CS 395/495-26: Spring 2002

IBMR: Week 9B

The Camera Matrix and World Geometry

Chapter 7 (finish)

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Reminders

CTEC Online – please add your comments…

- Homework 1 return
- Proj3 Due Thurs May 23

  HW2 posted on website.
- HW2 due Thurs May 30

  Proj4 posted on website.

  ~~HW 3~~ CANCELLED
- Proj4 Due Tues June 11
- ~~HW3 Due Tues June 11~~

The Simplest Camera

P·X = x World space **X** → image space **x**

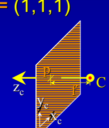
The full planar camera has **11** DOF:

P = ([K|0]·R·T) =

$$\begin{bmatrix} \alpha_x f & s & p_x & 0 \\ 0 & \alpha_y f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & -\sin\phi ? & t_x \\ \sin\theta\sin\phi & \cos\theta & -\sin\theta\cos\phi & t_y \\ -\cos\theta\sin\phi & -\sin\phi ? & \cos\theta\cos\phi & t_y \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Simplify:
  - Aim camera to match image and world axis directions; e.g. $(x_w,y_w,z_w,0) = (x_c,y_c,z_c,0)$,
  - Focal length f=1,no skew α=1, image center = (1,1,1)
  - Camera position at C = (a,b,c,d) yields

$$\mathbf{P}_{min} = \begin{bmatrix} a & 0 & 0 & -d \\ 0 & b & 0 & -d \\ 0 & 0 & c & -d \end{bmatrix}$$



The Simplest Camera

$P \cdot X = x$ World space **X** → image space **x**

The full planar camera has **11** DOF:

P

•

Rotations don't change image content

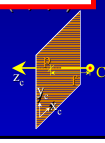
Same Center Point? Same image!
(just a planar reprojection in P^2)

- **The camera center C must move to change the image.**
- **3-D image data requires camera movement.**

no zooming, warping, rotation can change this!

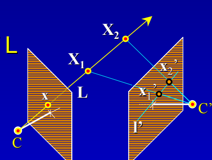
– Camera position C = (a,b,c,d), and…

$$P_{min} = \begin{bmatrix} a & 0 & 0 & -d \\ 0 & b & 0 & -d \\ 0 & 0 & c & -d \end{bmatrix}$$



Movement Detection?

- Can we do it from images only?
  - 2D projective transforms often LOOK like 3-D;
  - External cam. calib. affects all elements of **P**
- YES. Camera moved if-&-only-if Camera-ray points ($C \to x \to X_1, X_2, \ldots$) will map to LINE (not a point) in the other image
- 'Epipolar Line' == l' = image of L
- 'Parallax' == $x_1' \to x_2'$ vector



Cameras as Protractors

- Define world-space direction d:
  - From a P^3 infinity point $X_d = [x_d \ y_d \ z_d \ 0]^T$ define $d == [x_d \ y_d \ z_d]$
- Set camera at world origin, axes aligned
  - (e.g. C=(0,0,0,1), $(x_w, y_w, z_w) = (x_c, y_c, z_c)$)
  - (Danger! now mixing P2, P3…)
  - Link direction d to image pt. $x = (x_c, y_c, z_c)$: $PX_d = [K|I]X_d = Kd = x$
- Ray thru image pt. x has direction $d = K^{-1}x$

Cameras as Protractors

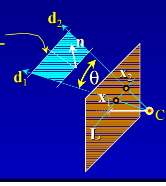
- Angle between C and 2 image points x_1, x_2

(see book pg 199)

$$\cos\theta = \frac{x_1^T (K^{-T}K^{-1})\, x_2}{\sqrt{(x_1^T (K^{-T}K^{-1})\, x_1)(x_2^T (K^{-T}K^{-1})\, x_2)}}$$

- Image line L defines a plane π_L
  - (Careful! P^3 world $\neq P^2$ camera axes here!)
  - Plane normal direction:

$$n = K^T L$$



Cameras as Protractors

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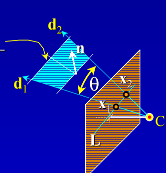
- Image line L defines a plane π_L
  - (Care… …ere!)
  - Plan…

Something Special here? Yes!

$$n = K^T L$$



Cameras as Protractors

What is $(K^{-T}K^{-1})$?

- Recall P^3 Conic Weirdness:
  - Plane at infinity π_∞ holds all 'horizon points' d ('universe wrapper')
  - Absolute Conic Ω_∞ is imaginary outermost circle of π_∞
- for ANY camera,
  Translation won't change 'Horizon point' images:

  $P X_d = x = KRd$ (pg200)
- Absolute conic is inside π_∞; it's all 'horizon points'
- for ANY camera,

  $P\,\Omega_\infty = (K^{-T}K^{-1}) = =$ 'Image of Absolute Conic'

Why do we care?

P Ω_∞ = $(K^{-T}K^{-1})$ = = 'I mage of A bsolute C onic'

- IAC is a 'magic tool' for camera calibration K
- Recall Ω_∞ let us find H from perp. lines.
- Much better than 'vanishing pt.' methods

- With IAC, find P matrix from an image of just 3 (non-coplanar) squares…

END