Graph Search

CS 214, Fall 2019

Questions we might ask about graphs

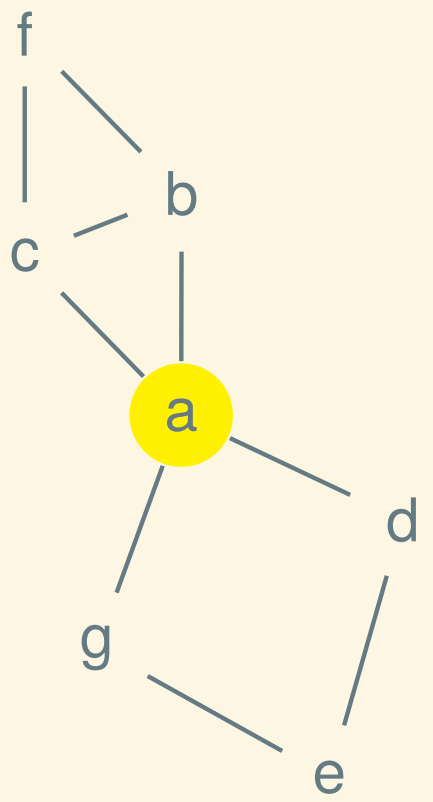
- Is there a path from v to u?
- What's the shortest path from v to u?
- Are there any cycles?

Graph search: basic idea

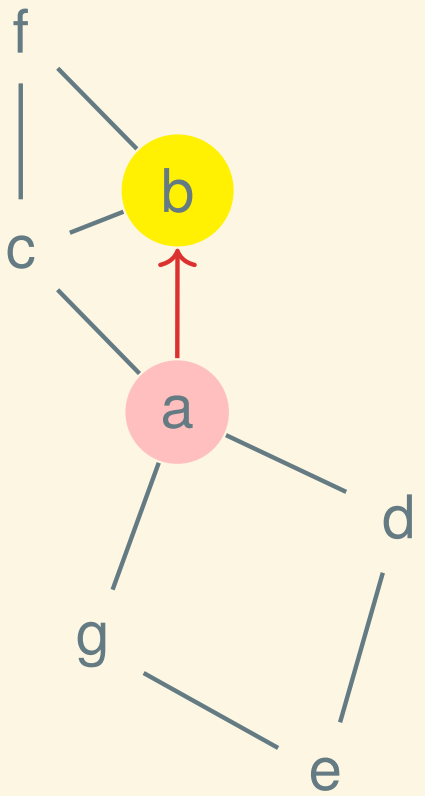
To answer whether there's a path (among other things), we can use:

- Depth-first search (DFS): go as far as you can along a path, then go back and try anything you haven't tried yet
- Breadth-first search (BFS): explore all the successors of a vertex before exploring their successors in turn

