Self-Balancing BSTs

EECS 214

November 6, 2015

Take-aways

- What is the *BST property*?
- How do BST lookup, insertion, and deletion work?
- Why does balance matter?

A basic binary tree

A binary tree, describing structure but not content:

```
; An [BinTree X] is one of:
; -- (leaf)
; -- (branch [BinTree X] X [BinTree X])
(define-struct leaf [])
(define-struct branch [left element right])
```

The *BST property*

To be a BST, a binary search tree needs to be ordered:

```
; [BST Integer] -> Boolean
(define (int-bst? tree)
  (int-bst-within? -INF.0 tree +INF.0))

; Number IntBST Number -> Boolean
(define (within? min tree max)
  (or
   (leaf? tree)
   (and
    (< min (element tree) max)
    (within? min (left tree) (element tree))
    (within? (element tree) (right tree) max))]))
```

Two helpful definitions

```
; An [Ord X] is a function [X X -> Boolean]
; Invariant: must be a total order on Xs
```

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; A [Maybe X] is one of:
; -- X
; -- #false
```

Binary search

The BST property enables binary search:

```
; [Ord X] X [BST X] -> [Maybe X]
(define (lookup lt? needle haystack)
  (cond
    [(leaf? haystack) #false]
    [(lt? needle (element haystack))
     (lookup lt? needle (left haystack))]
    [(lt? (element haystack) needle)
     (lookup lt? needle (right haystack))]
    [else
     (element haystack)]))
```

Insertion is similar

```
; [Ord X] X [BST X] -> [BST X]
(define (insert lt? new tree)
  (cond
    [(leaf? tree) (make-branch LEAF new LEAF)]
    [(lt? new (element tree))
     (make-branch (insert lt? new (left tree))
                  (element tree)
                  (right tree))]
    [(lt? (element tree) new)
     (make-branch (left tree)
                  (element tree)
                  (insert lt? new (right tree)))]
    [else
     (make-branch (left tree) new (right tree))]))
```

Binary search is $\mathcal{O}(\log n)$, right?

Start with the empty tree.

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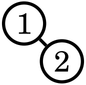
Start with the empty tree. Insert 1.

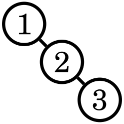
Binary search is $\mathcal{O}(\log n)$, right?

Start with the empty tree. Insert 1. Insert 2.



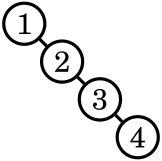
Binary search is $\mathcal{O}(\log n)$, right?

Start with the empty tree. Insert 1. Insert 2. Insert 3.



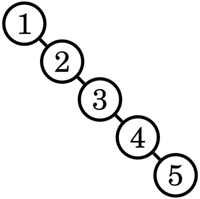
Binary search is $\mathcal{O}(\log n)$, right?

Start with the empty tree. Insert 1. Insert 2. Insert 3. Insert 4.



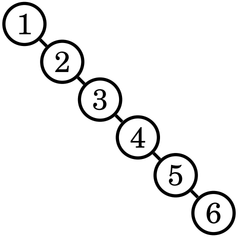
Binary search is $\mathcal{O}(\log n)$, right?

Start with the empty tree. Insert 1. Insert 2. Insert 3. Insert 4. Insert 5.



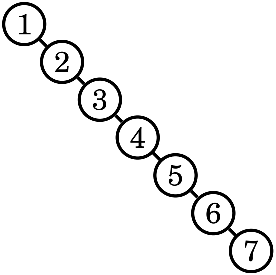
Binary search is $\mathcal{O}(\log n)$, right?

Start with the empty tree. Insert 1. Insert 2. Insert 3. Insert 4. Insert 5. Insert 6.



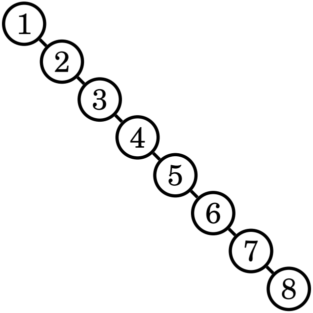
Binary search is $\mathcal{O}(\log n)$, right?

Start with the empty tree. Insert 1. Insert 2. Insert 3. Insert 4. Insert 5. Insert 6. Insert 7.



Binary search is $\mathcal{O}(\log n)$, right?

Start with the empty tree. Insert 1. Insert 2. Insert 3. Insert 4. Insert 5. Insert 6. Insert 7. Insert 8.



We need some balance!

There are a variety of self-balancing trees:

- Red-black trees
- Splay trees
- 2-3 trees
- 2-4 trees
- B trees
- and so on...

The *AVL property*

An AVL tree is *height balanced*: For every node, the heights of its left and right subtrees can differ by at most 1

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An AVL tree is *height balanced*: For every node, the heights of its left and right subtrees can differ by at most 1

We keep the balance in each node:

```
; An [AVLTree X] is one of:
; -- (leaf)
; -- (branch Balance [AVLTree X] X [AVLTree X])
; where Balance is the integer interval [-1, 1]
;
; Invariant: for all nodes n,
;   (= (balance n)
;      (- (height (right n)) (height (left n))))
(define-struct leaf [])
(define-struct branch [balance left element right])
```

Big theme!

Local properties induce global properties.

Take-aways

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