Abstract Data Types

CS 214, Fall 2019

What is an ADT?

An ADT defines:

- A set of (abstract) values
- A set of (abstract) operations on those values

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- A set of (abstract) values
- A set of (abstract) operations on those values

An ADT omits:

- How the values are concretely represented
- How the operations work

ADT: Stack

Looks like: $\boxed{|3, 4, 5\rangle}$

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Signature:

- *push*(Stack, Element)
- *pop*(Stack): Element
- *empty?*(Stack): Bool

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Signature:

- *push*(Stack, Element)
- *pop*(Stack): Element
- *empty?*(Stack): Bool

```
interface STACK:
    def push(self, element)
    def pop(self)
    def empty?(self)
```

Stack Interface, with Contracts

Looks like: $\boxed{|3, 4, 5\rangle}$

Signature:

```
interface INT_STACK:
    def push(self, element: int?) -> NoneC
    def pop(self) -> int?
    def empty?(self) -> bool?
```

Stack Interface, with Contracts

Looks like: $\boxed{|3, 4, 5\rangle}$

Signature:

```
interface INT_STACK:
    def push(self, element: int?) -> NoneC
    def pop(self) -> int?
    def empty?(self) -> bool?

interface STACK[T]:
    def push(self, element: T) -> NoneC
    def pop(self) -> T
    def empty?(self) -> bool?
```

ADT: Queue (FIFO)

Looks like: $\boxed{\langle 3, 4, 5 \langle}$

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```
interface QUEUE[T]:
    def enqueue(self, element: T) -> NoneC
    def dequeue(self) -> T
    def empty?(self) -> bool?
```

Stack versus Queue

```
interface STACK[T]:
    def push(self, element: T) -> NoneC
    def pop(self) -> T
    def empty?(self) -> bool?

interface QUEUE[T]:
    def enqueue(self, element: T) -> NoneC
    def dequeue(self) -> T
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Adding laws

$$\{p\}\ f(x) \Rightarrow y\ \{q\}$$

means that if precondition p is true when we apply f to x then we will get y as a result, and postcondition q will be true afterward.

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means that if precondition *p* is true when we apply *f* to *x* then we will get *y* as a result, and postcondition *q* will be true afterward.

Examples:

$$\{a = [2, 4, 6, 8]\}\ a[2] \Rightarrow 6\ \{a = [2, 4, 6, 8]\}$$

$$\{a = [2, 4, 6, 8]\}\ a[2] = 19 \Rightarrow \texttt{None}\ \{a = [2, 4, 19, 8]\}$$

ADT: Stack

Looks like: $\boxed{|3, 4, 5\rangle}$

Signature:

```
interface STACK[T]:
    def push(self, element: T) -> NoneC
    def pop(self) -> T
    def empty?(self) -> bool?
```

Laws:

$$\{\}\ \boxed{|\rangle}.\mathit{empty?}() \Rightarrow \texttt{True}\ \{\}$$

$$\{\}\ \boxed{|e_1, \ldots, e_k, e_{k+1}\rangle}.\mathit{empty?}() \Rightarrow \texttt{False}\ \{\}$$

$$\left\{s = \boxed{|e_1, \ldots, e_k\rangle}\right\}\ s.\mathit{push}(e) \Rightarrow \texttt{None}\ \left\{s = \boxed{|e_1, \ldots, e_k, e\rangle}\right\}$$

$$\left\{s = \boxed{|e_1, \ldots, e_k, e_{k+1}\rangle}\right\}\ s.\mathit{pop}() \Rightarrow e_{k+1}\ \left\{s = \boxed{|e_1, \ldots, e_k\rangle}\right\}$$

ADT: Queue (FIFO)