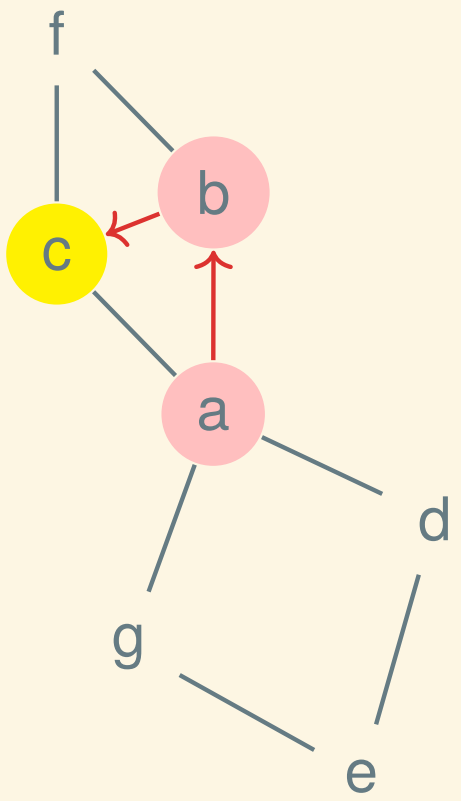
DFS example



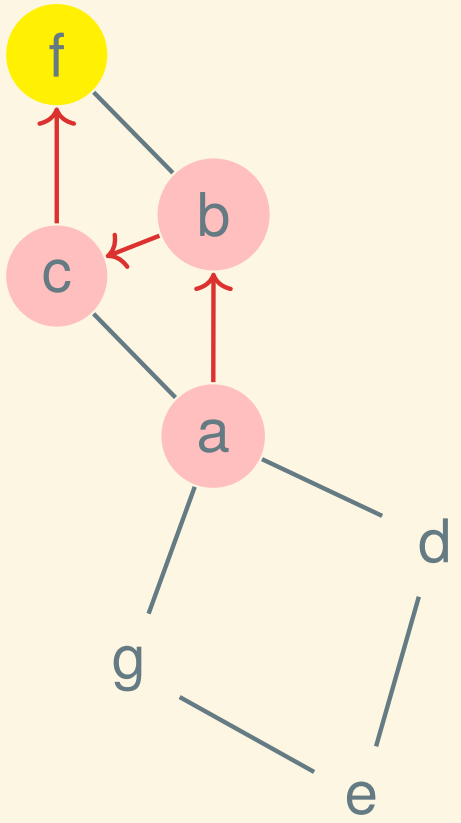
DFS example



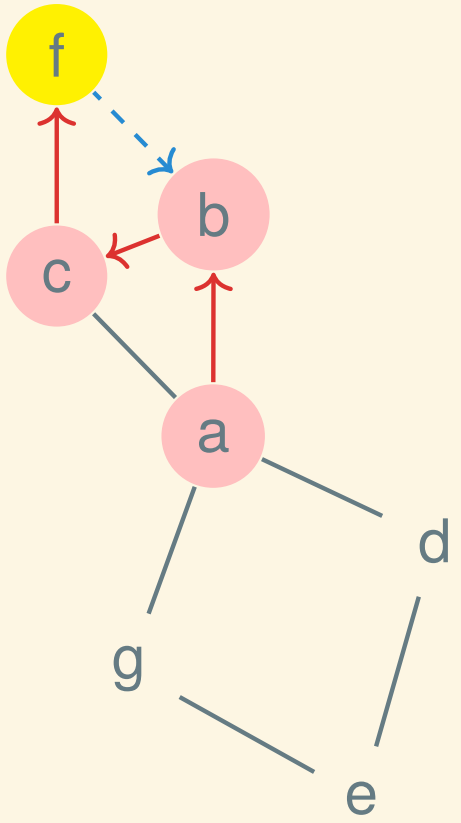
DFS example



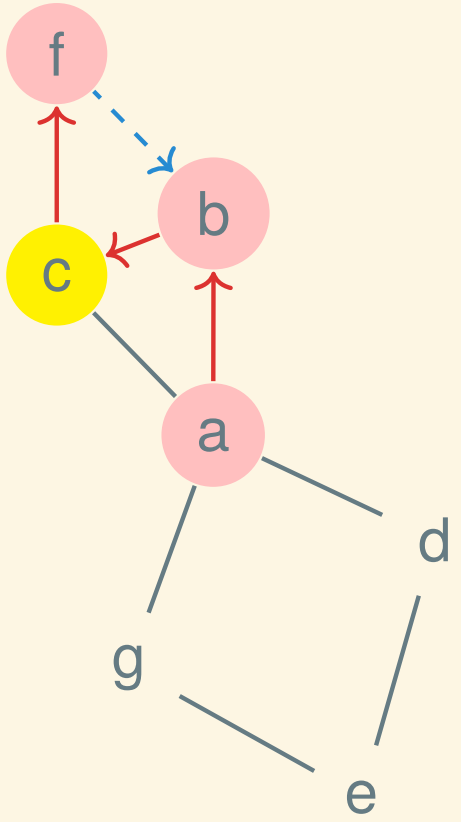
DFS example



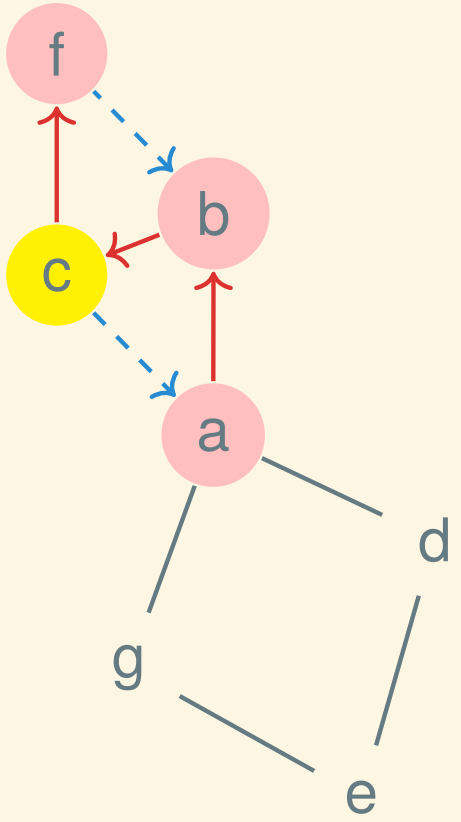
DFS example



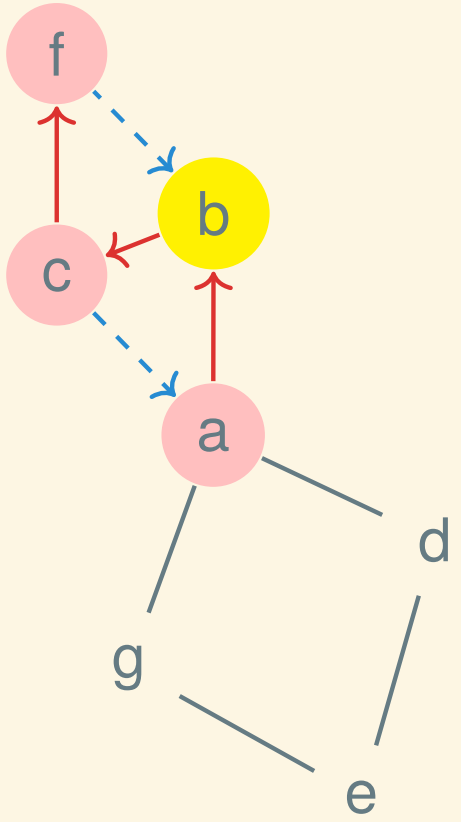
DFS example



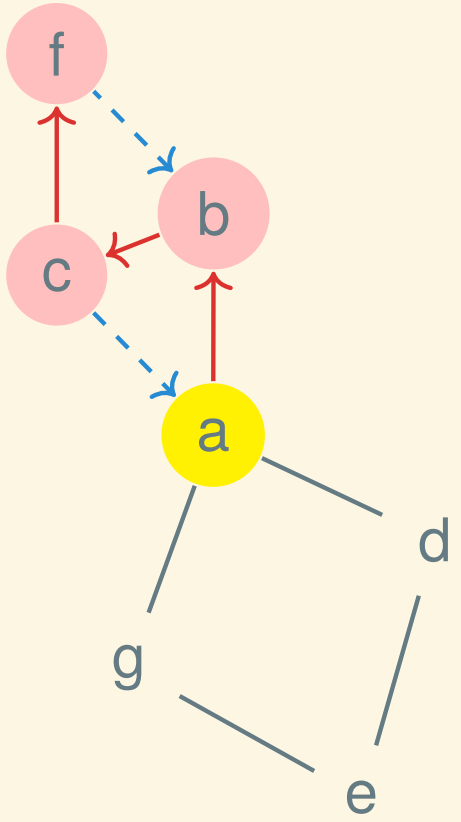
DFS example



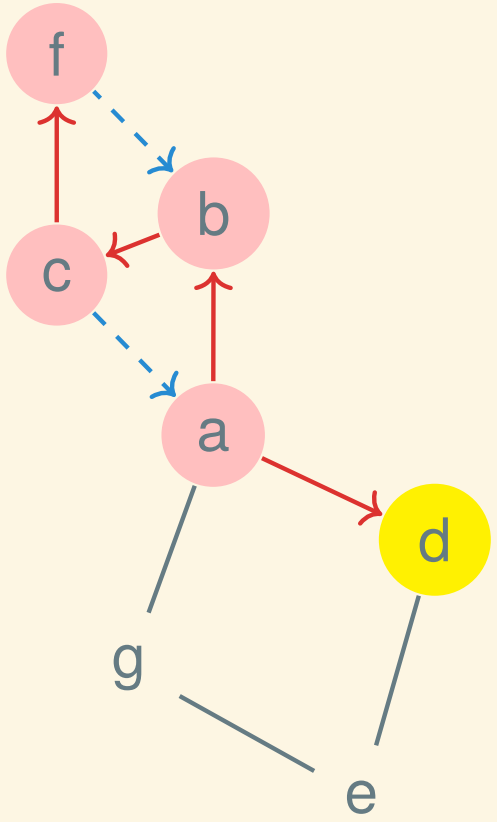
DFS example



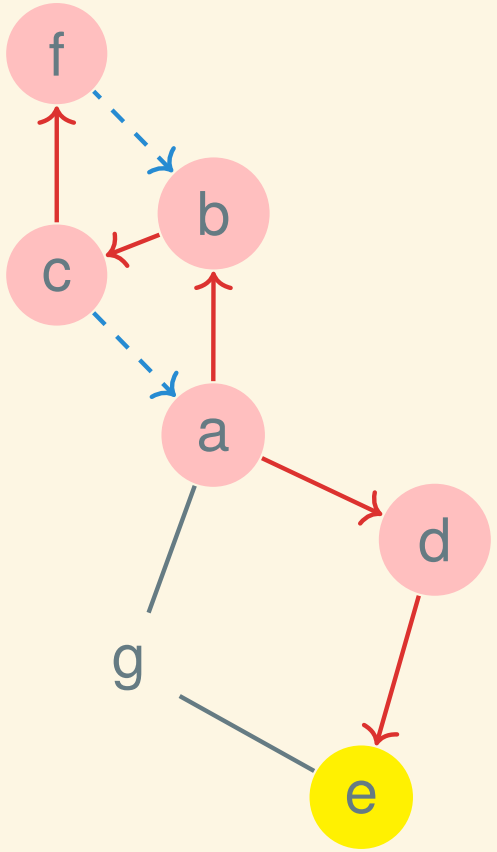
DFS example



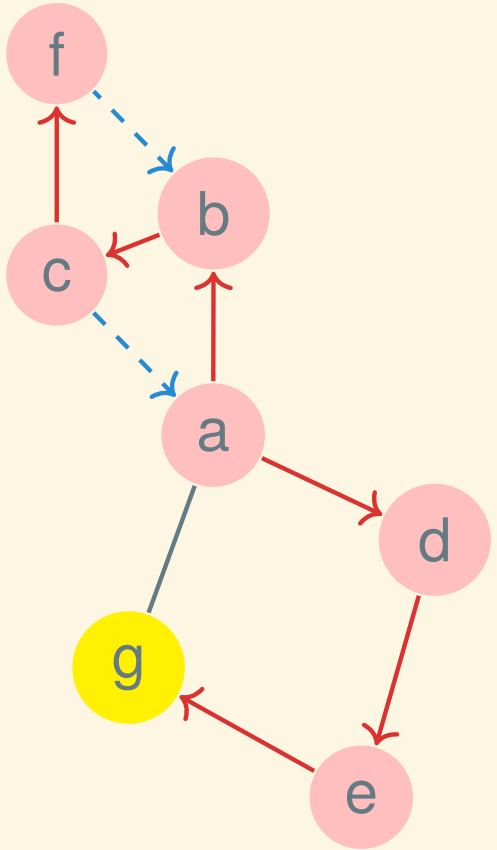
DFS example



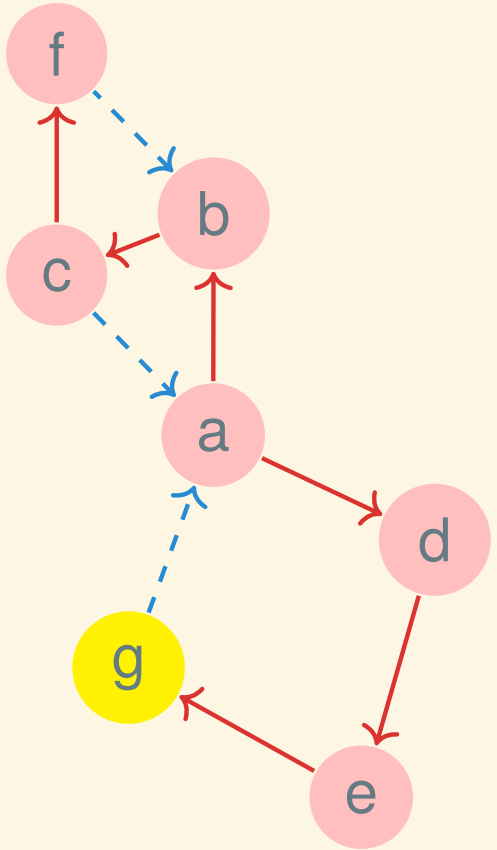
DFS example



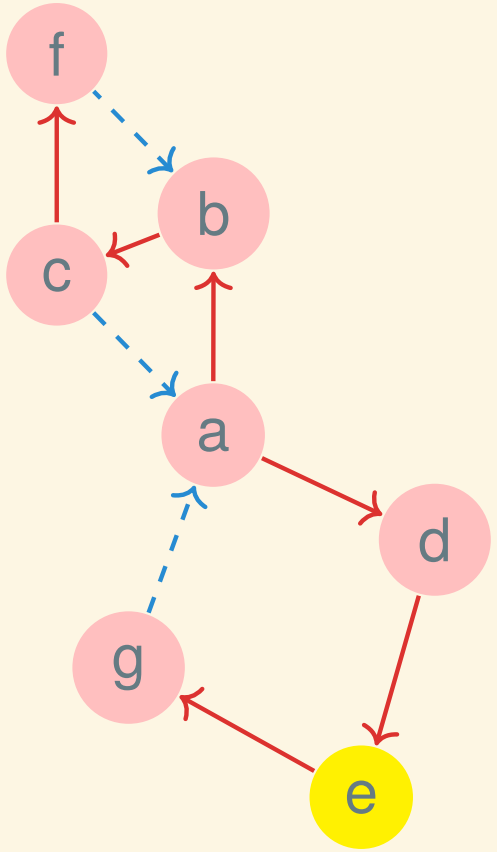
DFS example



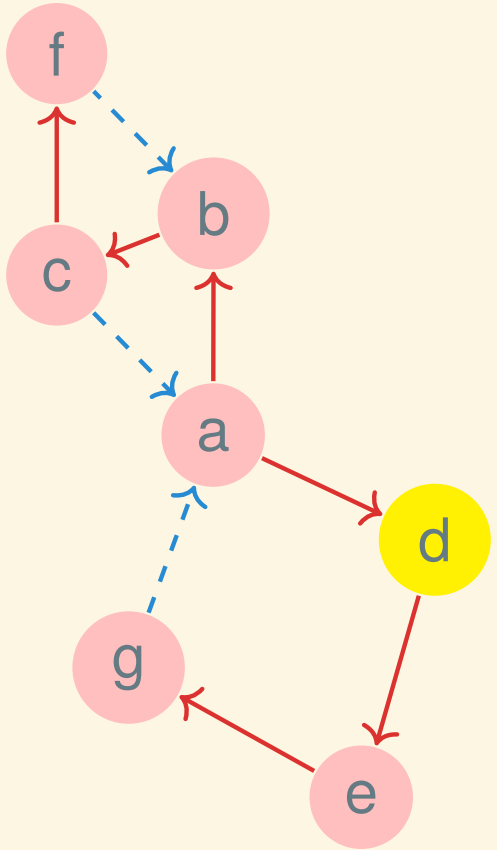
DFS example



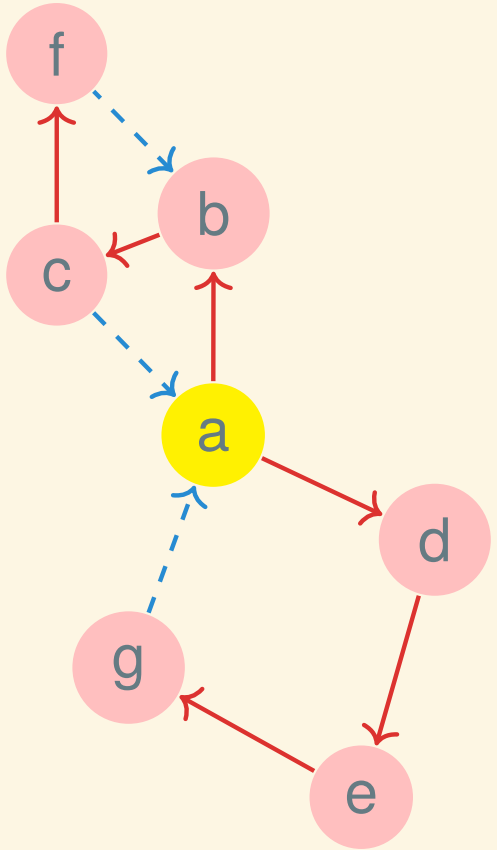
DFS example



DFS example



DFS example



Recursive DFS algorithm (one source)

```
Procedure DFS(graph, start) is
    seen ← new array (same size as graph, filled with false);
    Procedure Visit(v) is
        if not seen[v] then
            seen[v] ← true;
            for u in Successors(graph, v) do
                Visit(u)
            end
        end
    end
    Visit(start);
    return seen
end
```

Recursive DFS algorithm (one source, lifted)

```
Procedure Visit(graph, seen, v) is
    if not seen[v] then
        seen[v] ← true;
        for u in Successors(graph, v) do
            Visit(graph, seen, u)
        end
    end
end

Procedure DFS(graph, start) is
    seen ← new array (same size as graph, filled with false);
    Visit(graph, seen, start);
    return seen
end
```

Recursive DFS algorithm (1 src., builds tree)

```
Procedure DFS(graph, start) is
    preds ← new array (same size as graph, filled with false);
    Procedure Visit(pred, v) is
        if not preds[v] then
            preds[v] ← pred;
            for u in Successors(graph, v) do
                Visit(v, u)
            end
        end
    end
    Visit(true, start);
    return preds
end
```

Recursive DFS algorithm (full)

```
Procedure DFS(graph) is
    preds ← new array (same size as graph, filled with false);

    Procedure Visit(pred, v) is
        if not preds[v] then
            preds[v] ← pred;
            for u in Successors(graph, v) do
                Visit(v, u)
            end
        end
    end

    for v in Vertices(graph) do
        Visit(true, v)
    end
    return preds
end
```

Iterative DFS algorithm

```
Procedure DFS(graph, start) is
    preds ← new array (same size as graph, filled with false);
    todo ← new stack;

    preds[start] ← true;
    Push(todo, start);

    while todo is not empty do
        v ← Pop(todo);
        for u in Successors(graph, v) do
            if not preds[u] then
                preds[u] ← v;
                Push(todo, u)
            end
        end
    end
    return preds
end
```

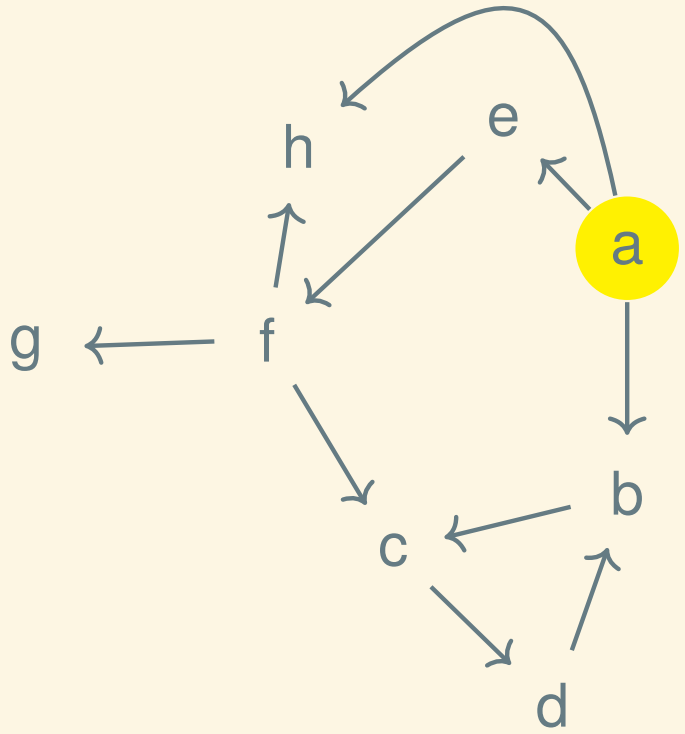
Running DFS on a digraph

tree

back

cross

forward



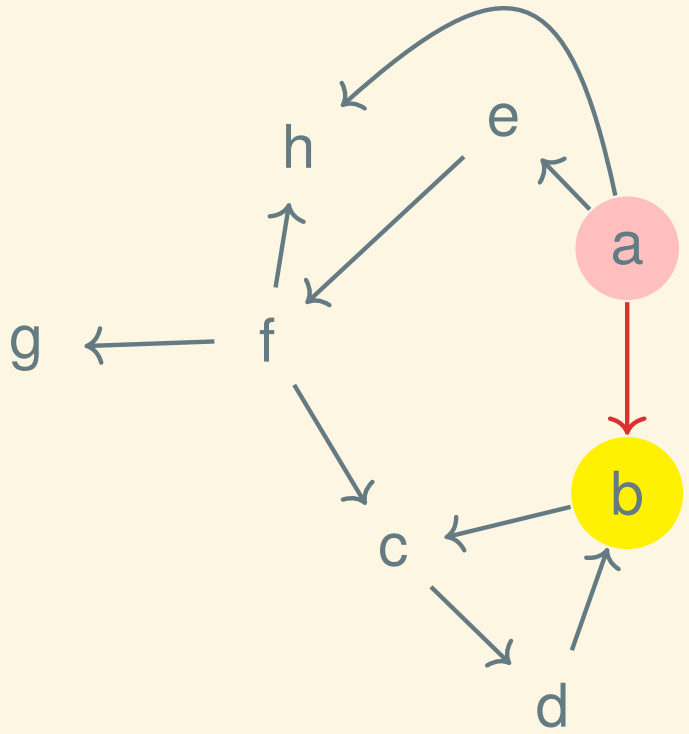
Running DFS on a digraph

tree

back

cross

forward



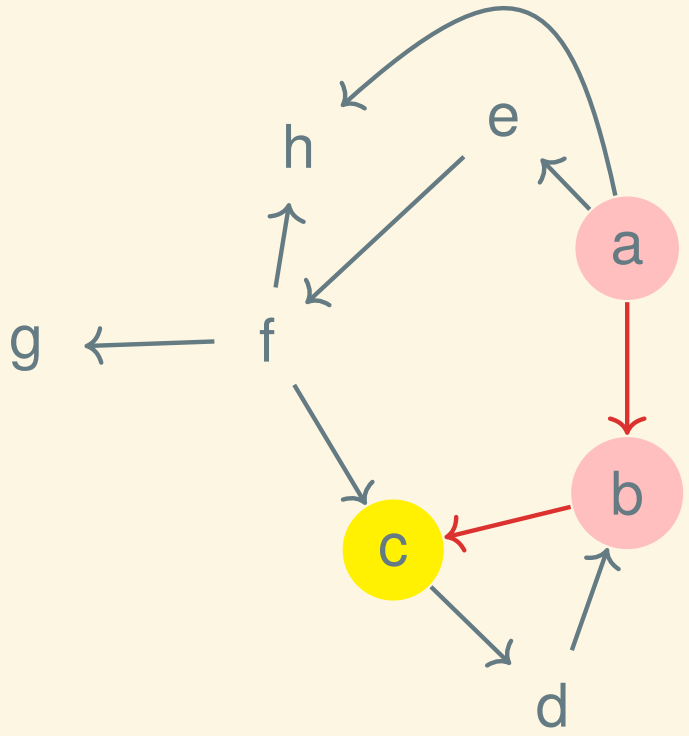
Running DFS on a digraph

tree

back

cross

forward



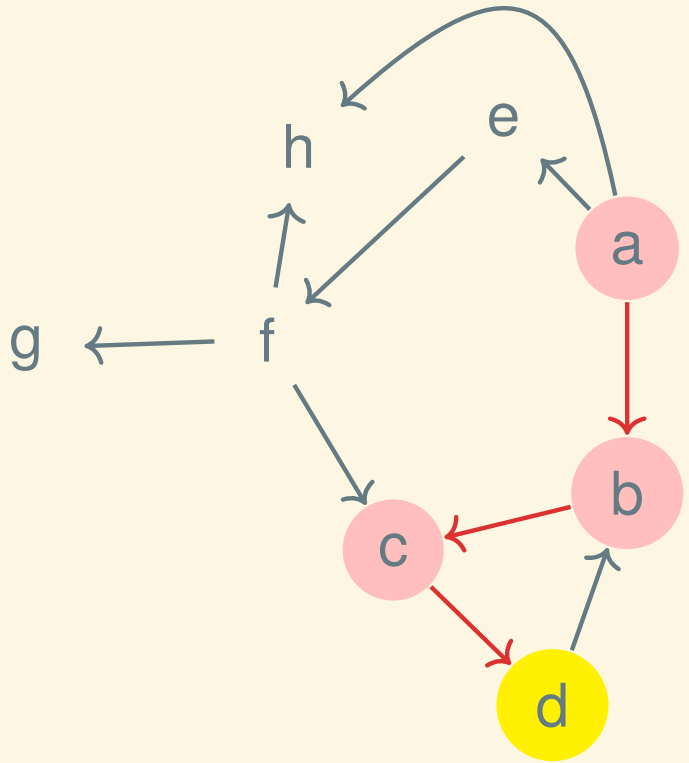
Running DFS on a digraph

tree

back

cross

forward



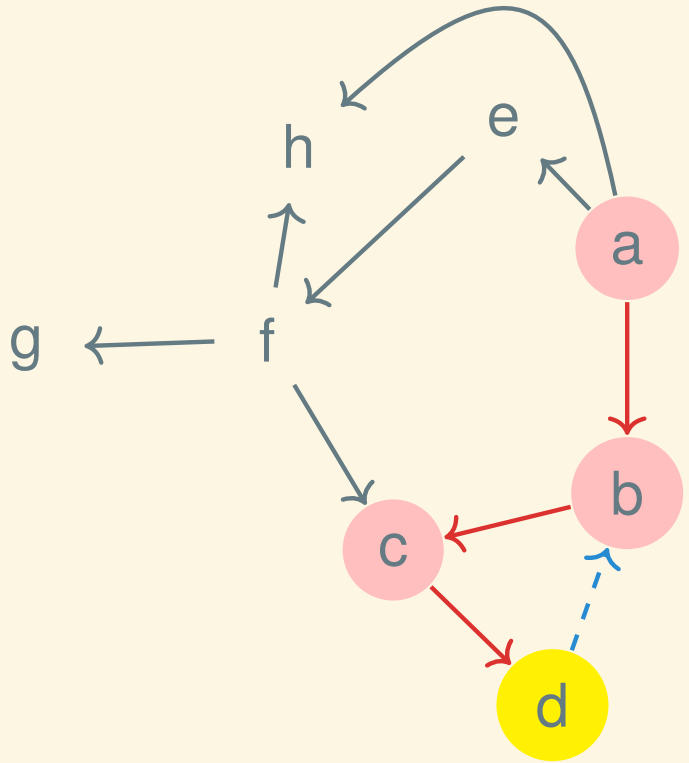
Running DFS on a digraph

tree

back

cross

forward



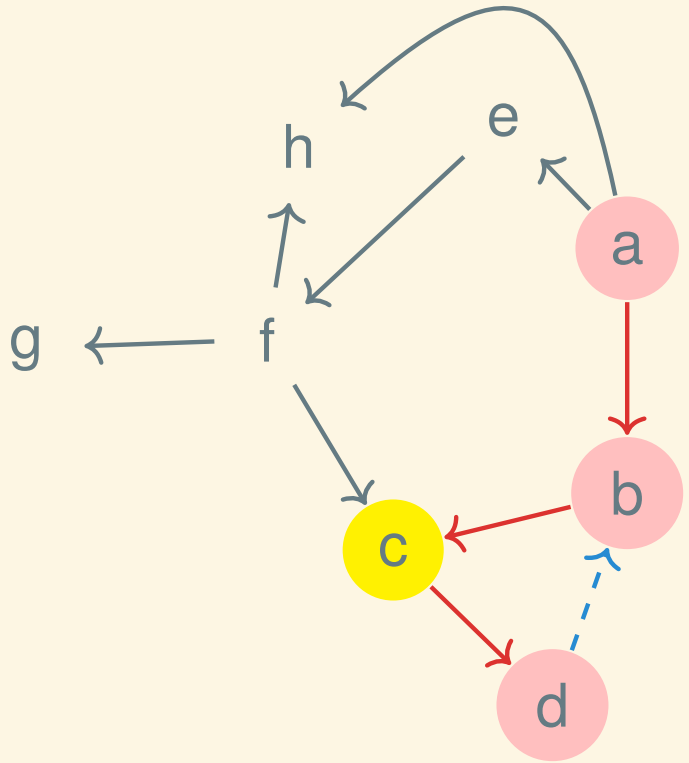
Running DFS on a digraph

tree

back

cross

forward



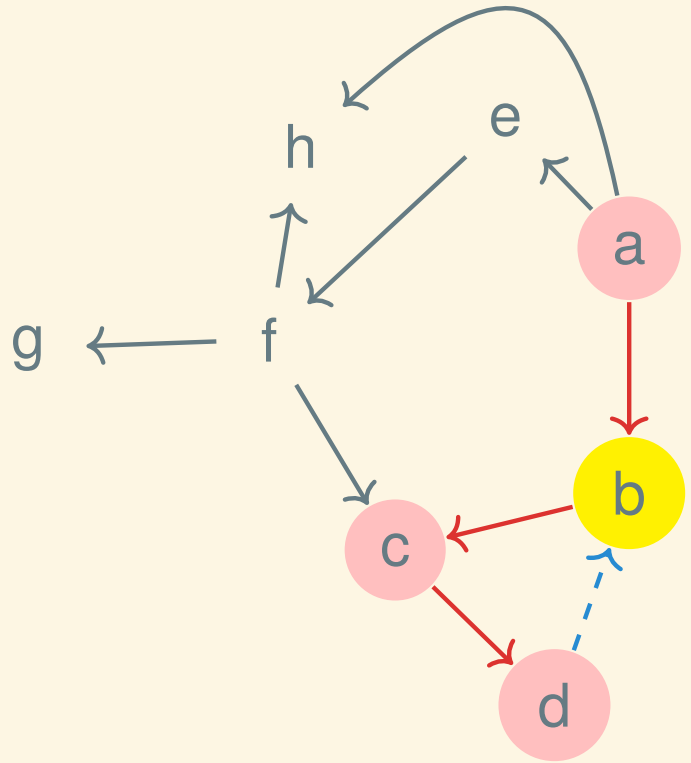
Running DFS on a digraph

tree

back

cross

forward



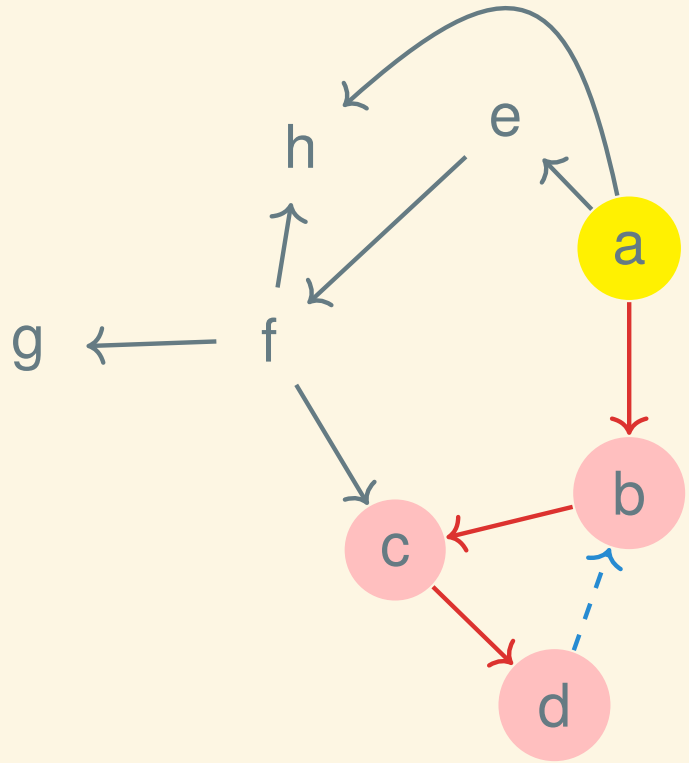
Running DFS on a digraph

tree

back

cross

forward



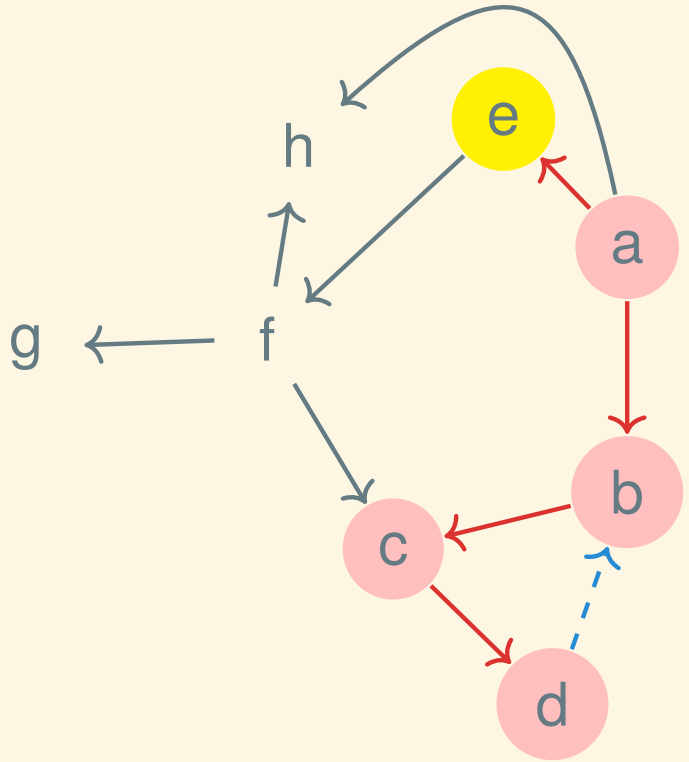
Running DFS on a digraph

tree

back

cross

forward



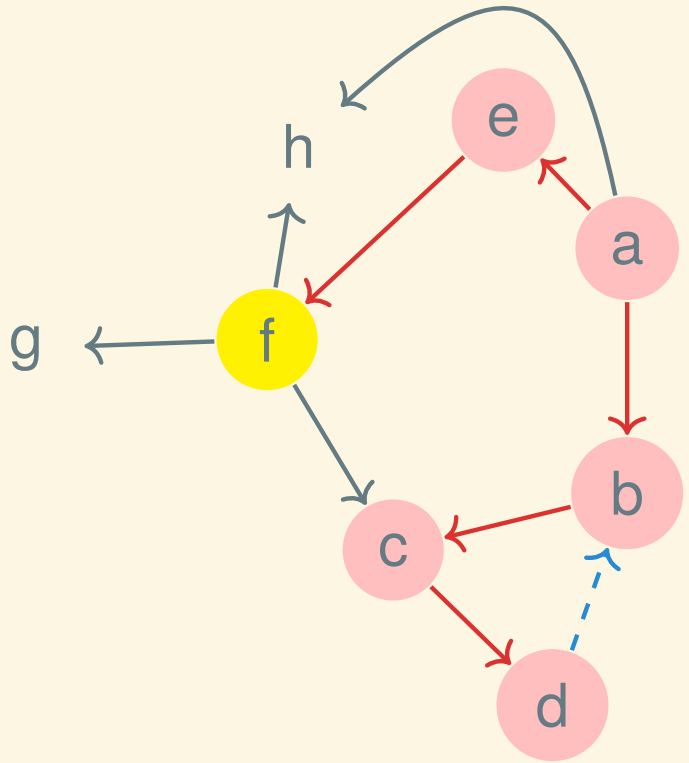
Running DFS on a digraph

tree

back

cross

forward



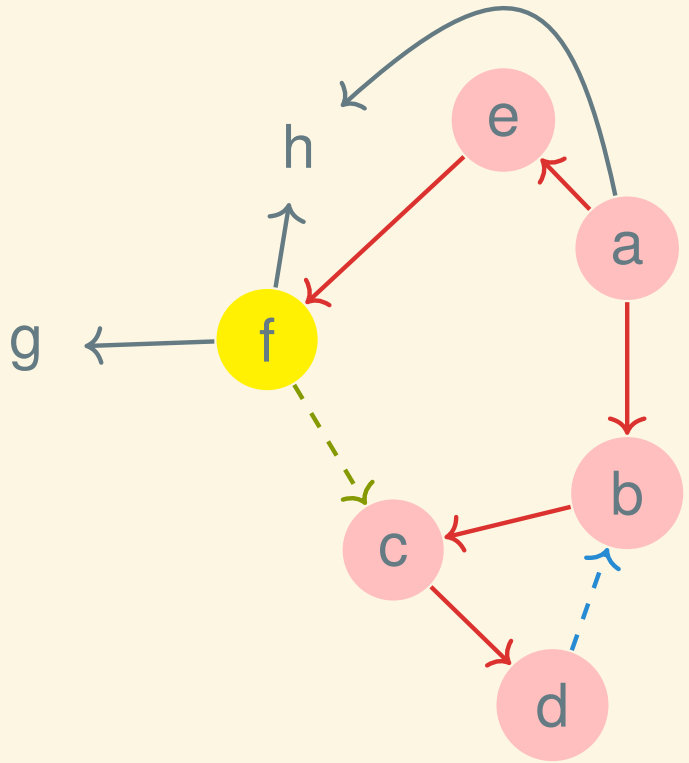
Running DFS on a digraph

tree

back

cross

forward



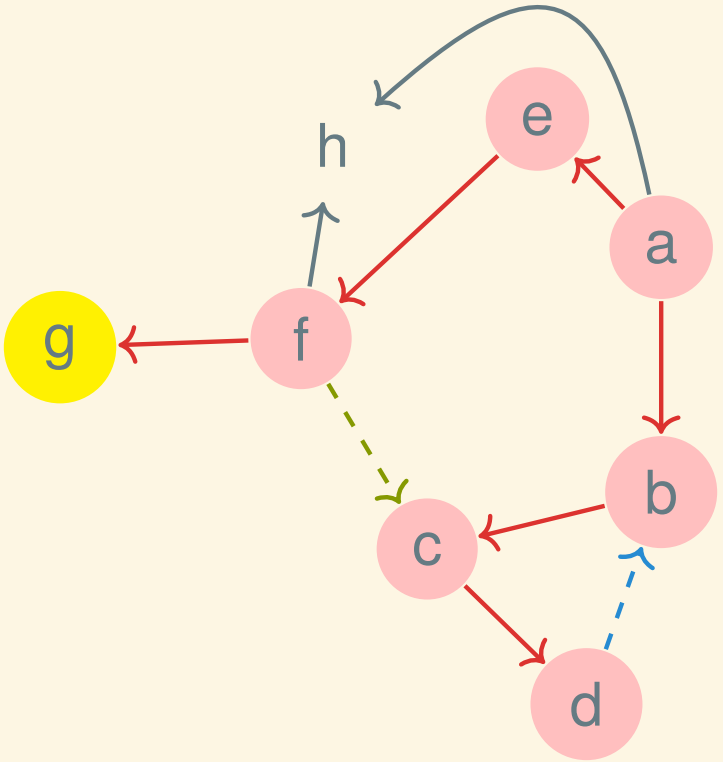
Running DFS on a digraph

tree

back

cross

forward



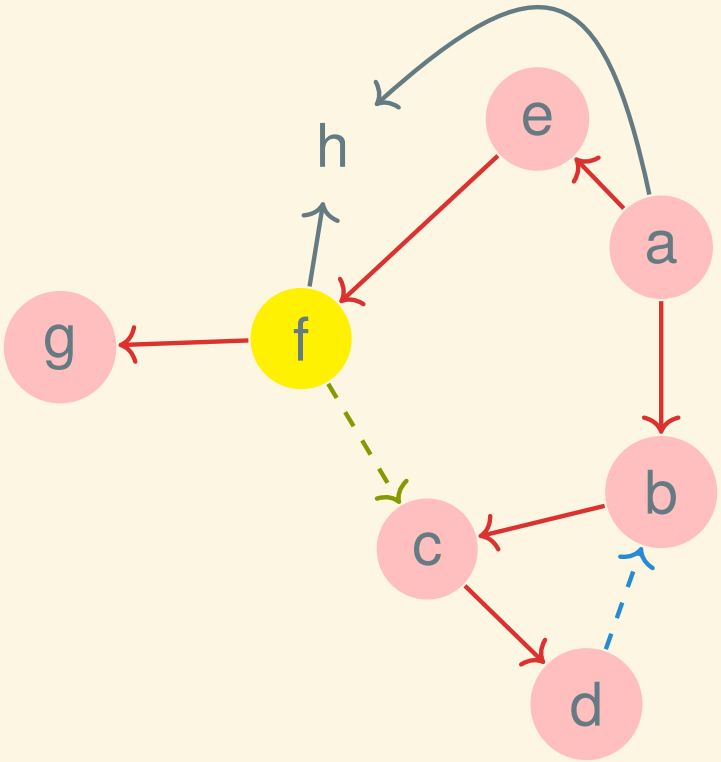
Running DFS on a digraph

tree

back

cross

forward



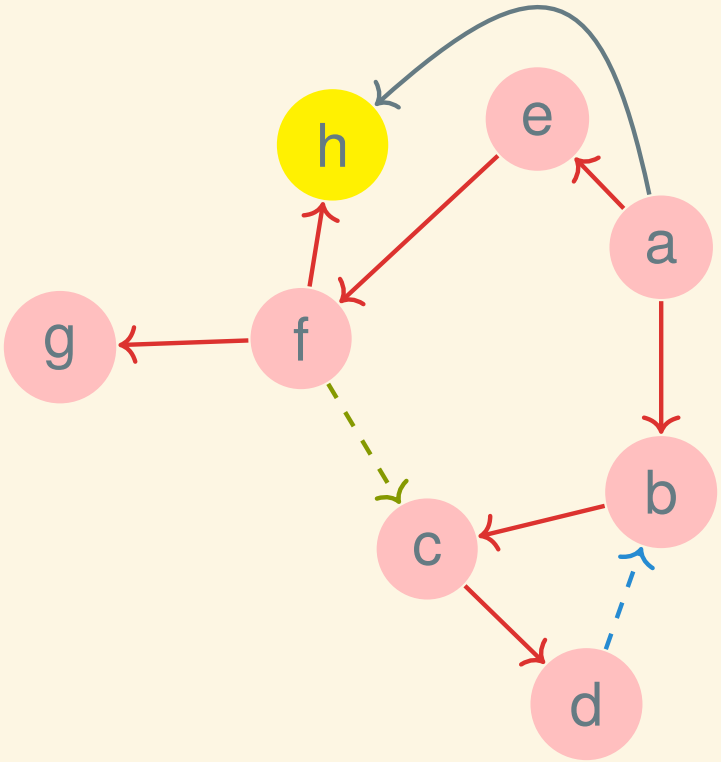
Running DFS on a digraph

tree

back

cross

forward



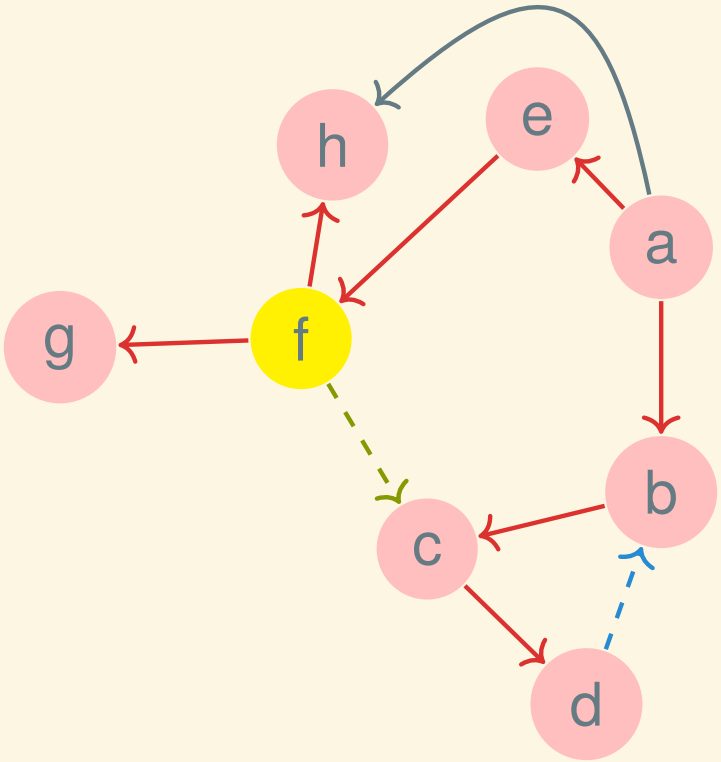
Running DFS on a digraph

tree

back

cross

forward



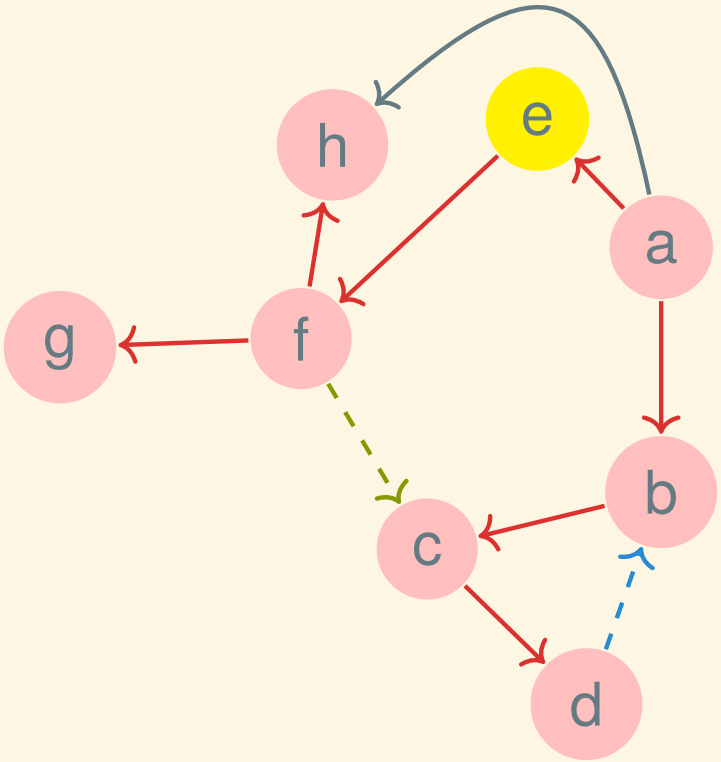
Running DFS on a digraph

tree

back

cross

forward



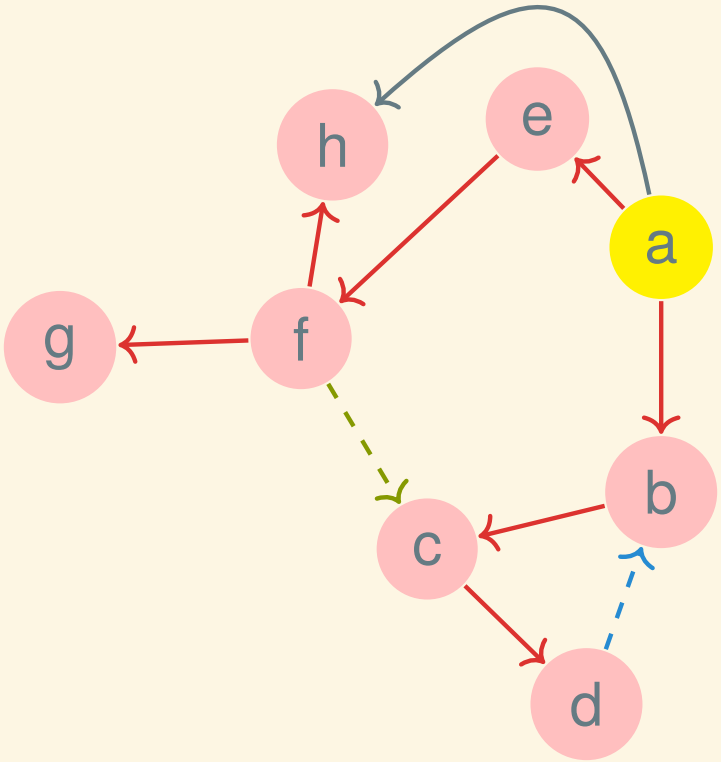
Running DFS on a digraph

tree

back

cross

