Solving Everyday Physical Reasoning Problems by Analogy using Sketches

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Abstract

Understanding common sense reasoning about the physical world is one of the goals of qualitative reasoning research. This paper describes how we combine qualitative mechanics and analogy to solve problems posed as sketches. The problems are drawn from the Bennett Mechanical Comprehension Test, which is used to evaluate technician candidates. We discuss *sketch annotations*, which define conceptual quantities in terms of visual measurements, how modeling decisions are made by analogy, and how analogy can be used to frame comparative analysis problems. Experimental results are presented indicating that this approach has promise.

1 Introduction

Understanding common sense reasoning about the physical world is one of the goals that motivated qualitative reasoning (QR) research from the beginning. Despite its success at capturing many important aspects of reasoning about technical domains, little progress has been made on applying QR ideas to common sense reasoning *per se*. A key difference between common sense problems and reasoning in technical domains is *breadth.* In domains such as electronics or thermodynamics, a small library of components and relationships between them suffice to describe the systems of interest. This is not true for everyday reasoning, where the number of types of entities that can potentially be involved is at least in the tens of thousands. A second important feature of common sense reasoning is *robustness*, by which we mean the ability to draw conclusions even with partial knowledge. QR already provides one piece of the puzzle, by enabling natural conclusions to be drawn without detailed numerical information. However, existing QR techniques tend to assume complete and correct domain theories, which are applied to construct situation-specific models as needed to solve a given problem. By contrast, (1) mental models research [12] suggests that people's models are often incomplete and incorrect, and (2) psychological evidence suggests that people often miss opportunities to apply relevant principles in everyday life. How then can we explain the robustness of common sense reasoning?

Forbus & Gentner [6] suggest that the use of analogy provides a missing piece of the puzzle. Here we do not refer to cross-domain analogies (e.g., seeing heat as a liquid), which are rare, can be risky, and are prized precisely because good ones require considerable insight. Instead, we focus on within-domain analogies, where a new situation is understood in terms of a prior example (e.g., seeing a person pushing a wheelbarrow as like another person pushing a different wheelbarrow, or a shopping cart). The prior example might have been understood in terms of the person's domain theory, but it might have also been understood in terms of an explanation that is completely specific to that example (e.g., the stability of this building decreases as its height increases). One is reminded of similar experiences, and the explanation of those experiences is applied to the current problem. Common sense is learned via experience, accumulating examples which can be used directly as analogs, and generalized to form more abstract knowledge.

Using sketches in common sense reasoning is a particularly good venue for exploring these ideas because sketches are concrete. A sketch depicts a particular system, and general principles are articulated in terms of how they apply to this specific situation. Sketches and diagrams are heavily used in teaching and learning about physical domains. For example, their importance in physical thinking is indicated by the structure of the Bennett Mechanical Comprehension Test (BMCT), an examination used to evaluate applicants for technical positions. BMCT problems consist of diagrams depicting physical scenarios, with multiple-choice questions about their qualitative properties. The BMCT is extremely broad, including questions about statics, dynamics, heat, and electricity, all stated in terms of everyday situations. The BMCT is also used by cognitive psychologists as an independent measure of mechanical aptitude and spatial ability. In QR terms, BMCT problems can be divided into two aspects: model formulation and computing the answer from the model. As indicated below, computing the answer can typically be done by existing QR techniques, with one or two extensions. The most serious difficulty is in formulating the model. The compositional modeling methodology [3, 18] assumes complete and correct domain theories, and says little about the mapping from structural descriptions to structural abstractions. We claim that the problem of mapping from the broad vocabulary of entities and relationships used in the everyday world to a more refined set that can be used to describe conceptual models is central to understanding common sense reasoning.

This paper describes a system we have constructed which solves problems from the BMCT, using the similarity-based qualitative reasoning model outlined above. It uses a new cognitive architecture, *Companion Cognitive Systems* [9], which is applying these ideas more broadly. Here we focus on three novel qualitative modeling ideas that were needed to build this system: (1) *sketch annotations* define conceptual properties in terms of visual quantities, (2) *use analogy to derive structural abstractions from structural descriptions*, and (3) *use analogy to frame comparative analyses.* We start with a brief review to ground the discussion, then describe each idea in turn. The overall architecture of the system is described next, followed by some experimental results. We end with related work not mentioned elsewhere and a discussion of future work.

2 Background

Sketching is a powerful way to work out and communicate ideas. The nuSketch model [10] takes sketching to be a combination of interactive drawing and conceptual labeling. While most sketch understanding systems focus on recognition, nuSketch systems are based on the insight that recognition is not necessary in human-to-human sketching. The *sketching Knowledge Entry Associate* (sKEA) is the first open-domain sketch understanding system, built on a large knowledge base (1.2 million facts about 39,000 concepts). The breadth of this KB makes it an excellent platform for exploring common sense reasoning.

Glyphs are the basic constituent of sketches. A glyph consists of its *ink*, which indicates its visual properties, and its *entity*, which is the thing depicted by the glyph. Entities can be instances of any of the concepts in the KB. Sketches are further structured into *bundles* and *layers*. Bundles formally capture the way that people divide up a complex sketch into subsketches. In this paper all of the sketches involve only a single bundle, so we ignore bundles in this paper. Layers decompose different aspects of a subsketch, e.g., two systems being compared side by side would be drawn in the same bundle, but each system on a different layer. sKEA computes a variety of visual relationships between glyphs based on ink [20]. For example, Cohn's RCC8 qualitative topology relationships [1] are computed on every pair of glyphs in a layer.

We use Gentner's *structure-mapping* theory of analogy and similarity [11]. In structure-mapping, analogy and similarity are defined in terms of structural alignment processes operating over structured representations. The output of this comparison process is one or more *mappings*, constituting a construal of how the two entities, situations, or concepts (called *base* and *target*) can be aligned. A mapping consists of a set of *correspondences*, a set of *candidate inferences*, and a *structural evaluation score.* A correspondence maps an item (entity or expression) from the base to an item in the target. A candidate inference is the surmise that a statement in the base might hold in the target, based on the correspondences. The structural evaluation score indicates overall match quality.

We use two cognitive simulations based on structure-mapping theory here. The Structure-Mapping Engine (SME) does analogical mapping [2]. SME uses a greedy algorithm to compute approximately optimal mappings in polynomial time [7]. The base and target descriptions can be pre-stored cases, or dynamically computed based on queries to a large knowledge base [16]. MAC/FAC [8] models similarity-based retrieval. The first stage uses a special kind of feature vector, automatically computed from structural descriptions, to rapidly select a few (typically three) candidates from a large memory. The second stage uses SME to compare these candidates to the probe description, returning one (or more, if very close) of them as what the probe reminded it of. As performance systems, both SME and MAC/FAC have been used successfully in a variety of different domains, and as cognitive models, both have been used to account for a variety of psychological results [5].

3 Sketch Annotations

In everyday sketching, people annotate sketches of physical entities with conceptual information, depicting information that would not appear in the actual situation. In architectural drawings, annotations indicate distances between walls and the widths of windows. In sketches explaining principles, annotations indicate important properties, such as physical quantities (e.g., width of the base of a ladder) and where forces are applied. We introduced *annotation glyphs* to provide this capability. Like other glyphs, an annotation glyph consists of its ink and the entity it is representing. However, annotation glyphs also refer to one or more other glyphs in the sketch, depicting the entity (or entities) that they are providing information about. We call these the *references* for the annotation glyph.

We have found four kinds of annotation glyphs useful to date. Linear annotations indicate linear distances, either along a single reference or between two references. Two special subclasses of linear annotations are X-coordinate and Y-coordinate annotations, which refer to the projection of the glyph to the appropriate axis. Force annotations indicate where, and in what direction, a force is applied to the reference. Rotational annotations indicate the references direction of rotation. Figures 1a and 1b illustrate each type of annotation.

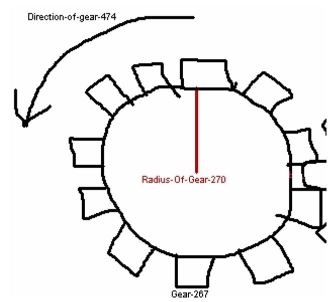

Figure 1a: Gear with length and counter clockwise rotation annotations

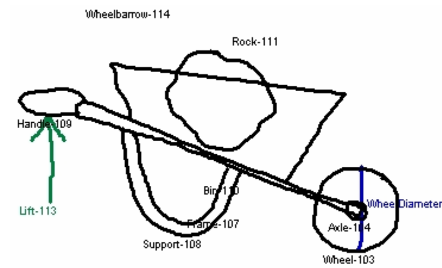

Figure 1b: Wheelbarrow with assumed force and length annotations

Annotation glyphs express relationships about their reference(s) in three ways. For annotations indicating directions (i.e., forces and rotations), the ink of the glyph is interpreted as an arrow which indicates what direction is being referred to. In the case of force glyphs, the intersection of the head of the arrow with the recipient of the force also indicates the location of the applied force. Thirdly, the rotational motion annotations assume qualitative rotational motion information about their reference. For annotations indicating distances, *anchor points* are used to specify which part of the reference that the annotation is tied to. Each glyph has nine anchor points: Its centroid, the leftmost, rightmost, bottommost, and topmost points, and the left topmost, right topmost, etc. Anchor points provide symbolic descriptions that can be projected as candidate inferences from an example to a new situation (e.g., the distance from the left bottommost point to the right bottommost point). For example, the symbolic expression of the linear annotation glyph on the gear in Figure 1a is `(startPointOf (AnnotationGlyphFn Object-270 Layer-241) (CentroidPointFn (GlyphFn Object-267 Layer-241)))` and `(endPointOf (AnnotationGlyphFn Object-270 Layer-241) (LeftmostTopPointFn (GlyphFn Object-267 Layer-241)))`.