Looks like: $\boxed{\langle 3, 4, 5 \langle}$

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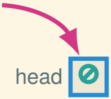
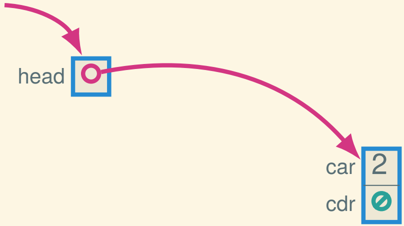
$$\left\{q = \boxed{\langle e_1, e_2, \ldots, e_k \langle}\right\}\ q.\mathit{dequeue}() \Rightarrow e_1\ \left\{q = \boxed{\langle e_2, \ldots, e_k \langle}\right\}$$

Stack implementation: linked list



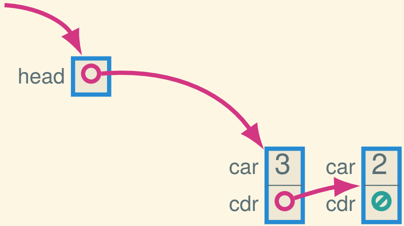
```
let s = ListStack()
```

Stack implementation: linked list



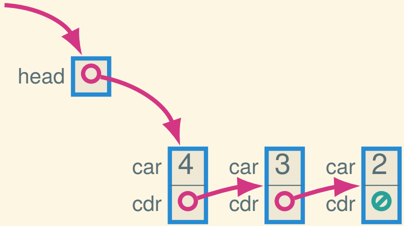
```
let s = ListStack()
s.push(2)
```

Stack implementation: linked list



```
let s = ListStack()
s.push(2)
s.push(3)
```

Stack implementation: linked list



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let s = ListStack()
s.push(2)
s.push(3)
s.push(4)
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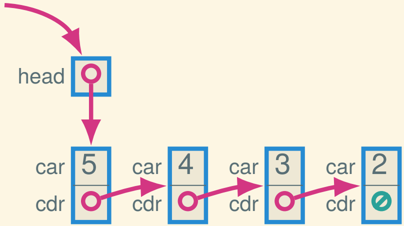

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let s = ListStack()
s.push(2)
s.push(3)
s.push(4)
s.push(5)
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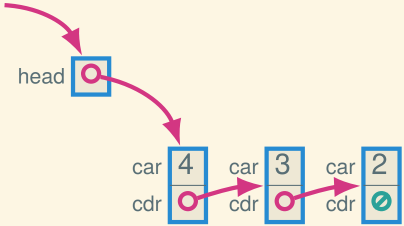

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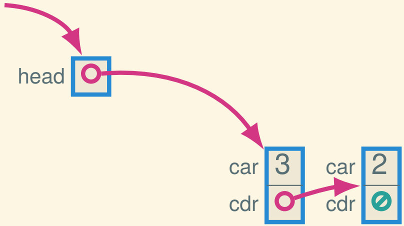
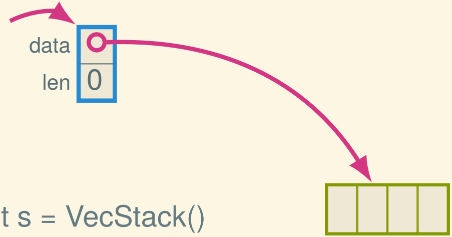
let s = ListStack()
s.push(2)
s.push(3)
s.push(4)
s.push(5)
s.pop()
```

Stack implementation: linked list



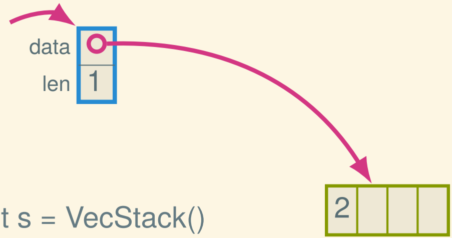
```
let s = ListStack()
s.push(2)
s.push(3)
s.push(4)
s.push(5)
s.pop()
s.pop()
```

Stack implementation: array



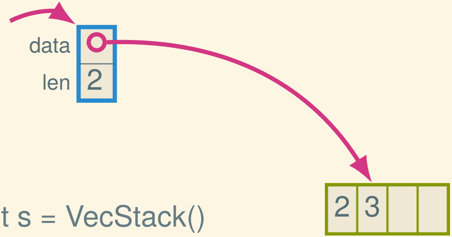
let s = VecStack()

Stack implementation: array



let s = VecStack()
s.push(2)

Stack implementation: array



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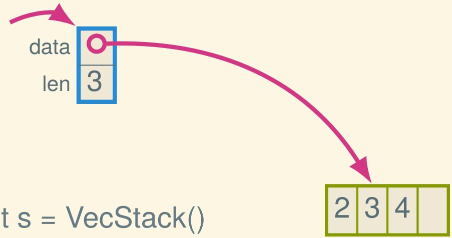
s.push(2)
s.push(3)
```

Stack implementation: array



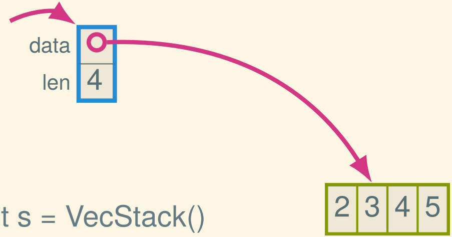
data

len 3

2 3 4

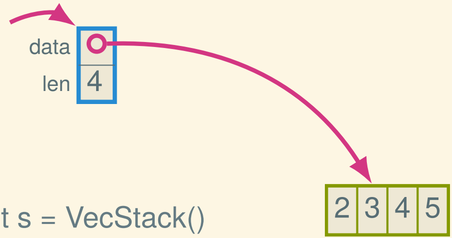
```
let s = VecStack()
s.push(2)
s.push(3)
s.push(4)
```

Stack implementation: array



```
let s = VecStack()
s.push(2)
s.push(3)
s.push(4)
s.push(5)
```

Stack implementation: array



```
let s = VecStack()
s.push(2)
s.push(3)
s.push(4)
s.push(5)
s.push(6)
```

ADT: Stack

Looks like: $\boxed{|3, 4, 5\rangle}$

Signature:

```
interface STACK[T]:
    def push(self, element: T) -> NoneC  # O(1)
    def pop(self) -> T                   # O(1)
    def empty?(self) -> bool?            # O(1)
```

Laws:

$$\{\}\ \boxed{|\rangle}.\mathit{empty?}() \Rightarrow \texttt{True}\ \{\}$$

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Trade-offs: linked list stack versus array stack

- Linked list stack only fills up when memory fills up, whereas array stack has a fixed size (or must reallocate)
- Array stack has better constant factors: cache locality and no (or rare) allocation
- Array stack space usage is tighter; linked list is smoother

ADT: Queue (FIFO)

Looks like: $\boxed{\langle 3, 4, 5 \langle}$

Signature:

```
interface QUEUE[T]:
    def enqueue(self, element: T) -> NoneC  # O(1)
    def dequeue(self) -> T                  # O(1)
    def empty?(self) -> bool?               # O(1)
```

Laws:

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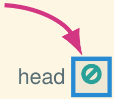
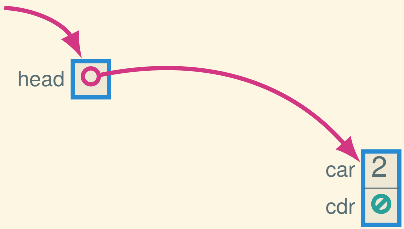
Queue implementation: linked list?



head

```
let q = LinkedListQueue()
```

Queue implementation: linked list?



```
let q = LinkedListQueue()
q.enqueue(2)
```

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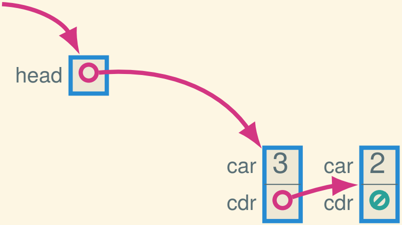

