Naïve Bayes Classifiers
Naïve Bayes Classifiers

- Combines all ideas we’ve covered
  - Conditional Independence
  - Bayes’ Rule
  - Statistical Estimation

- …in a simple, yet accurate classifier
  - Classifier: Function $f(x)$ from $X = \{<x_1, \ldots, x_d>\}$ to $Class$
  - E.g., $X = \{<\text{GRE, GPA, Letters}>\}$, $Class = \{\text{yes, no, wait}\}$
Classification task
- Learn function $f(x)$ from $X = \{<x_1, \ldots, x_d>\}$ to Class
- Given: Examples $D=\{(x, y)\}$

Probabilistic Approach
- Learn $P(Class = y \mid X = x)$ from $D$
- Given $x$, pick the maximally probable $y$
More formally

\[ f(x) = \arg \max_y P(Class = y \mid X = x, \theta_{\text{MAP}}) \]

\( \theta_{\text{MAP}} \): MAP parameters, learned from data
  - That is, parameters of \( P(Class = y \mid X = x) \)
  - …we’ll focus on using MAP estimate, but can also use ML or Bayesian

Predict next coin flip? Instance of this problem

- \( X = \text{null} \)
- Given \( D = \text{hhht...tht} \), estimate \( P(\theta \mid D) \), find MAP
- Predict \( Class = \text{heads} \) iff \( \theta_{\text{MAP}} > \frac{1}{2} \)
Dear Sir/Madam,
We are pleased to inform you of the result of the Lottery Winners International programs held on the 30/8/2004. Your e-mail address attached to ticket number: EL-23133 with serial Number: EL-123542, batch number: 8/163/EL-35, lottery Ref number: EL-9318 and drew lucky numbers 7-1-8-36-4-22 which consequently won in the 1st category, you have therefore been approved for a lump sum pay out of US$1,500,000.00 (One Million, Five Hundred Thousand United States dollars)

- SPAM

- NOT SPAM?
Representation

- $X = \text{document}$
- Task: Estimate $P(\text{Class} = \{\text{spam, non-spam}\} \mid X)$
- Question: how to represent $X$?
  - Lots of possibilities, common choice: “bag of words”

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...
Bag of Words

- Ignores Word Order, i.e.
  - No emphasis on title
  - No compositional meaning ("Cold War" -> "cold" and "war")
  - Etc.
  - But, massively reduces dimensionality/complexity

- Still and all...
  - Presence or absence of a 100,000-word vocab => $2^{100,000}$ distinct vectors
Naïve Bayes Classifiers

- \( P(Class \mid X) \) for \(|\text{Val}(X)| = 2^{100,000} \) requires \( 2^{100,000} \) parameters
  - Problematic.

- Bayes’ Rule:
  \[
P(Class \mid X) = \frac{P(X \mid Class) \cdot P(Class)}{P(X)}
  \]

- Assume presence of word \( i \) is independent of all other words given \( Class \):
  \[
P(Class \mid X) = \prod_i P(X_i \mid Class) \cdot P(Class) / P(X)
  \]

- Now only 200,001 parameters for \( P(Class \mid X) \)
Naïve Bayes Assumption

- **Features are conditionally independent given class**
  
  \[ \text{Not } P(\text{“Republican”, “Democrat”}) = P(\text{“Republican”})P(\text{“Democrat”}) \]
  
  but instead
  
  \[ P(\text{“Republican”, “Democrat” | Class = Politics}) = P(\text{“Republican” | Class = Politics})P(\text{“Democrat” | Class = Politics}) \]

- **Still, an absurd assumption**
  
  \[ (“Lottery” \perp “Winner” | \text{SPAM})? (“lunch” \perp “noon” | \text{Not SPAM})? \]

- **But: offers massive tractability advantages and works quite well in practice**
  
  Lesson: Unrealistically strong independence assumptions sometimes allow you to build an accurate model where you otherwise couldn’t
Getting the parameters from data

- Parameters $\theta = \langle \theta_{ij} = P(w_i \mid \text{Class} = j) \rangle$
- Maximum Likelihood: Estimate $P(w_i \mid \text{Class} = j)$ from $D$ by counting
  - Fraction of documents in class $j$ containing word $i$
  - But if word $i$ never occurs in class $j$?
- Commonly used MAP estimate:
  - $\frac{(\text{# docs in class } j \text{ with word } i) + 1}{(\text{# docs in class } j) + 2}$
Caveats

- Naïve Bayes effective as a classifier
- Not as effective in producing probability estimates
  - \( \prod_i P(w_i | \text{Class}) \) pushes estimates toward 0 or 1
- In practice, numerical underflow is typical at classification time
  - Compare sum of logs instead of product
Reading

- Elements of Statistical Learning, Ch 7: