The Binary Heap

CS 214, Fall 2019

Implementing a priority queue

A (min-)priority queue provides these operations:

- `insert`: adds an element
- `remove_min`: removes the smallest element

Some implementation complexities

	`insert`	`remove_min`
sorted list	$\mathcal{O}(n)$	$\mathcal{O}(1)$
unsorted list	$\mathcal{O}(1)$	$\mathcal{O}(n)$

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	insert	remove_min
sorted list	$\mathcal{O}(n)$	$\mathcal{O}(1)$
unsorted list	$\mathcal{O}(1)$	$\mathcal{O}(n)$
binary heap	$\mathcal{O}(\log n)$	$\mathcal{O}(\log n)$

Introducing the binary heap

A *binary heap* is complete binary tree that is *heap-ordered*

A tree is heap-ordered if every element is *less than or equal* to its children

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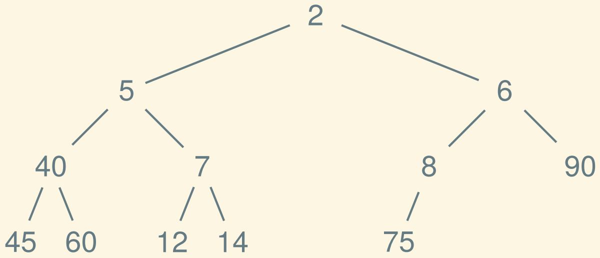
Which of these is a binary heap?:

```
        2                 2                 5
      /   \             /   \             /   \
     5     97          5     97          2     97
    / \      \        / \    /          / \    /
  40   7      99    40   7  99        40   7  99
```

Binary heap insertion

1. Add the new element at the end
2. Bubble up to restore invariant



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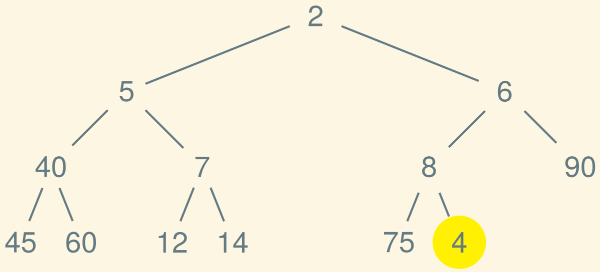

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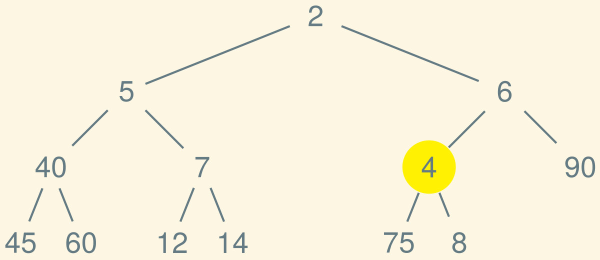

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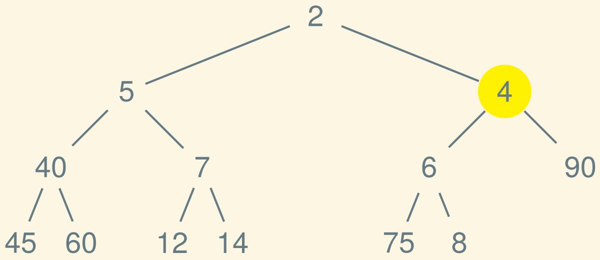

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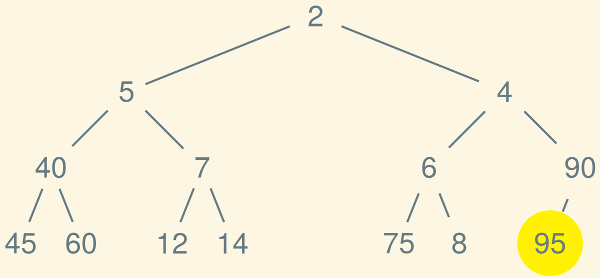

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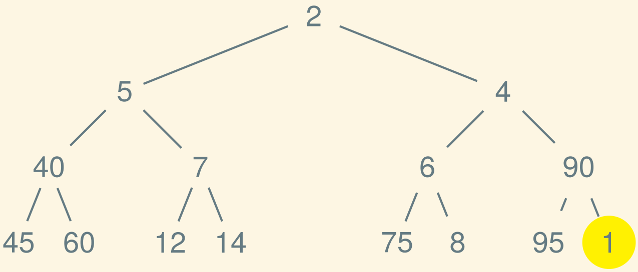