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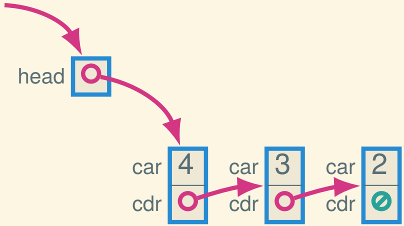

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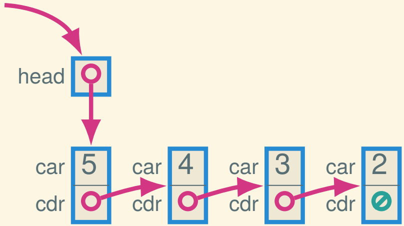

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let q = LinkedListQueue()
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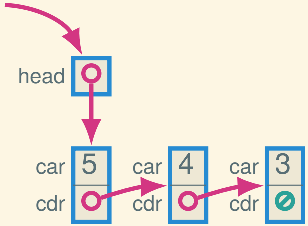

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let q = LinkedListQueue()
q.enqueue(2)
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q.enqueue(4)
q.enqueue(5)
q.dequeue()
```

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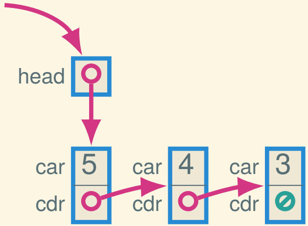

```
let q = LinkedListQueue()
q.enqueue(2)
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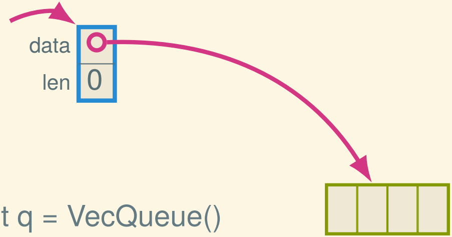
q.enqueue(5)
q.dequeue() — $\mathcal{O}(n)$???
```

Queue implementation: array?



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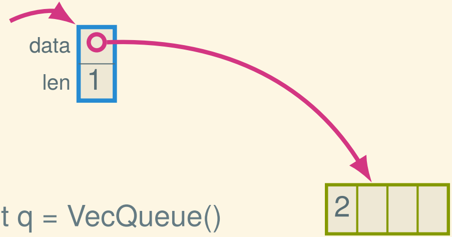
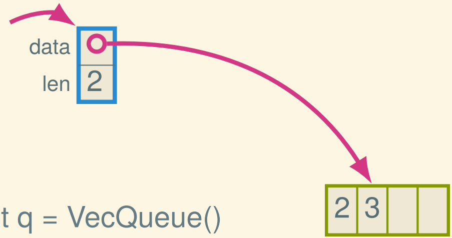
let q = VecQueue()
```

Queue implementation: array?



```
let q = VecQueue()
q.enqueue(2)
```

Queue implementation: array?



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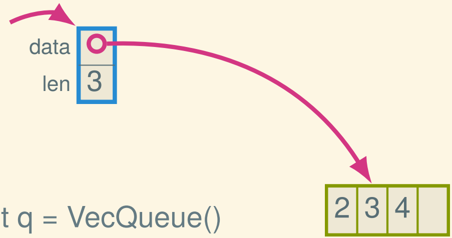

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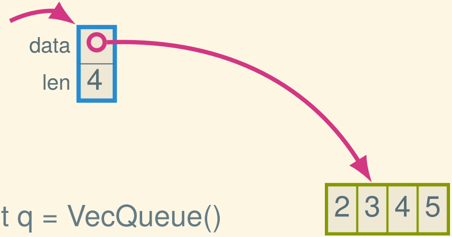