forward



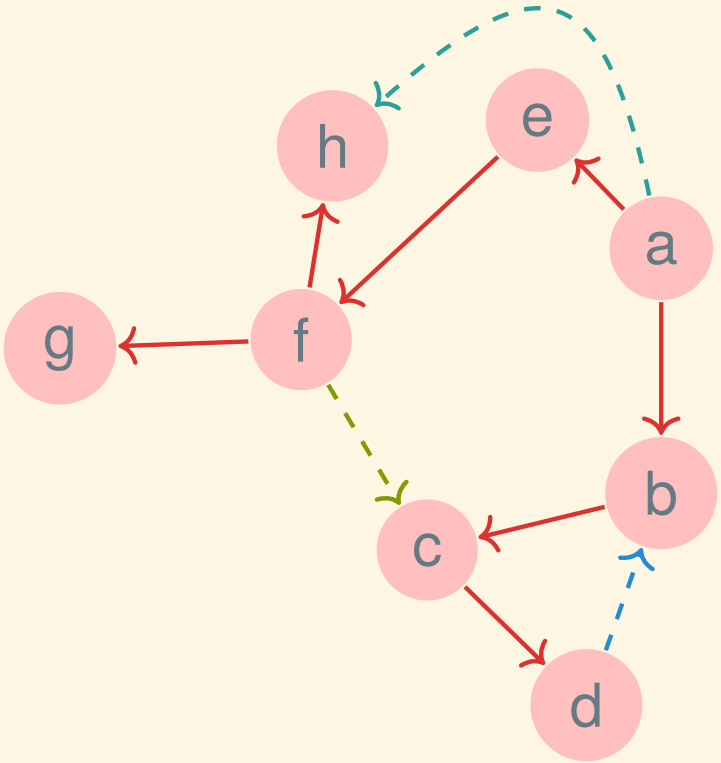
Running DFS on a digraph

tree

back

cross

forward



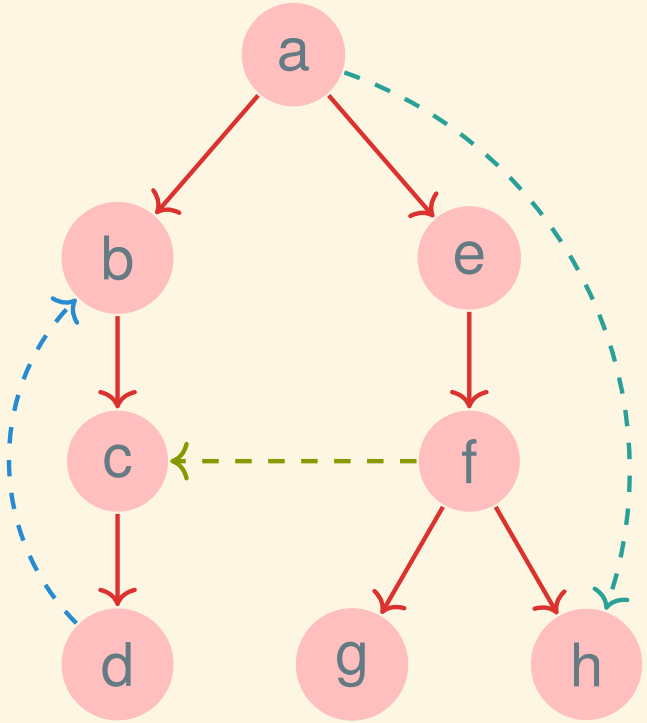
A DFS tree

tree

back

cross

forward



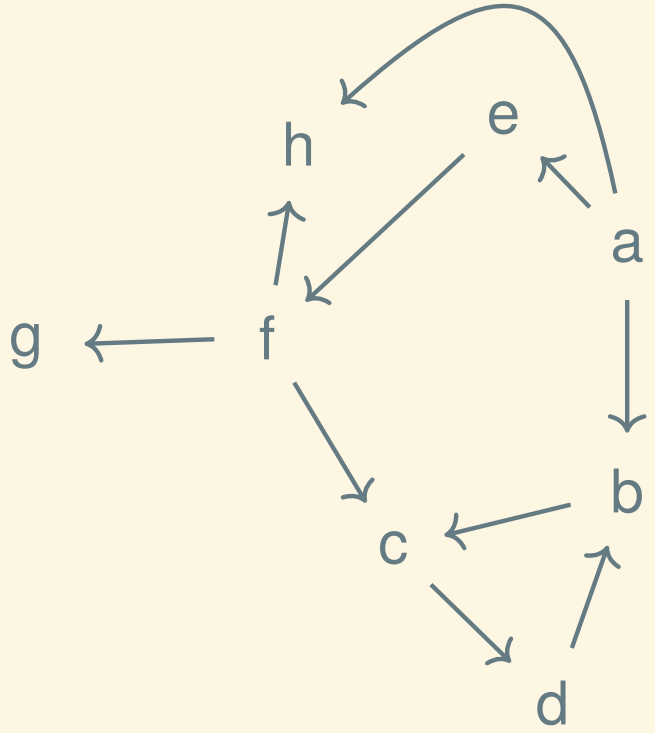
DFS for cycle detection

```
Procedure FindCycle(graph)is
    started ← new array (same size as graph, filled with false);
    finished ← new array (same size as graph, filled with false);

    Procedure Visit(v)is
        if not finished[v] then
            if started[v] then
                we found a cycle!
            end
            started[v] ← true;
            for u in Successors(graph, v) do
                Visit(u)
            end
            finished[v] ← true;
        end
    end

    for v in Vertices(graph) do
        Visit(v)
    end
end
```

Breadth-first search

