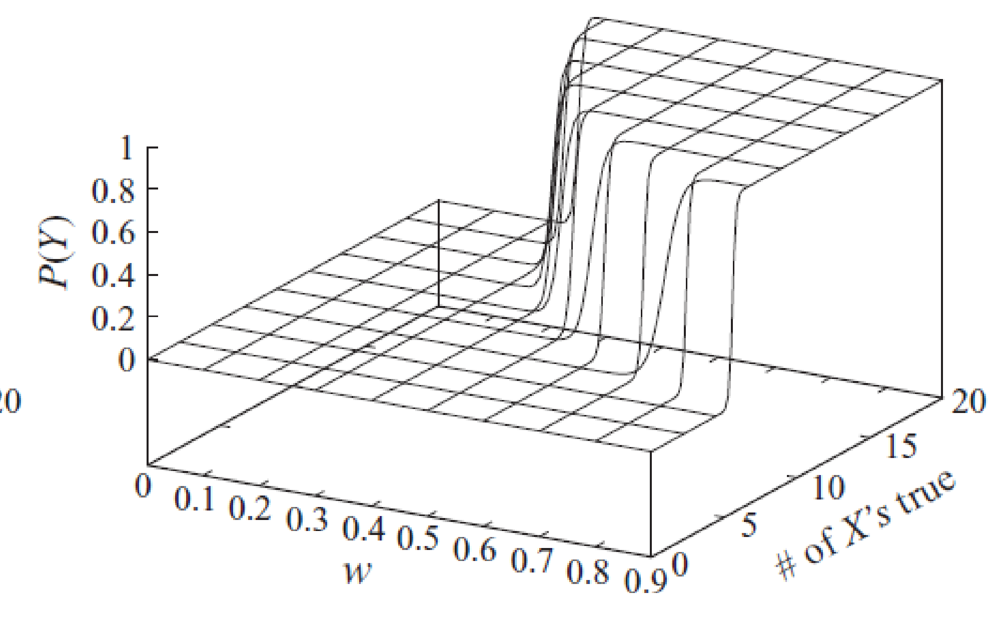

P(Z)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
−10
−5
0
5
10
z

(a)


P(Y)
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
w
0
5
10
15
20
# of X's true

(b)


P(Y)
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
w
0
5
10
15
20
# of X's true


P(Y)
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
w
0
5
10
15
20
# of X's true

# Naïve Bayes vs. LR

- Naïve Bayes "trusts its assumptions" in training
- Logistic Regression doesn't – recovers better when assumptions violated

# NB vs. LR: Example

Training Data

| SPAM | Lottery | Winner | Lunch | Noon |
|---|---|---|---|---|
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |

- Naïve Bayes will classify the last example incorrectly, even after training on it!
- Whereas Logistic Regression is perfect with e.g.,
  $w_0 = 0.1$ $w_{\text{lottery}} = w_{\text{winner}} = w_{\text{lunch}} = -0.2$ $w_{\text{noon}} = 0.4$

# Logistic Regression in practice

- Can be employed for any numeric variables $X_i$
  - or for other variable types, by converting to numeric (e.g. indicator) functions
- "Regularization" plays the role of priors in Naïve Bayes
- Optimization tractable, but (way) more expensive than counting (as in Naïve Bayes)

# Discriminative Training

- Naïve Bayes vs. Logistic Regression one illustrative case
- Applicable more broadly, whenever queries P(***Y*** | ***X***) known *a priori*