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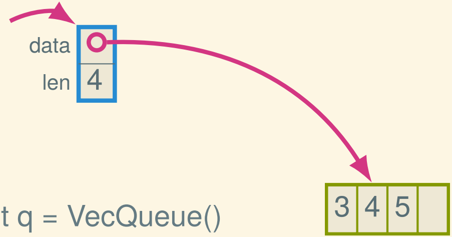

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let q = VecQueue()
q.enqueue(2)
q.enqueue(3)
q.enqueue(4)
q.enqueue(5)
q.dequeue()
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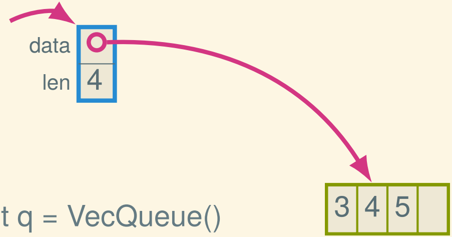
Queue implementation: array?



let q = VecQueue()
q.enqueue(2)
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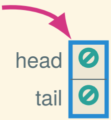
q.dequeue() — $\mathcal{O}(n)$???

Queue impl.: linked list with tail pointer



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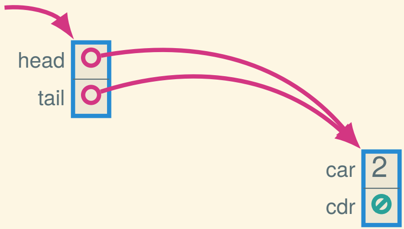

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let q = LinkedListQueue()
q.enqueue(2)
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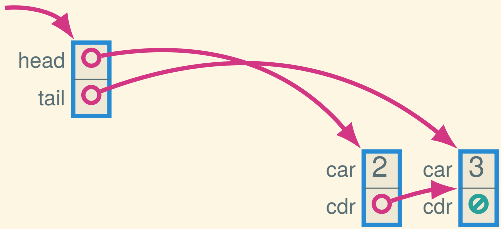

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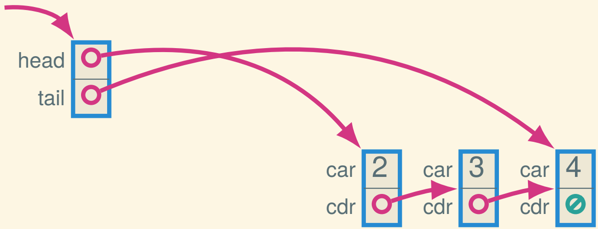

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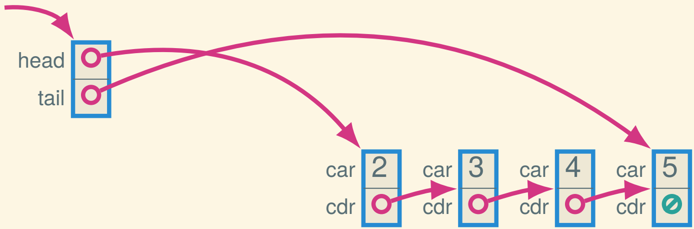

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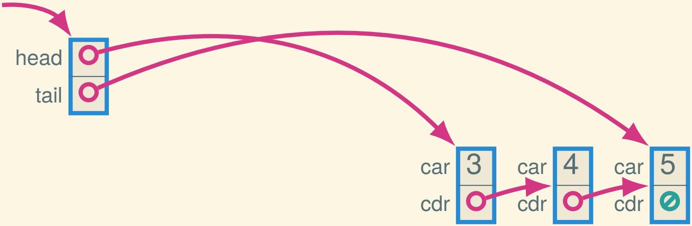