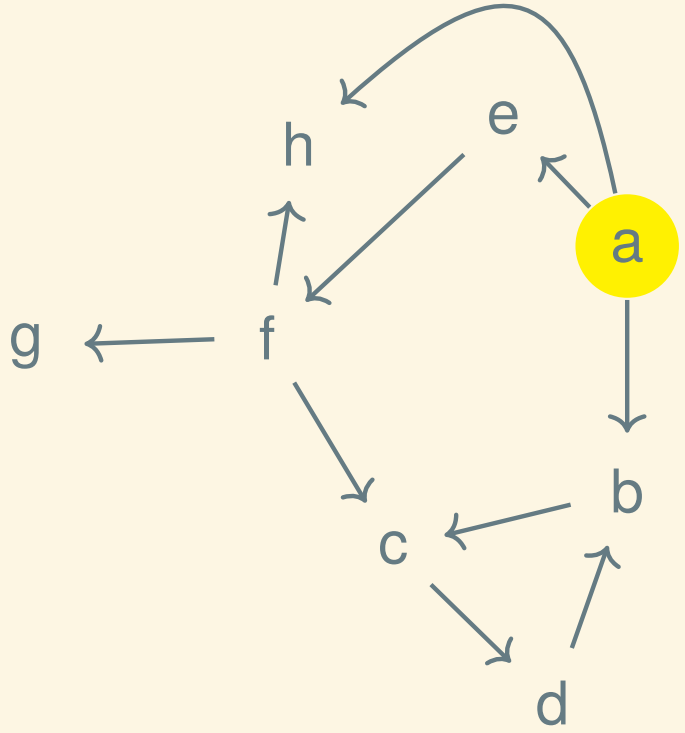
```
Procedure BFS(graph, start) is
    preds ← new array (same size as graph, filled with false);
    todo ← new queue;

    preds[start] ← true;
    Enqueue(todo, start);

    while todo is not empty do
        v ← Dequeue(todo);
        for u in Successors(graph, v) do
            if not preds[u] then
                preds[u] ← v;
                Enqueue(todo, u)
            end
        end
    end

    return preds
end
```

Running BFS on a digraph



a

Running BFS on a digraph



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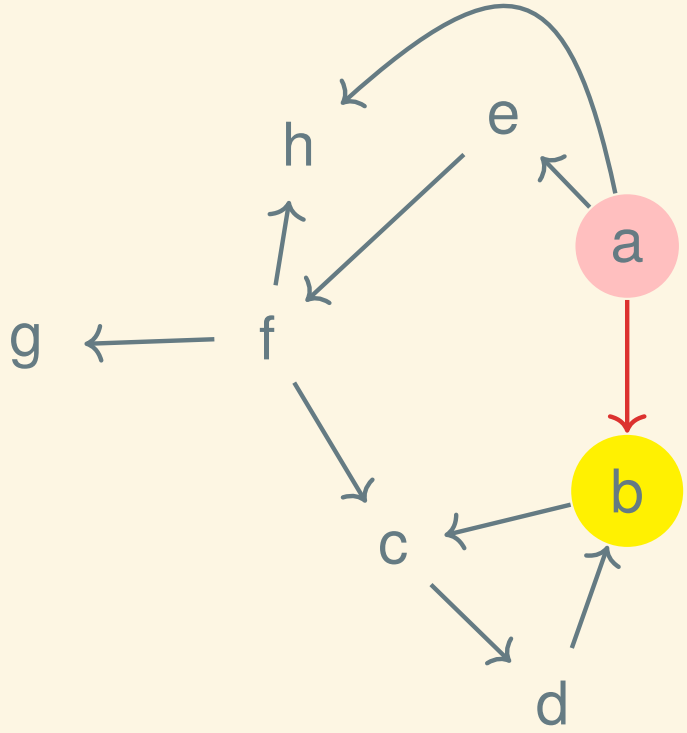
Running BFS on a digraph



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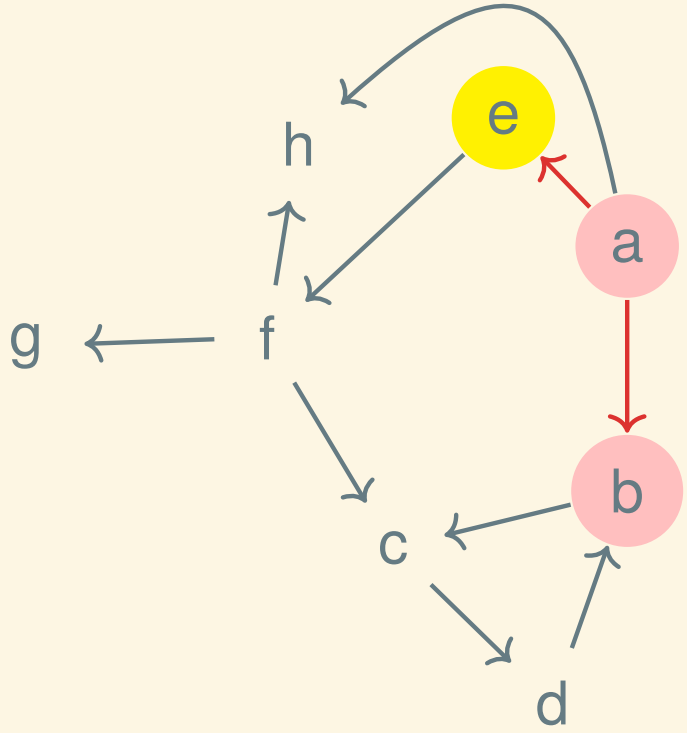
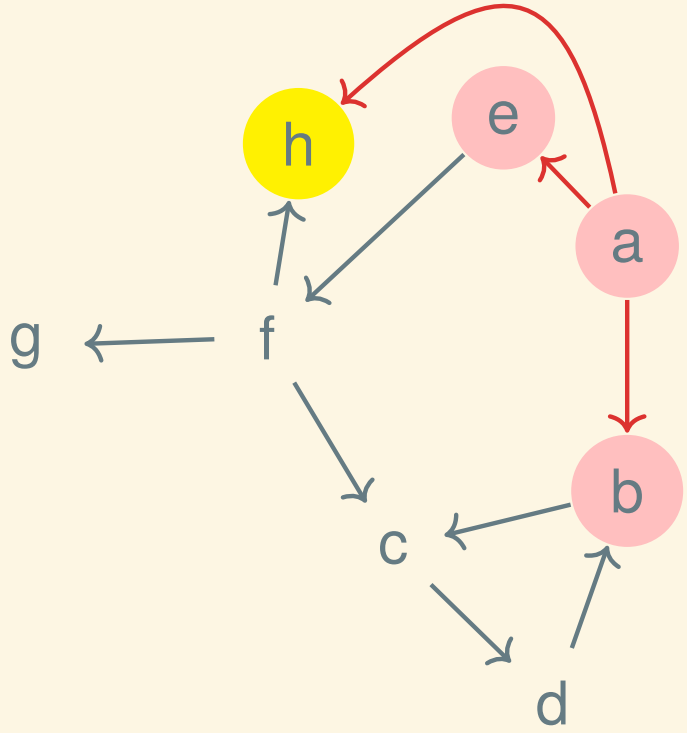
Running BFS on a digraph



