Probabilistic Data Structures

EECS 214

November 18, 2015

Take-aways

- What's a hash function? What makes a hash function good?
- What's the purpose of a hash table? How does it work, and how can it "go wrong"?
- What's the purpose of a Bloom filter? How does it work, and how can it "go wrong"?
- What does it mean for a data structure to be *probabilistic*?

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Notation note: We will write $\mathbb{N}_k$ for the set $\{0, 1, \ldots, k\}$

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How can we represent this?

We can't use strings to index into an array—we need a *hash function*

Definition: *hash function*

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$$hash_k(c_1 \ldots c_n) = \sum_{i=1}^{n} ord(c_i) \mod k$$

It adds up the character values

How do we use a hash function?

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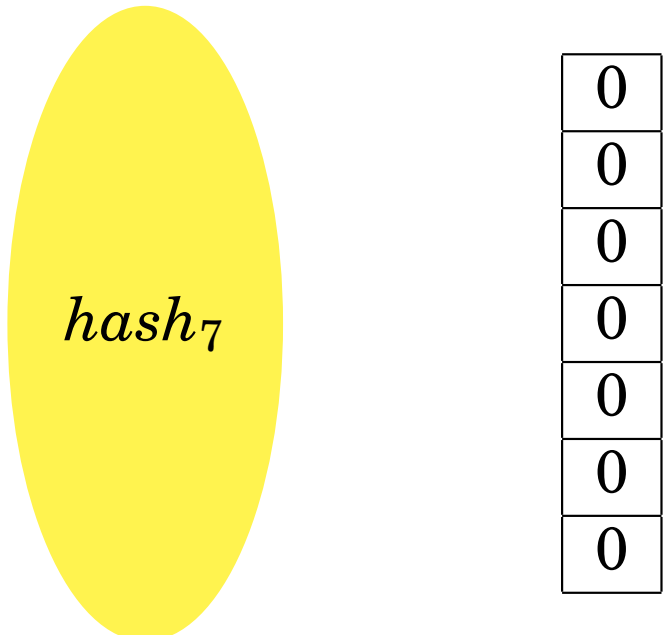
A *hash function* for some type maps values of that type to $\mathbb{N}_k$
We then store our value v at the index given by $hash(v)$.

Example (using shitty string hash function)

Store word frequencies at the index given by the hash of the word:



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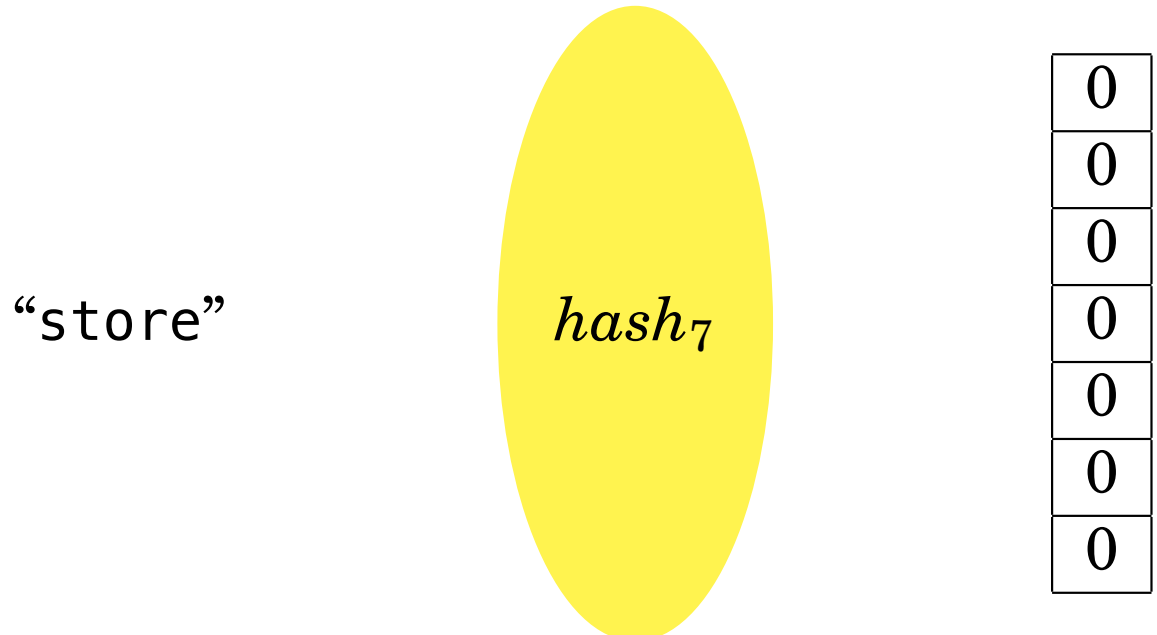

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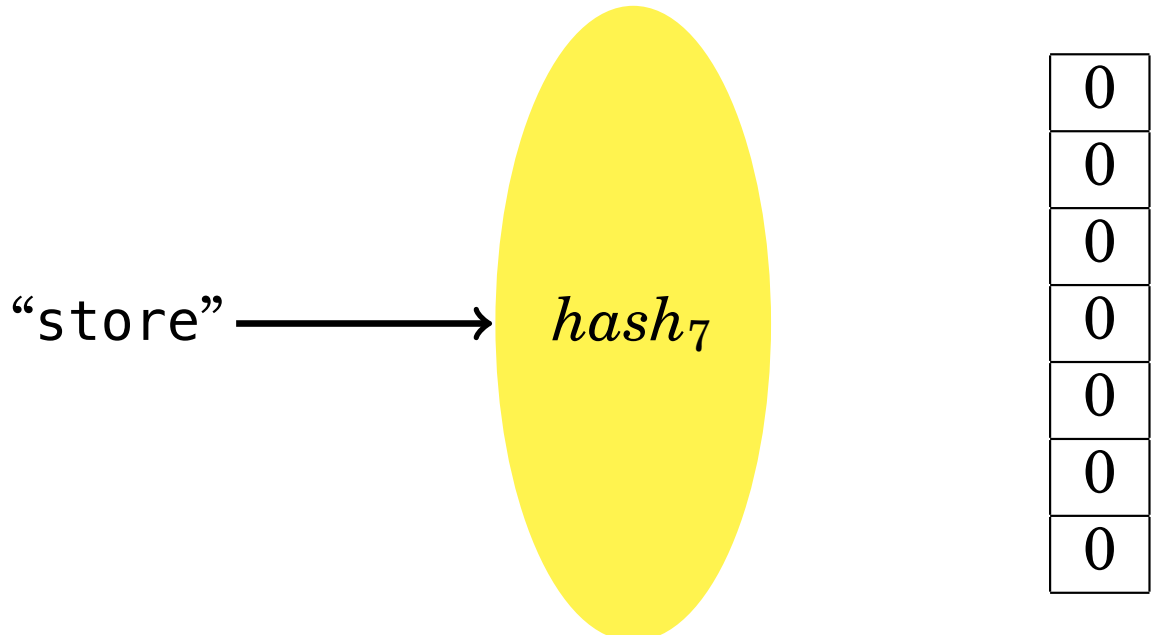

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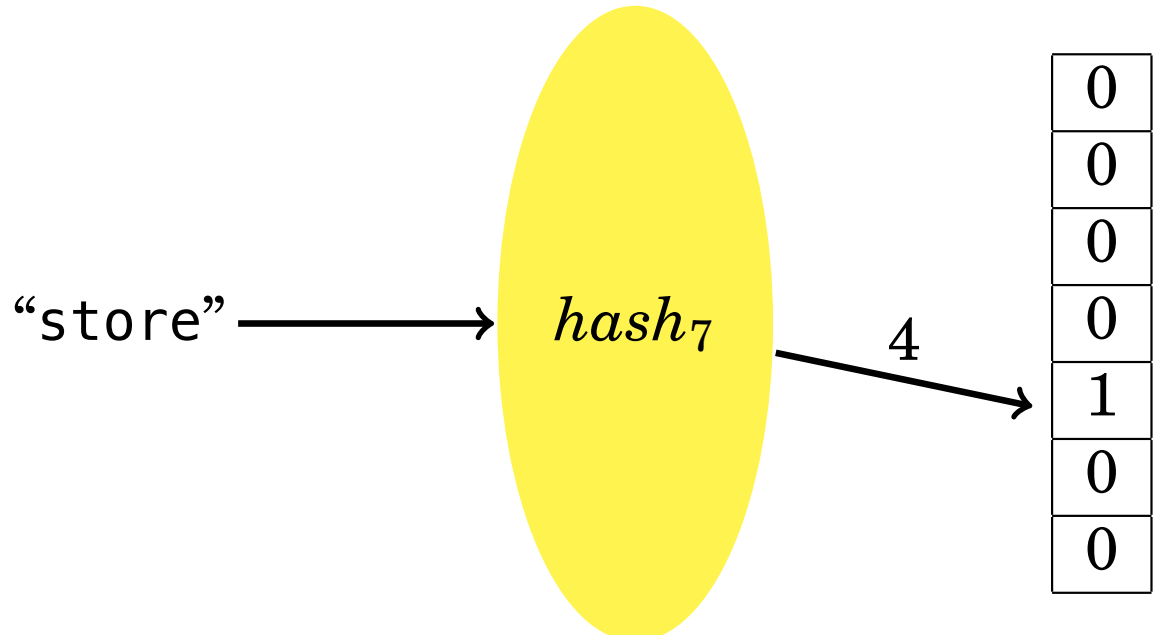

