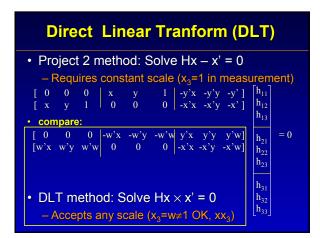
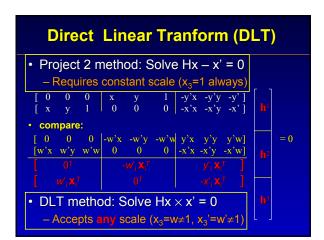
# CS 395/495-26: Spring 2002 **IBMR: Week 5 B Chapter 3: Estimation & Accuracy** Jack Tumblin jet@cs.northwestern.edu **Homework 1** • Paper-and-pencil exercises • Shouldn't be very hard—mostly practice and reminders of important properties Not comprehensive (skips good stuff) • Due in 2 weeks: May 16 • (Turn it in! only got 2 Proj 1's...) **Overview** Perfect Math .vs. Imperfect measurements Many ways to recover H: Points, lines, planes, ||, ⊥, conics, quadrics, cross-ratios, vanishing points, twisted cubics... - Vectorize ('flatten, stack, null space') to solve most / all - Robustness, Accuracy .vs. # of measurements (data-rich images: quantity easier than quality more is easier than better) ?What links measurement errors ← → H errors? ?How can more measurements reduce error?

# Recall: Project 2 Hints 4 point correspondence: - Book shows: $x'(h_{31}x + h_{32}y + h_{33}) = (h_{11}x + h_{12}y + h_{13})$ $y'(h_{31}x + h_{32}y + h_{33}) = (h_{11}x + h_{12}y + h_{13})$ - Rearrange: known vector (dot) unknown vector $\begin{bmatrix} x & y & 1 & 0 & 0 & 0 & -x^2x & -x^2y & -x^2 \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \end{bmatrix} = 0$ - stack, solve for null space... But this assumes $x_1 = x$ , $x_2 = y$ , $x_3 = 1$ , Book has a better way (DLT)...





### **Direct Linear Tranform (DLT)**

- DLT method: Solve  $H \times x = 0$ 
  - Accepts any scale, any point  $(x_3=w\neq 1 \text{ is OK})$
  - 'Pure', Compatible -- P<sup>2</sup> terms only
  - Much better suited to error measurements.
- Subtlety:
  - Won't divide-by-zero if w=0 or h<sub>33</sub>=0 (tt happens!)
  - has a 3<sup>rd</sup> row; it is not degenerate if w'=0
  - OK to use it... (Solve 8x12)

$$\begin{bmatrix} 0^{T} & -w_{1}\mathbf{X}_{1}^{T} & y_{1}\mathbf{X}_{1}^{T} \\ w_{1}\mathbf{X}_{1}^{T} & 0^{T} & -x_{1}^{T}\mathbf{X}_{1}^{T} \\ -y_{1}^{T}\mathbf{X}_{1}^{T} & -x_{1}^{T}\mathbf{X}_{1}^{T} & 0^{T} \end{bmatrix} \begin{bmatrix} \mathbf{h}^{1} \\ \mathbf{h}^{2} \\ \mathbf{h}^{3} \end{bmatrix} = \mathbf{0}$$

### **Deceptive 'Robustness'**

- Suppose we have 4 pt-correspondences:
  - Use DLT to write 8x9 (or 12x9) matrix A: Ah=0
  - Solve for h null space. ALWAYS gives H matrix
- But what if points are bad / fictional?
  - − 3 collinear input pts, crooked out: IMPOSSIBLE!
  - Yet we get an **H** solution! Why?
- A matrix rank is 7 or 6→ rank 1 H result(s)
  - 'Null Space' of **A** may contain >1 **H** solution!
  - 'Degenerate' H solution of form  $aH_1 + bH_2$ ...
- Answer: SVD ranks A; reject bad point sets.

### **Actual 'Robustness'**

 Vectorizing ('Flatten, Stack, Null Space' method) works for almost ANY input!

(Points, lines, planes, ||,  $\bot$ , conics, quadrics, cross-ratios, vanishing points, twisted cubics...

• Use DLT formulation:  $(Hx \times x') = 0$ 

- Rearrange as dot product: (known)·(unknown) = 0
- Be careful to have ENOUGH constraints tricky when you mix types: points, lines,... (99 76)

### **Adding More Measurements**

How can we use >4 point correspondences?

- Easy:
  - Add more rows to our 8x9 matrix A: A·h = 0
  - Use SVD to find Null space (Always gives an answert
  - Result: 'Least squares' solution
    - minimizes  $||\mathbf{A}\mathbf{h}||^2 = \sum_i \mathbf{\varepsilon}_i$

where  $\mathcal{E}_i$  is error for i-th pt. correspondence:

- $\mathcal{E}_i = || Hx_i \times x_i' ||^2 = || (2 \text{ rows of A})^*h ||^2 = 'algebraic distance'$
- 'Algebraic Distance' ? No geometric meaning!

### **Adding More Measurements**

- 2D'Algebraic Distance' ? No geometric meaning!
- 2D 'Geometric Distance' d(a,b)² is Better: measurable length in input or output space

if  $a = (a_1 a_2 a_3)$  and  $b = (b_1 b_2 b_3)$ , then define

$$\mathbf{d}(a,b)^2 = \left(\frac{a_1}{a_3} - \frac{b_1}{b_3}\right)^2 + \left(\frac{a_2}{a_3} - \frac{b_2}{b_3}\right)^2$$

**Turns out that:** 

$$\mathbf{d}(a,b)^2 = \underbrace{\mathbf{d}_{\text{algebraic}}(a,b)}_{\mathbf{a_1} \cdot \mathbf{b_1}}$$

(Not very surprising)

# **Adding More Measurements**

### **Overall Strategy:**

- Overconstrain the answer H
  - Collect extra measurements (>4 point pairs, etc. ...)
  - expect errors; call them 'estimates' x
- Compute a 1<sup>st</sup> solution (probably by SVD)
- Compute error  $d(H\hat{x}, \hat{x}')^2$ , and use this to...
- 'Tweak' answer H and estimates x̂
- · Compute new answer
- Stop when error < useful threshold</li>


## **Using Estimates**

- Simplest: 'one image' transfer method:
  - Assume inputs are a perfect test pattern:
     only output pts are in error
  - Adjust output estimates  $\hat{x}'$  until d(Hx,  $\hat{x}'$ )<sup>2</sup> →0 (note we re-compute H as x' changes)
- · Better: 'Symmetric' transfer method:
  - Assume BOTH inputs and outputs have error.
  - Adjust BOTH input and output ests  $\hat{x}$   $\hat{x}'$  (note we re-compute H as x, x' change)
  - Stop when  $d(H\hat{x},\hat{x}')^2 + d(H^{-1}\hat{x}',\hat{x})^2 \rightarrow 0$

# END