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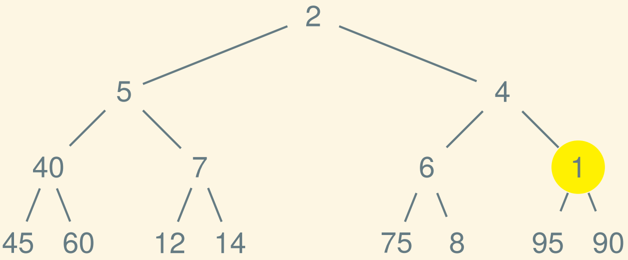

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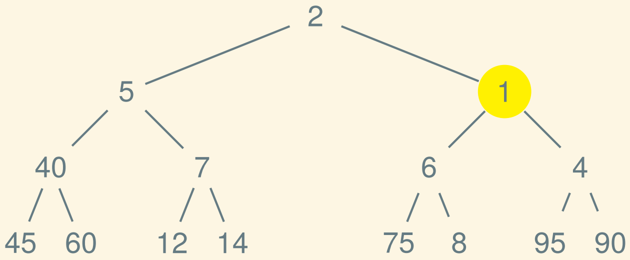

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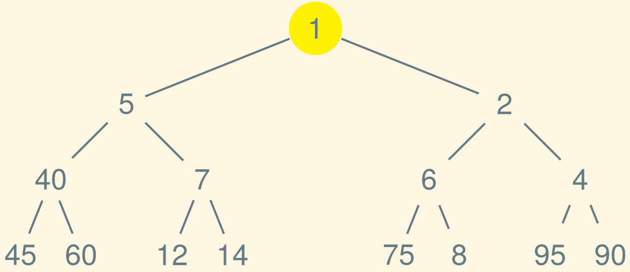

Binary heap removal

1. Replace the root with the last element of the heap
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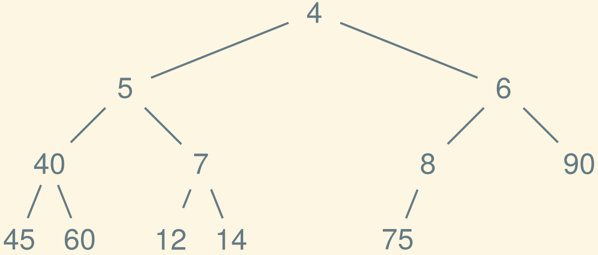

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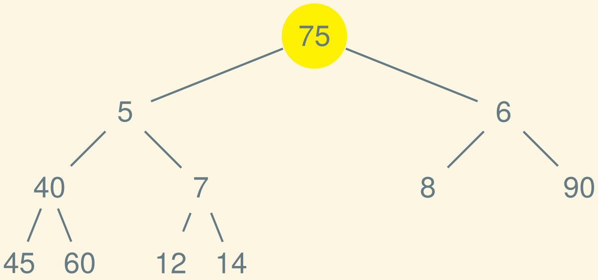

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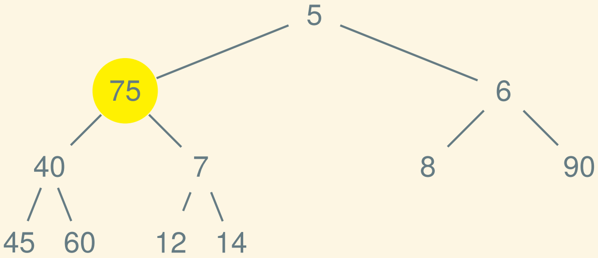

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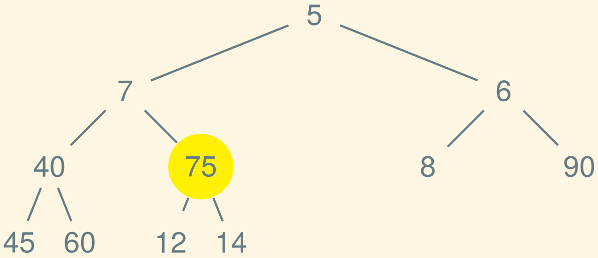

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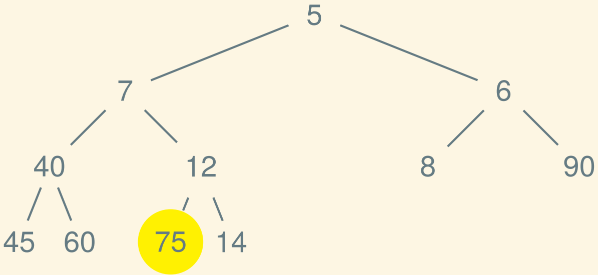