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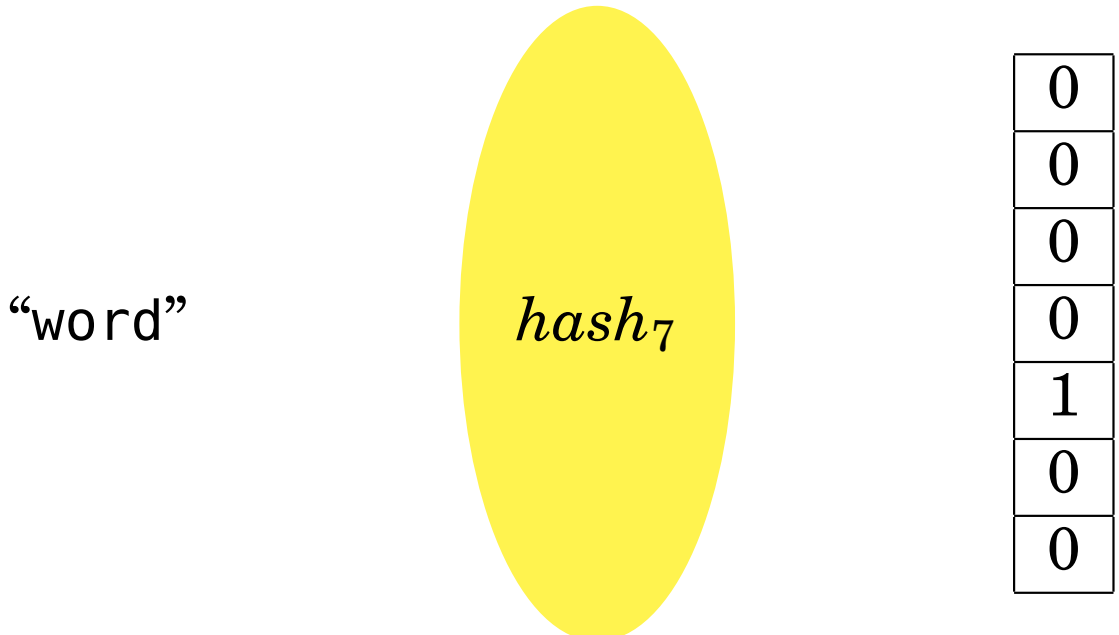

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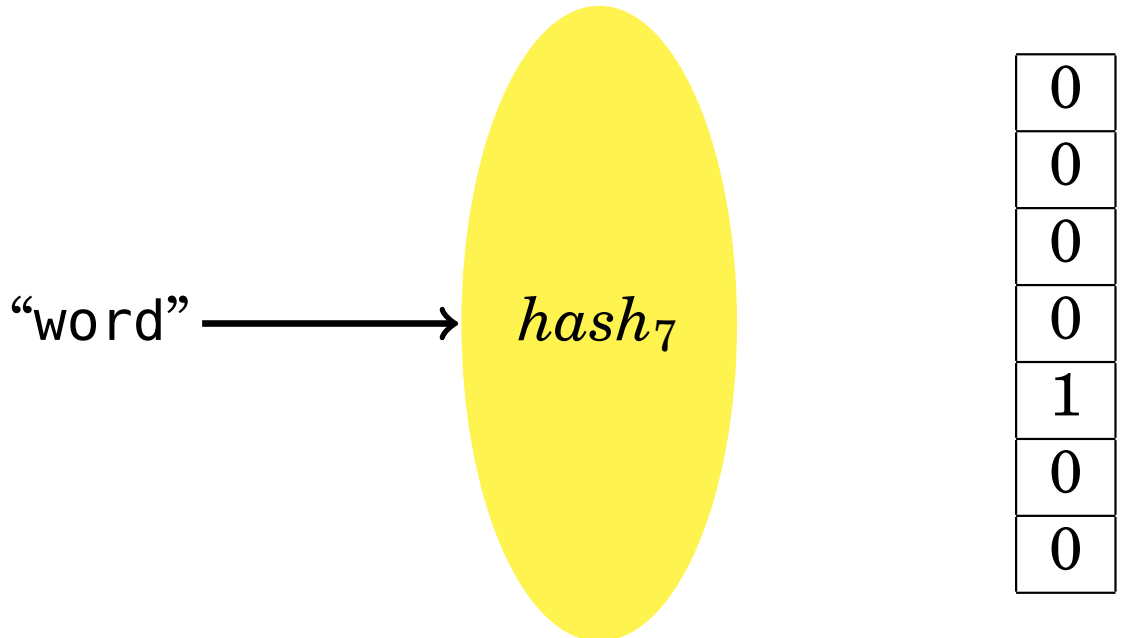

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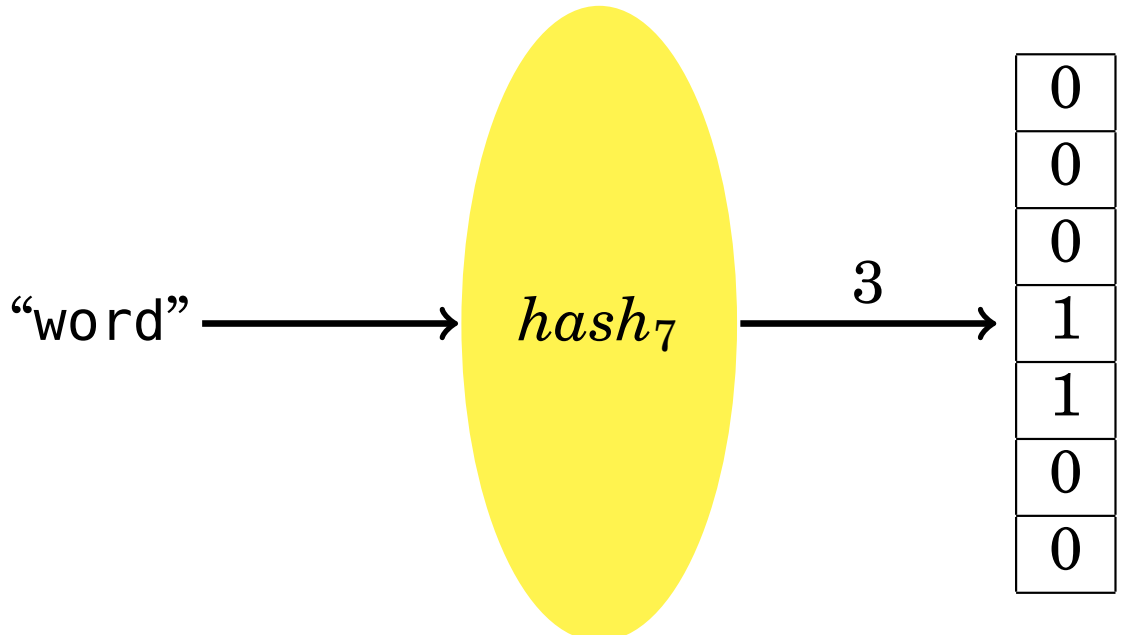