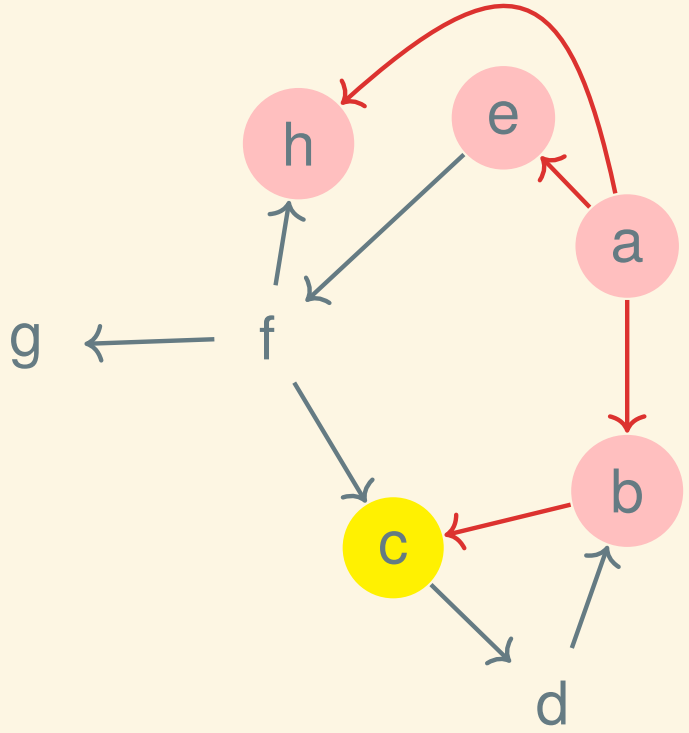
h c f

Running BFS on a digraph



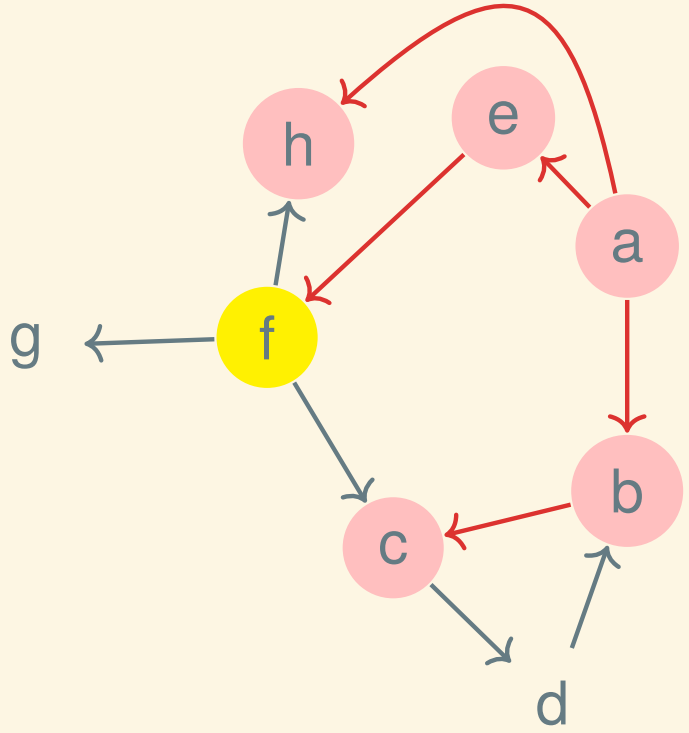
c f

Running BFS on a digraph



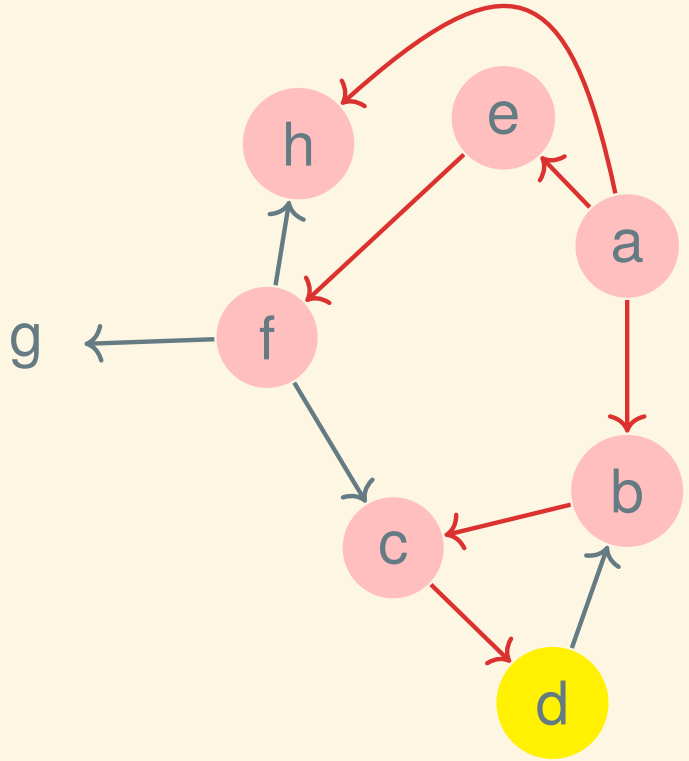
f d

Running BFS on a digraph



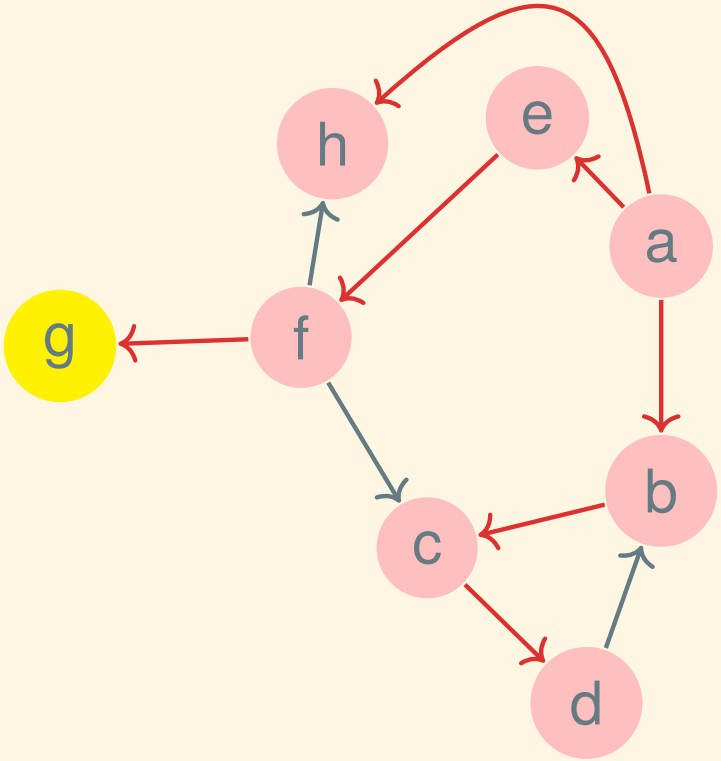
d g

Running BFS on a digraph

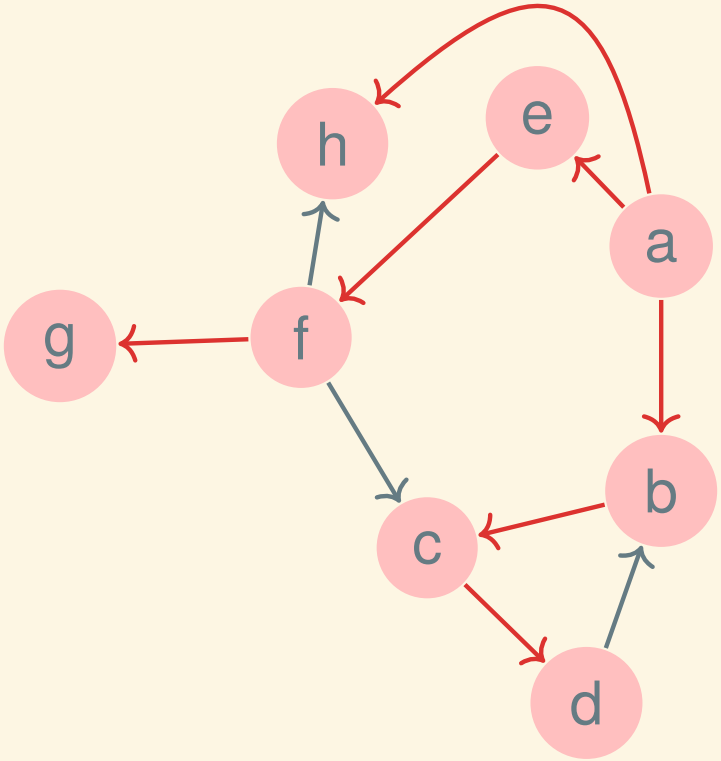


g

Running BFS on a digraph



Running BFS on a digraph



Generic graph search

If *todo* is a stack we get DFS; if *todo* is a queue we get BFS:

```
Procedure Search(graph, start) is
    preds ← new array (same size as graph, filled with false);
    todo ← new collection;

    preds[start] ← true;
    Add(todo, start);

    while todo is not empty do
        v ← Remove(todo);
        for u in Successors(graph, v) do
            if not preds[u] then
                preds[u] ← v;
                Add(todo, u)
            end
        end
    end

    return preds
end
```

Next time: shortest paths