Graph representations and algorithms

EECS 214

November 23, 2015

Take-aways

- How are graphs commonly represented?
- How can we find shortest paths?

Definitions

A *graph* is a pair (V, E), where V is the set of *vertices* and symmetric relation $E \subseteq V^2$ is the set of *edges*.

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A *directed graph* is a pair (V, E) where V is the set of *vertices* and relation $E \subseteq V^2$ is the set of edges.

Weighted graph definitions

Let W be a set of *weights*. (Often $W = \mathbb{R}$.) Then:

A *weighted directed graph* is a pair (V, E), where V is the set of *vertices* and $E : V^2 \rightharpoonup W$ is a partial map from edges to their weights.

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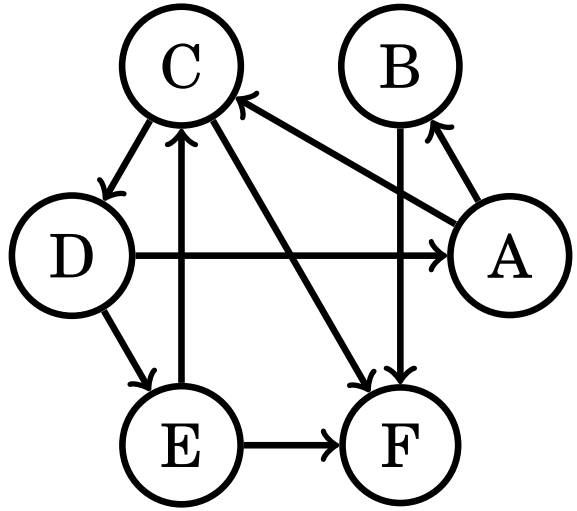
A *weighted (undirected) graph* is a weighted directed graph where $E(v, u) = E(u, v)$ for all $v, u \in V$.

Two major graph representations



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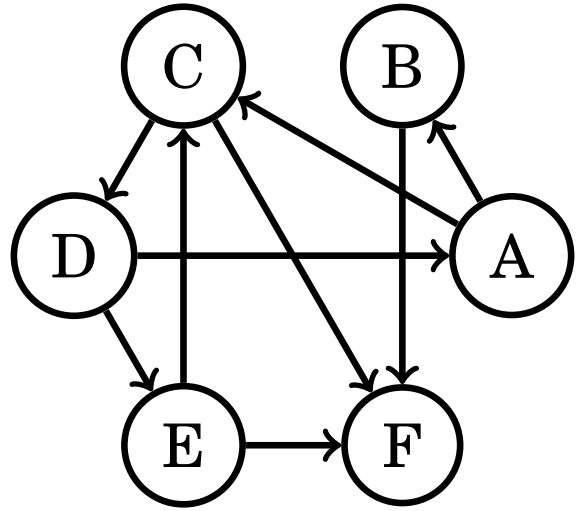

Adjacency list

V	Successors
A	B, C
B	F
C	D, F
D	A, E
E	F, C
F	

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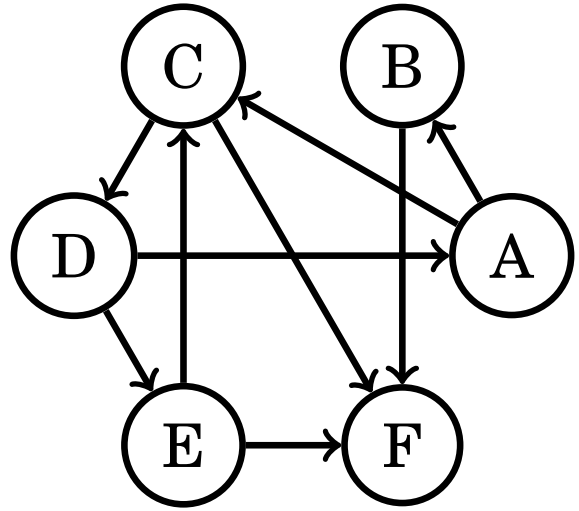


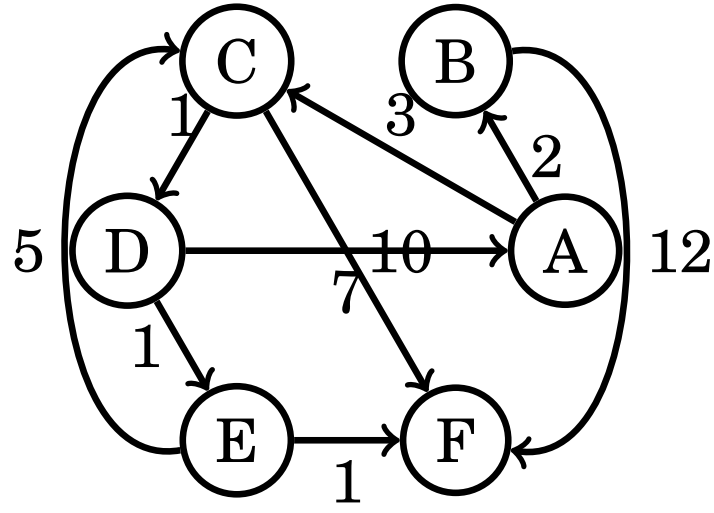
Adjacency matrix

	A	B	C	D	E	F
A		•	•			
B						•
C				•		•
D	•				•	
E			•			•
F						

Adjacency list

V	Successors
A	B, C
B	F
C	D, F
D	A, E
E	F, C
F	

Two major weighted graph representations



Adjacency matrix

	A	B	C	D	E	F
A		2	3			
B						12
C				1		7
D	10				1	
E			1			5
F						

Adjacency list

V	Successors
A	(B,2), (C,3)
B	(F,12)
C	(D,1), (F,7)
D	(A,10), (E,1)
E	(F,1), (C,5)
F	

Dijkstra's algorithm

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