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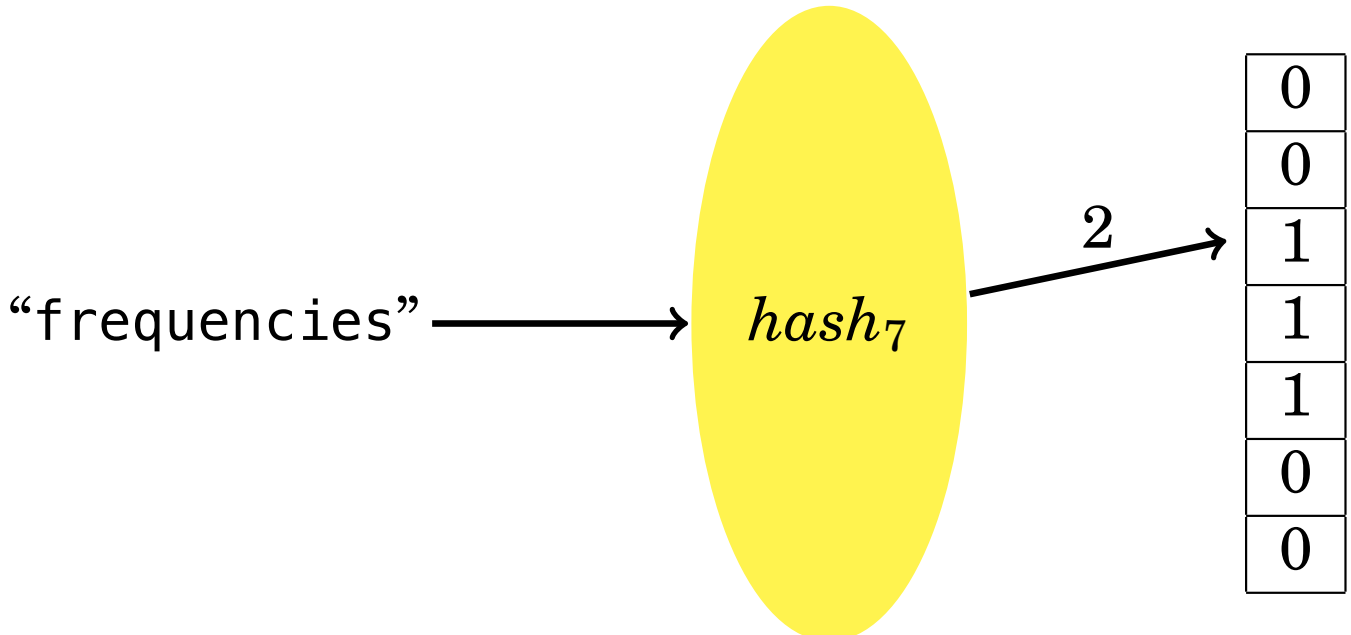

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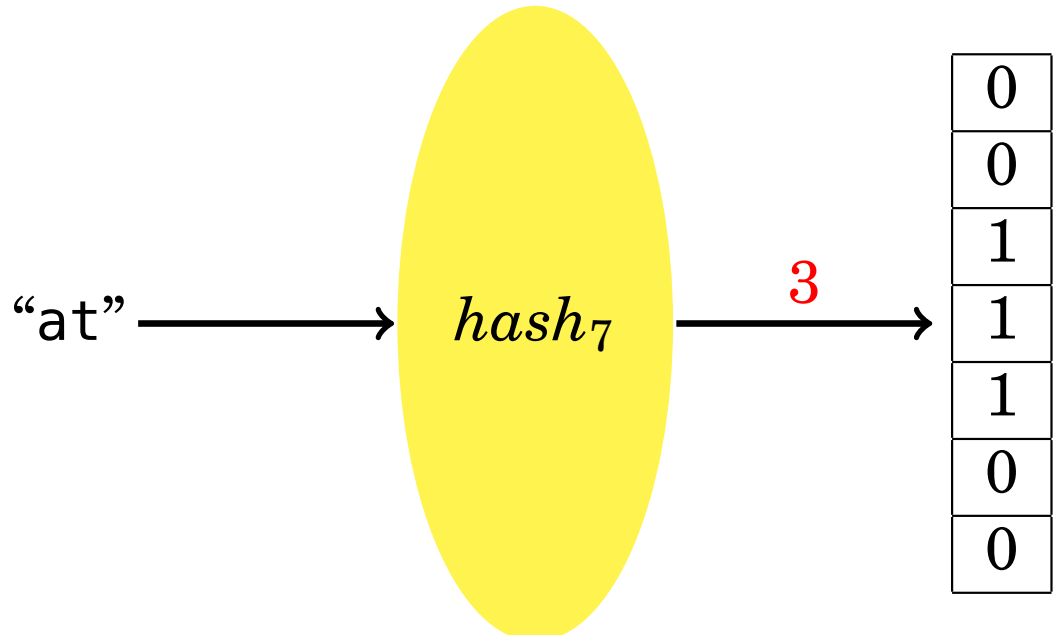