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let q = LinkedListQueue()
q.enqueue(2)
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q.dequeue()
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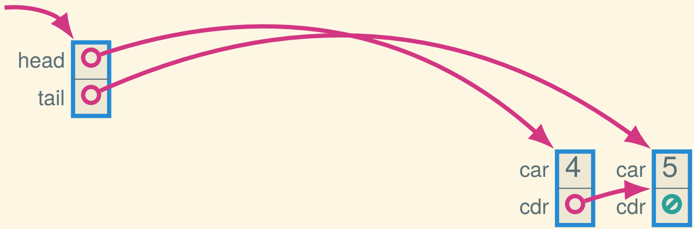
Queue impl.: linked list with tail pointer



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q.enqueue(2)
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q.dequeue()
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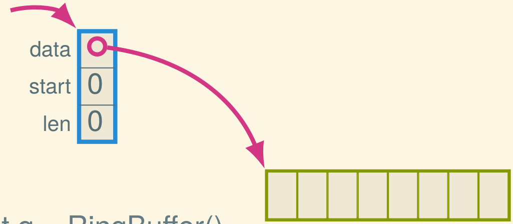
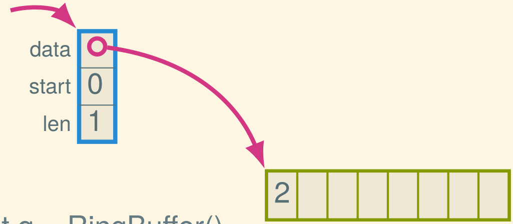
```

Queue implementation: ring buffer



let q = RingBuffer()

Queue implementation: ring buffer



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q.enqueue(2)
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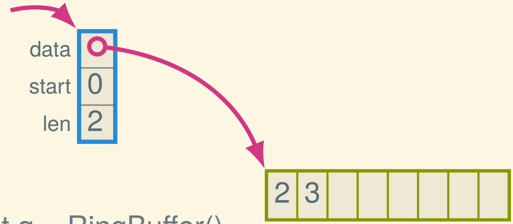
Queue implementation: ring buffer



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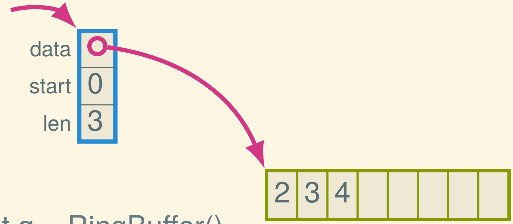

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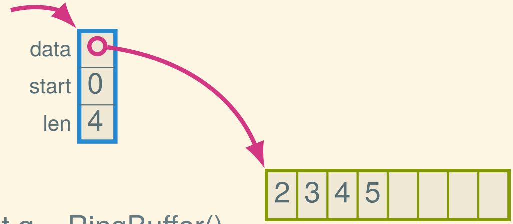

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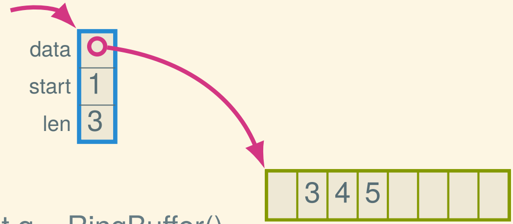

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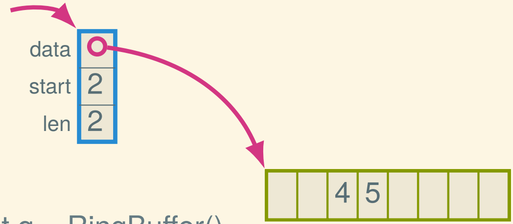
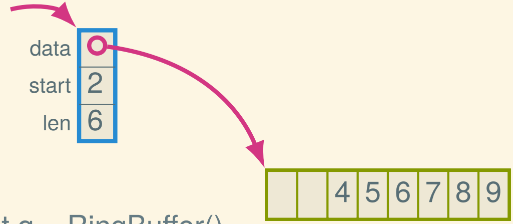
q.enqueue(4)
q.enqueue(5)
q.dequeue()
```