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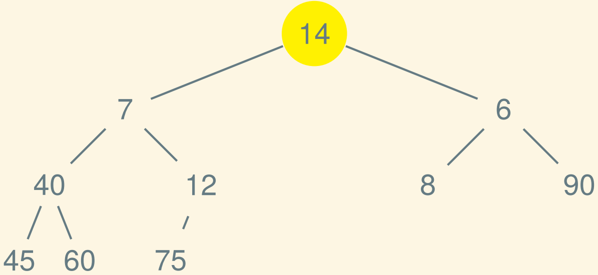

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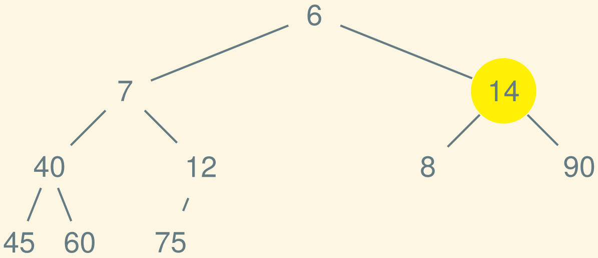

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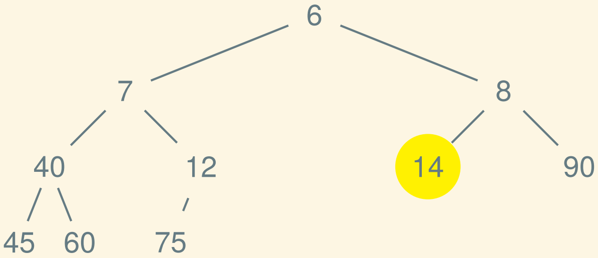
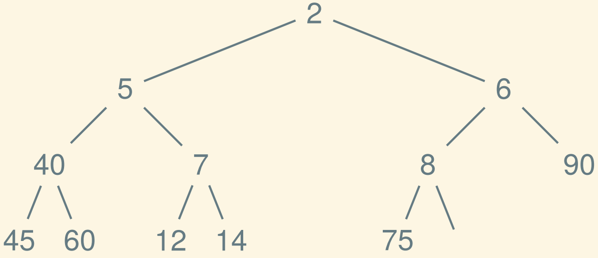


The super cool thing about binary heaps

Instead of storing it as an actual tree with pointers:



a binary heap is stored in level-order in an array:

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
2	5	6	40	7	8	90	45	60	12	14	75												

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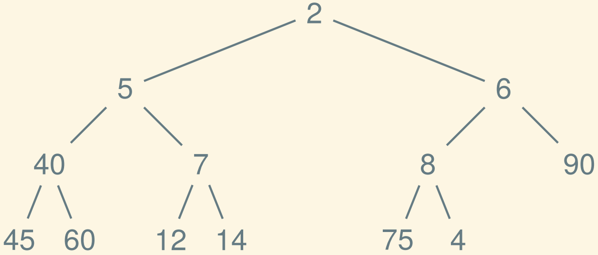

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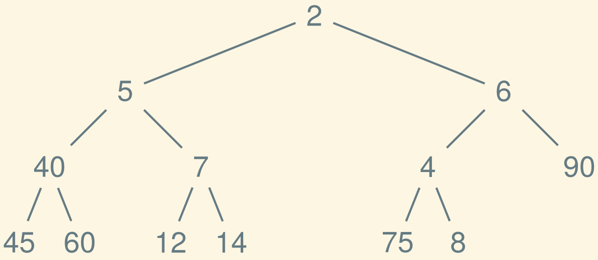

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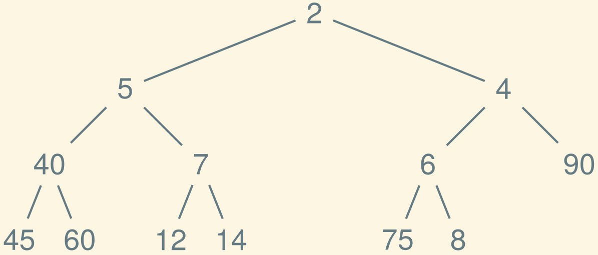

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Finding parents and children

Because the structure is *implicit*, we can't just follow pointers

Suppose i is the index of a node:

- How can we find its parent (if any)?
- How can we find its children (if any)?

Next time: another graph algorithm and another data structure to go with it