CS 395/495 Image Based Modeling and Rendering **Spring, 2004**
Take-Home Midterm Exam **Jack Tumblin**

Assigned: 4/29/2004
Due: 5/6/2004

1(10 pts) This line in P2 space, $L = [1\ \ 2\ \ 3]^T$ is like any other line—it is infinitely long. Find the 'ideal points' that exist on each 'end' of the line (the two endpoints are infinitely far away), and write them in P2 coordinates. Would you say the 'universe' of P2 is a circular?

2(10pts) In P2 space, what is the line that passes through each of these point pairs? Express your answer as a 3-element column vector, whose last element (x3) is 1.0.
p0 = [2 1 -3]
p1 = [-4 3 -2]

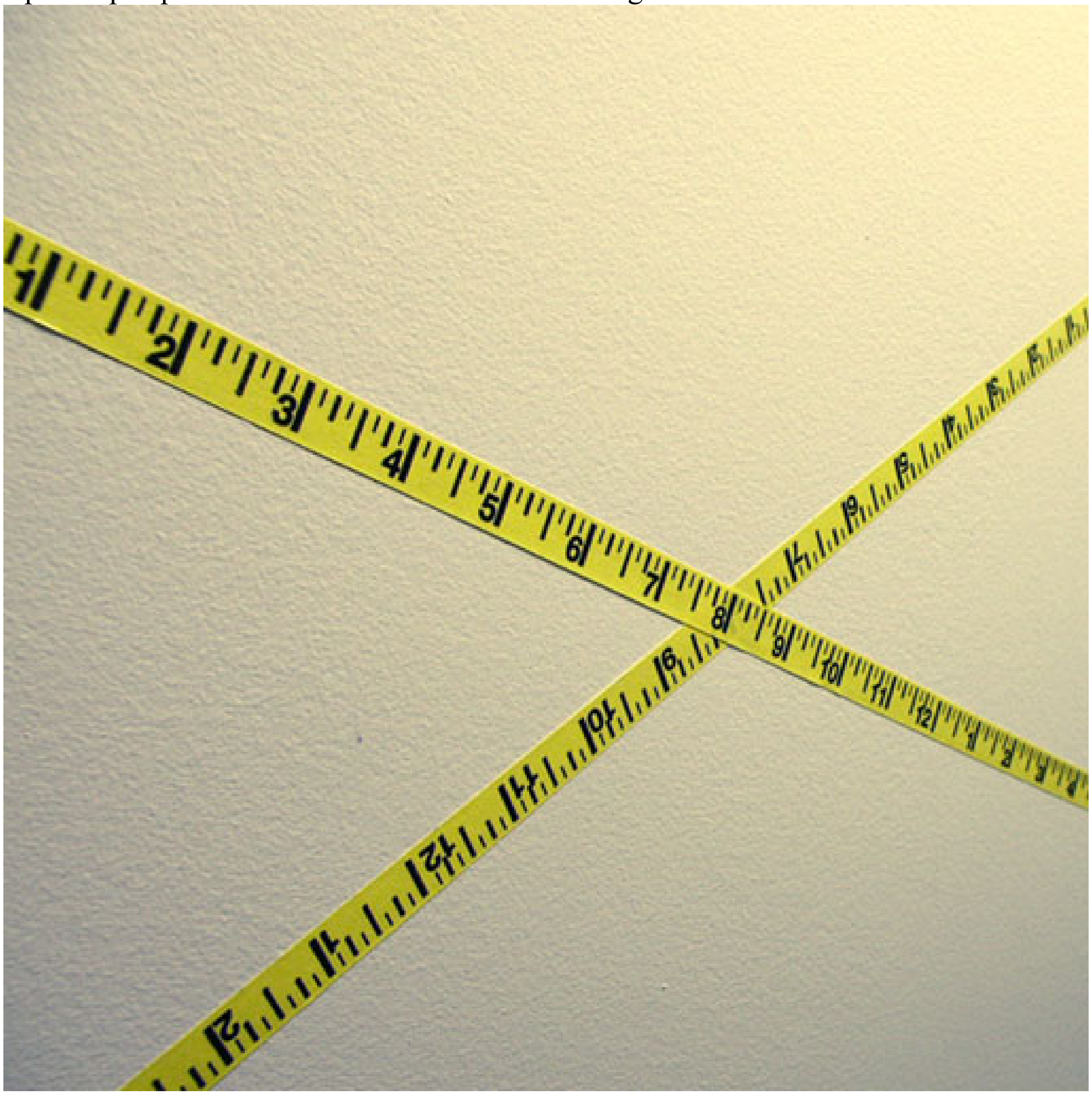
3(10pts) In P2 space, what is the point found at the intersection of lines L0 and L1? Express your answer as a 3-element column vector, whose last element (x3) is 1.0.
L0 = [2 1 -3]
L1 = [-4 3 -2]