Queue implementation: ring buffer



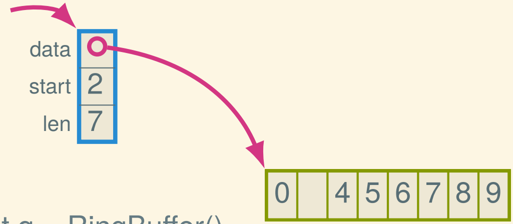
```
let q = RingBuffer()
q.enqueue(2)
q.enqueue(3)
q.enqueue(4)
q.enqueue(5)
q.dequeue()
q.dequeue()
```

Queue implementation: ring buffer



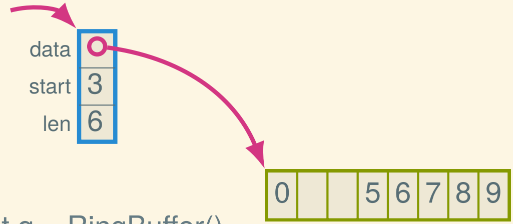
```
let q = RingBuffer()
q.enqueue(2)
q.enqueue(3)
q.enqueue(4)
q.enqueue(5)
q.dequeue()
q.dequeue()
⋮
```

Queue implementation: ring buffer



```
let q = RingBuffer()
q.enqueue(2)
q.enqueue(3)
q.enqueue(4)
q.enqueue(5)
q.dequeue()
q.dequeue()
⋮
q.enqueue(0)
```

Queue implementation: ring buffer



```
let q = RingBuffer()
q.enqueue(2)
q.enqueue(3)
q.enqueue(4)
q.enqueue(5)
q.dequeue()
q.dequeue()
⋮
q.enqueue(0)
q.dequeue()
```

Trade-offs: linked list queue versus ring buffer

Basically the same as for the stack implementations:

- Ring buffer has better constant factors and uses less space (potentially)
- Linked list doesn’t fill up

Ring buffer in DSSL2

Signature, with `full?`

```
interface QUEUE[T]:
    def enqueue(self, element: T) -> NoneC
    def dequeue(self) -> T
    def empty?(self) -> bool?
    def full?(self) -> bool?
```

Representation and initialization

```
class RingBuffer (QUEUE):
    let data: VecC
    let start: nat?
    let size: nat?

    def __init__(self, capacity):
        self.data = [None; capacity]
        self.start = 0
        self.size = 0

    …
```

Size stuff

```
class RingBuffer (QUEUE):
    let data: VecC
    let start: nat?
    let size: nat?
    …

    def cap(self):
        self.data.len()

    def len(self):
        self.size

    def empty?(self):
        self.len() == 0

    def full?(self):
        self.len() == self.cap()

    …
```

Enqueueing

```
class RingBuffer (QUEUE):
    let data: VecC
    let start: nat?
    let size: nat?
    …

    def enqueue(me, value):
        if me.full?():
            error('RingBuffer.enqueue: full')
        let ix = (me.start + me.size) % me.cap()
        me.data[ix] = value
        me.size = me.size + 1



    …
```

Dequeueing

```
class RingBuffer (QUEUE):
    let data: VecC
    let start: nat?
    let size: nat?
    …

    def dequeue(me):
        if me.empty?():
            error('RingBuffer.dequeue: empty')
        let result = me.data[me.start]
        me.start = (me.start + 1) % me.cap()
        me.size = me.size - 1
        result


    …
```

Dequeueing without Leaking

```
class RingBuffer (QUEUE):
    let data: VecC
    let start: nat?
    let size: nat?
    …

    def dequeue(me):
        if me.empty?():
            error('RingBuffer.dequeue: empty')
        let result = me.data[me.start]
        me.data[me.start] = None
        me.start = (me.start + 1) % me.cap()
        me.size = me.size − 1
        result

    …
```

Next time: BSTs and the Dictionary ADT