CS395/495: IBMR Take-Home Midterm Exam

Assigned: 3:30pm,Thurs May 1, 2003
Due: 3:30pm,Thurs May 8, 2003

You may turn in your exam work either on paper brought to class, or by e-mail to me (jet (at) cs (dot) northwestern.edu). If you submit your work on paper, be sure to number the pages and put your name on each page. Please do your own work—do not discuss the problems or your solutions with other students until after the due date.

Projective 2-D Exercises:

1) What 3-vector describes the line in P^2 that intersects the x axis at (xp,0) and the y axis at (yp,0) ?

2) What point is at the intersection of the lines [a b c]^T and [d e f]^T?

3) Show mathematically that two parallel lines in P2 space really do intersect at infinity, and their intersection point is part of the L_∞ line.

4) Vanishing points and panoramas: Suppose we visit a farmer's field in Kansas to take pictures, where the ground is flat and level for many miles. The farmer's field plowed in long, straight, parallel furrows that reach towards the horizon. However, one set of railroad tracks cut across the farmer's field. The tracks are perfectly straight and level for many miles. The furrows are not parallel to the railroad tracks. Now suppose we stand in one place, aim the camera north, take one picture, turn about 20 degrees eastward, take another picture, turn another 20 degrees eastward, and take a third picture. There is considerable overlap in each picture, because the camera's horizontal field of view is 38 degrees. In each picture, we can see both furrows and the railroad tracks, and it is easy to define parallel line pairs for each of them. We took the photographs in springtime, and the machine-planted crop is just beginning to sprout, so that each furrow has a small green spot spaced equally along each furrow.

   On the second digital photograph, we define two pairs of parallel lines, one pair that matches the two railroad tracks, given by **La**=(La1,La2,La3), **Lb** (Lb1,Lb2,Lb3), and the other pair is aligned with two furrows in the farmer's field, given by **Lc =** (Lc1,Lc2,Lc3) and **Ld** = (Ld1,Ld2,Ld3).

   a) Find the transformation matrix H needed that will transform these into parallel lines aligned with the y axis (x2/x3) in P2. After this transformation, the image would appear to be an aerial view: all furrows will be parallel to each other, and the railroad tracks will be parallel.

   b) Explain how you would find transformation matrices H1 and H3 needed to create a 3-panel panorama. In the panorama, image 2 is not transformed at all—it is centered on the x3 axis (as it would be in your ProjectA and Project B software), and images 1 and 3 are positioned to the left and right of it by applying transformations H1 and H3. For an illustration, see page 196 (but you can do this in P2 only—there is no need for the P3-based method described there). Explain how you would find H1 and H3 in sufficient detail to implement it in your ProjectA or B code.

Conics in P^2:

5) Suppose you have photograph of a planar table-top, photographed from an unknown position. Within that image you have measured N line pairs that are linearly independent (e.g. not redundant) and named (L1a, L1b), (L2a, L2b), ...(LNa, LNb).

   a) Line pairs 1-5 are known to form 90-degree angles. Find the 3x3 C^*_∞' matrix for this photograph.

   b) In a photograph taken from a different position, we measured the angle between lines L7a and L7b as 42 degrees. If the C^*_∞' matrix for this image is:

5.0	2.5	17.5
2.5	1.25	8.75
17.5	8.75	61.25

   then what is the angle between these lines if measured in the plane of the tabletop? (in degrees, not radians).

6) a) Write the point-conic matrix C for a circle of radius r centered at (x,y) location (a,b).
   b) Write the point-conic matrix Ch for a hyperbola that intersects the x axis at (-1,0) and (1,0).
   c) Find the homography H that converts this circle C to a hyperbola Ch.

Singular Value Decomposition:

7) P^2 Conic Correspondence: Describe a method for finding a homography between an input image and an output by matching one or more conic features in the images, such as the circles formed by two wheel rims in the side view of a car or truck, or the image of several CDs placed randomly on a tabletop. Do this in two steps:

   a) Describe how you would convert points or lines positioned on a conic curve in the image into the C matrix that describes the conic. (Be specific: your explanation must be sufficiently detailed for someone to implement it a matrix class that includes an SVD-finding function).

   b) Describe how you would find a good estimate of homography H from one or more conic pairs (C, C'). It is not necessary to use the C^*_∞' in your solution.

Estimation/Optimization:

8) Construct a specific example of image rectification where the DLT solution of Chapter 3 (e.g. $Hx \times x' = 0$) works well, but the 'naïve' method (pg. 13,14: $Hx - x' = 0$) works poorly or not at all. Your answer should list at least 6 point pairs (**x**,**x'**). Describe the method you used to find this example; it should (Hint: trial and error isn't a good idea; this question doesn't require you to implement the naïve solution in software or compute it by hand).

9) Section 3.4 in the book (pp. 88--93) and the last slide of Lecture 7 describe another subtlety of the DLT method: its precision depends on the placement of the origin. Though will not need to

implement 'normalization' in your DLT code to answer this question, I want to be sure you know how to do it by solving this problem. Given these 6 noisy point pairs: (input)(output)

(100.0, 50.0, -1.0) (300.0, 250.0, 2.0)
(101.0, 50.0, -1.0) (300.6, 251.0, 2.0)
(101.0, 52.0, -1.0) (301.3, 252.0, 2.0)
(100.0, 49.0, -1.0) (299.5, 248.0, 2.0)
(105.0, 60.0, -1.0) (305.4, 259.7, 2.0)

find the normalizing and de-normalizing matrices T and T', and explain how to compute and apply them to improve the accuracy of our DLT solution. Give sufficient detail in your explanation to allow an easy implementation.

Open-Ended Discussion Questions:

10) Write a two-column list that compares IBMR to computer graphics methods, listing as many advantages and disadvantages as you can find. Your goal is to find many *reasons* as possible, not to fill many pages. Write just enough to convey the idea; most ideas can be written in one line, or just a few.

11) Explain as clearly as you can all the ways that projective transformations in P^2 differ from Cartesian transformations in either R^2 or R^3.