Using the points and lines defined in questions 2 and 3, find the answers to these questions mathematically—show your work:

- **4a)(5pts)** Is point p0 found on line L0? On line L1?
- **4b)(5pts)** Is point p1 found on line L0? On line L1?
- **4c)(5pts)** How far away from L0 is point p0 measured perpendicular to L0?

Hint: write the equation of a line in Cartesian coordinates (Ax + By +C=0). What is the distance from the origin to the nearest point on that line? What homography (H matrix) will move a point [x1,x2,x3] to the origin [0,0,1]?

'Project' part of the exam: you will need to use the 'starter code' from the website, and you can find this image as a BMP file "wallTapeRuler.bmp" on the website.
I have a narrow roll of masking tape that is marked in inches like a ruler (from American Science and Surplus). I attached two strips of this tape on a flat portion of a CS department hallway wall, and placed them carefully to be sure each one formed a straight line (note that the tape strips are not perpendicular to each other!) Here is a 512x512 photograph of that wall, taken with a planar perspective camera. The camera is not aligned with the wall.



5a)(15pts) Use the cross-ratio on these two strips of tape in the image to calculate the horizon line L_h for wall, expressed in P2 coordinates for the image. Note that the two strips of tape shown are NOT perpendicular to each other, but they ARE in the same plane. You may make any measurements you wish on this image; assume the image is in the x_3=-1 plane, rows and columns of pixels align with x1 and x2 respectively, and the center of the image is at [0,0,-1],

L_h = [________, ________, ________]?

If you plotted the horizon line L_h on the photo above, you would find it is entirely outside of the photo itself (and not even on the same page of paper). But if you could somehow extend this image, leaving its current set of pixels unchanged, but adding more pixels to extend the width, height, and field-of-view of the image, then you would find that the line L_h location is approximately the end of the hallway in the image—the line where an infinitely large wall vanishes into the distance.

5b)(5pts) Write the H_p matrix that would 'rectify' this image up to a similarity (you can do this by inspection from your 5a result). Remember, 'up to a similarity' means you're not solving for the entire H matrix; you don't have enough information to solve for rotation and absolute scale.

5c)(10pts) Using the 'starter code' from the website, transform the images above by the H_p matrix you computed for each of them; make a 'before' and 'after' picture; it should look like the camera moved so that the wall is parallel to the image plane.

The two strips to tape shown in the image above have ¼-inch marks along only one edge (the other is marked in 1-inch marks); use only the ¼-inch-marked edge. Estimate the image coordinates of the location of these 5 points on the image as best you can:

P0: the point where the two edges cross.

P1 through P4: the points that are a distance of 3 inches of the location P0 as measured by the tape. You know that on the wall (but not in the image) we could use P0 as the center of a circle that passes through and P1 through P4. Call this the 'world-space' circle. In image space, it forms an ellipse.

6a)(5pts) List the points you just measured, and then ignore them. First, write the matrix C for a point-conic curve that forms a circle centered at position [x0, y0, 1] with a radius of 3.0. This is just an algebra problem to prepare you for the next steps.

To prepare for the next step, find point PC, the center of the image-space ellipse that passes through points P1,P2,P3,P4 in image space. (e.g. PC = ¼*(P1 + P2 +P3 +P4). Use PC to find 4 more points on the ellipse named P1', P2', P3', P4'. Note that a line through the center of an ellipse crosses the ellipse at two points, and both of those points are at the same distance from the ellipse center.

6d)(20pts) Explain how you could find the point-conic matrix C that best fits these 8 points in the least-squares sense by formulating it as a null-space problem; e.g. the elements of the unknown C matrix become a single vector 'c', and the 8 known point locations somehow become a big matrix A, and you find 'c' by solving Ac=0, using singular-value decomposition (SVD). Write A matrix and the c vector. Explain how you would use the U,S,V matrices that the SVD computed for you to find the 'c' vector.

6e) (20pts)EXTRA CREDIT: use the 'starter code' on the website, or use MATLAB (both give you an SVD routine) to solve for the C matrix that best fits your points. Plot a few points of the conic defined by C on the image above (better yet, draw the conic using OpenGL in the starter code), to see how well you did.