Let’s talk trees
Definition

A tree is a graph with no cycles:
Rooted trees

We can *root* a tree by choosing one vertex to be the root:

This lets us talk about *children* and *subtrees*
Rooted, ordered trees

An *ordered* tree assigns an order to the children of each node:

Now we can refer to the 1st child, 2nd child, etc.
In a $k$-ary tree, each node has at most $k$ children:

- a 3-ary tree
- a 2-ary tree
A rose tree is an \( \infty \)-ary tree:
Full trees

A $k$-ary tree is full if every non-leaf node has $k$ children:

full binary tree

not full
Complete trees

A tree is complete if every level is full of nodes except the last, which must be filled from the left:
Complete trees

A tree is complete if every level is full of nodes except the last, which must be filled from the left:
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What’s a tree walk?

A tree walk traverses a tree and linearizes the vertices in some order
Pre-order walk

Visit each node *before* its children:

Pre-order: 17
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7
Pre-order walk

Visit each node \textit{before} its children:

Pre-order: 17, 11, 6, 3, 7
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16
Pre-order walk

Visit each node before its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16, 33
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16, 33, 24
Pre-order walk

Visit each node \textit{before} its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16, 33, 24, 22
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16, 33, 24, 22
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16, 33, 24, 22
Pre-order walk

Visit each node before its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16, 33, 24, 22, 36
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16, 33, 24, 22, 36
Pre-order walk

Visit each node *before* its children:

Pre-order: 17, 11, 6, 3, 7, 15, 13, 16, 33, 24, 22, 36
Post-order walk

Visit each node after its children:

Post-order:
Post-order walk

Visit each node after its children:

Post-order:
Post-order walk

Visit each node *after* its children:

Post-order:
Post-order walk

Visit each node *after* its children:

Post-order: 3
Post-order walk

Visit each node \textit{after} its children:

Post-order: 3
Post-order walk

Visit each node *after* its children:

Post-order: 3, 7
Post-order walk

Visit each node *after* its children:

Post-order: 3, 7, 6
Post-order walk

Visit each node after its children:

Post-order: 3, 7, 6
Post-order walk

Visit each node *after* its children:

Post-order: 3, 7, 6
Post-order walk

Visit each node *after* its children:

Post-order: 3, 7, 6, 13
Post-order walk

Visit each node after its children:

Post-order: 3, 7, 6, 13
Post-order walk

Visit each node _after_ its children:

Post-order: 3, 7, 6, 13, 16
Post-order walk

Visit each node *after* its children:

Post-order: 3, 7, 6, 13, 16, 15
Post-order walk

Visit each node after its children:

Post-order: 3, 7, 6, 13, 16, 15, 11
Post-order walk

Visit each node *after* its children:

Post-order: 3, 7, 6, 13, 16, 15, 11
Post-order walk

Visit each node after its children:

Post-order: 3, 7, 6, 13, 16, 15, 11
Post-order walk

Visit each node after its children:

Post-order: 3, 7, 6, 13, 16, 15, 11
Post-order walk

Visit each node after its children:

Post-order: 3, 7, 6, 13, 16, 15, 11, 22
Post-order walk

Visit each node after its children:

Post-order: 3, 7, 6, 13, 16, 15, 11, 22, 24
Post-order walk

Visit each node after its children:

Post-order: 3, 7, 6, 13, 16, 15, 11, 22, 24
Post-order walk

Visit each node *after* its children:

Post-order: 3, 7, 6, 13, 16, 15, 11, 22, 24, 36
Post-order walk

Visit each node *after* its children:

Post-order: 3, 7, 6, 13, 16, 15, 11, 22, 24, 36, 33
Post-order walk

Visit each node after its children:

Post-order: 3, 7, 6, 13, 16, 15, 11, 22, 24, 36, 33, 17
In-order walk

Visit each node *between* its children:

In-order:
In-order walk

Visit each node *between* its children:

In-order:
**In-order walk**

Visit each node *between* its children:

In-order:
In-order walk

Visit each node *between* its children:

In-order: 3
In-order walk

Visit each node *between* its children:

In-order: 3, 6
In-order walk
Visit each node *between* its children:

In-order: 3, 6, 7
In-order walk

Visit each node *between* its children:

![Diagram of a binary tree with in-order traversal values: 3, 6, 7]

In-order: 3, 6, 7
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11
In-order walk

Visit each node *between* its children:

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In-order: 3, 6, 7, 11, 13
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In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16, 17
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16, 17
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16, 17
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16, 17, 22
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16, 17, 22, 24
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16, 17, 22, 24, 33
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16, 17, 22, 24, 33, 36
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16, 17, 22, 24, 33, 36
In-order walk

Visit each node *between* its children:

In-order: 3, 6, 7, 11, 13, 15, 16, 17, 22, 24, 33, 36
Tree walk pseudocode

Procedure
PreOrder(node) is
  if node is not null then
    visit node;
    PreOrder(node.left);
    PreOrder(node.right)
end

Procedure
PostOrder(node) is
  if node is not null then
    PostOrder(node.left);
    visit node
    PostOrder(node.right);
end

Procedure
InOrder(node) is
  if node is not null then
    InOrder(node.left);
    visit node;
    InOrder(node.right)
end
Level-order walk

Visit all of each level before the next level:

Level-order: 17
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11, 33
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11, 33, 6
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11, 33, 6, 15
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11, 33, 6, 15, 24
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11, 33, 6, 15, 24, 36
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11, 33, 6, 15, 24, 36, 3
Level-order walk

Visit all of each level before the next level:

Level-order:  17, 11, 33, 6, 15, 24, 36, 3, 7
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11, 33, 6, 15, 24, 36, 3, 7, 13
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11, 33, 6, 15, 24, 36, 3, 7, 13, 16
Level-order walk

Visit all of each level before the next level:

Level-order: 17, 11, 33, 6, 15, 24, 36, 3, 7, 13, 16, 22
Level-order pseudocode

We use a queue (FIFO) to visit the nodes level-by-level:

Procedure LevelOrder(root) is
    queue ← a new queue;
    Enqueue(queue, root);
    while queue is not empty do
        node ← Dequeue(queue);
        if node is not null then
            visit node;
            Enqueue(queue, node.left);
            Enqueue(queue, node.right);
        end
    end
end
Representing trees
Structs for $k$-ary trees
Rose trees using arrays
Complete binary trees in level order in an array

A very special case:
Complete binary trees in level order in an array

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A very special case:

```
  17
 /   \
11    33
 / \
 6 15
 /\  /
3 7 13 16
/ \     /
3 13 16 22
```

```
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
17 11 33 6 15 24
```
Complete binary trees in level order in an array

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Next time: graphs