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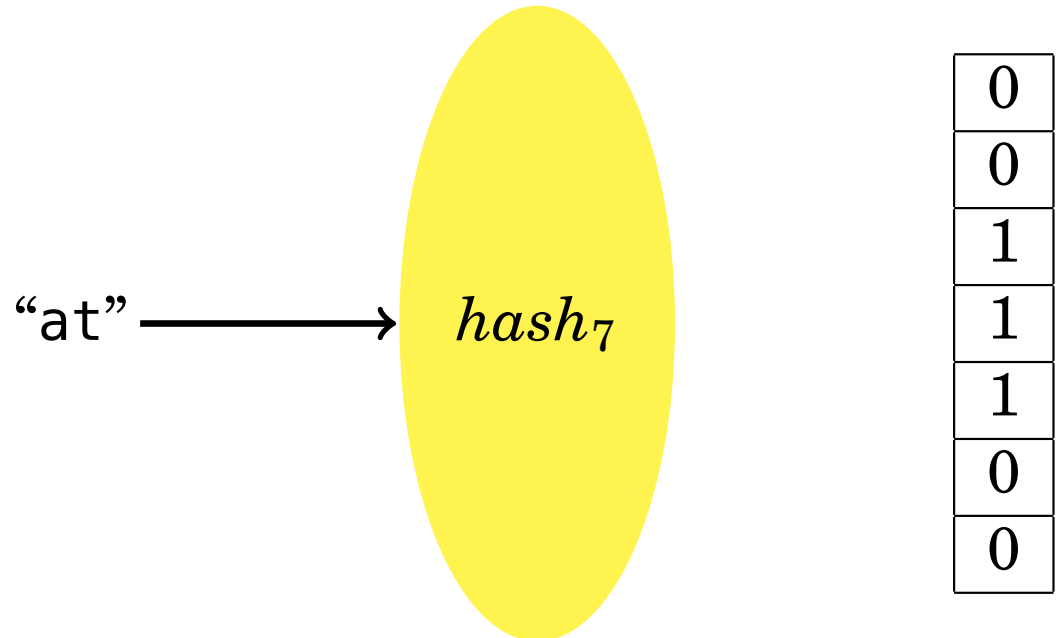

Hash collision!

Take two: separate chaining

Each bucket stores a linked list of associations:

(hash, 1)
(frequencies, 1) $\rightarrow$ (by, 1)
(word, 2) $\rightarrow$ (at, 1) $\rightarrow$ (of, 1)
(store, 1) $\rightarrow$ (index, 1)
(given, 1)
(the, 3)

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If we avoid collisions: $\mathcal{O}(1)$ *on average*

But too many collisions and the lists get too long: $\mathcal{O}(n)$

Probabilities of collisions

Number of **32-bit** hash values	Number of **64-bit** hash values	Number of **160-bit** hash values	Odds of a hash collision
77163	5.06 billion	1.42×10^{24}	1 in 2
30084	1.97 billion	5.55×10^{23}	1 in 10
9292	609 million	1.71×10^{23}	1 in 100
2932	192 million	5.41×10^{22}	1 in 1000
927	60.7 million	1.71×10^{22}	1 in 10000
294	19.2 million	5.41×10^{21}	1 in 100000
93	6.07 million	1.71×10^{21}	1 in a million
30	1.92 million	5.41×10^{20}	1 in 10 million
10	607401	1.71×10^{20}	1 in 100 million
	192077	5.41×10^{19}	1 in a billion
	60740	1.71×10^{19}	1 in 10 billion
	19208	5.41×10^{18}	1 in 100 billion
	6074	1.71×10^{18}	1 in a trillion
	1921	5.41×10^{17}	1 in 10 trillion
	608	1.71×10^{17}	1 in 100 trillion
	193	5.41×10^{16}	1 in 10^{15}
	61	1.71×10^{16}	1 in 10^{16}
	20	5.41×10^{15}	1 in 10^{17}
	7	1.71×10^{15}	1 in 10^{18}

Odds of a full house in poker
1 in 693

Odds of four-of-a-kind in poker
1 in 4164

Odds of being struck by lightning
1 in 576000

Odds of winning a 6/49 lottery
1 in 13.9 million

Odds of dying in a shark attack
1 in 300 million

Odds of a meteor landing on your house
1 in 182 trillion

10:31

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Stronger: changing any one bit of the input changes each bit of the output with probability $\frac{1}{2}$

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