Annotation glyphs link visual properties to conceptual properties. For example, in describing stability of a building, one wants to say that the stability of the building (a continuous conceptual property) decreases as its height (a continuous visual property) increases. Annotation glyphs allow us to do this by defining visual quantities in terms of measurable properties of glyphs. These visual quantities can be used in situation-specific causal models. In the building example, for instance, where `Object-226` represents the building:

```
(qprop-
  ((QPQuantityFn Stability)
     Object-226)
  ((ConceptKnownAsFn "Height")
    (GlyphFn Object-226 Layer-248)))
```

This example-specific statement can be applied by analogy to other situations, thus providing a means of formulating models even without a complete and correct domain theory.

4 Qualitative Mechanics

Qualitative mechanics (QM) concerns the same material as traditional mechanics, e.g., the effects of energy and forces on bodies, but from a qualitative perspective. We assume that people learn many aspects of qualitative mechanics in infancy, so we treat the structural abstractions and model fragments describing qualitative mechanics as part of the starting endowment of the system, rather than as something to be learned. Our QM domain theory is drawn from [14, 17]. Specifically, we use their qualitative representations of surfaces, force transfer, torque transfer, and center of rotation.

How these structural abstractions can and should be applied to everyday situations is learned by our system, via sketched examples. When a person is entering an example, they need to provide both the everyday concepts that are used to describe the entities in the system and the appropriate structural abstractions. This is done in two ways. First, the interface used for conceptual labeling includes both types of concepts. For example, the chassis of a wheelbarrow might be indicated to be an instance of `VehicleChassis` (an everyday concept) and as an instance of `RigidObject` (a QM abstraction). Second, sKEA automatically computes *visual/conceptual relationships* between pairs of glyphs if they are touching or one is inside another (as indicated by the computed RCC8 relationships). sKEA offers the user the opportunity to specialize these relationships to provide QM relationships, e.g., that the beam of a seesaw can pivot around the seesaw's base.

Sometimes systems must be viewed from multiple perspectives. In understanding how a wheelbarrow works, for example, it makes sense to draw the individual parts, since each contributes differently to how it functions. But if we are considering how hard it will be to lift a wheelbarrow, we need to consider these entities as a single rigid object. Consequently, we extended sKEA to include *group glyphs* to handle such situations.

By noting what QM concepts and relationships are applicable to each aspect of an example, analogy can be used to apply this information to new situations. The everyday properties and relationships provide the commonalities needed to be reminded of relevant examples and the information needed to align the new situation with the example. The technical vocabulary of QM can then be projected, via candidate inferences, to the new situation. This enables the first-principles QM domain theory to be applied to the new situation.

To ensure that users explain an example well, we have incorporated model formulation into the example-drawing process. When entering an example, users periodically ask the system to formulate a model for it, based on what it understands so far. If model fragment instances that they expect to occur are missing, that means they have not yet provided enough structural abstraction information to allow the system to understand it. They continue to add structural abstraction information until they are satisfied. Once satisfied, the example and explanation are entered into a case library for that user.

5 Problem Solving

There are two kinds of questions in the BMCT. *Outcome questions* ask the examinee to select which qualitative behavior will occur in a situation, e.g., "which way will the indicated gear turn: clockwise, counterclockwise, or not at all?" *Comparative analysis* questions concern relative values of the same (or similar) properties between two (or sometimes three) distinct scenarios. Both kinds of questions rely on the same process of model formulation via analogy, so we begin there.

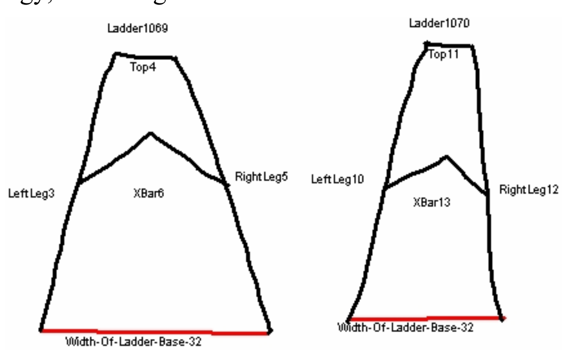

Figure 2: Ladder stability determined through base width annotation measurement

Solving a problem begins by using MAC/FAC to retrieve a relevant example. We found that retrieval worked best when only conceptual information and high-level visual information from the problem were included in the probe; including low-level visual detail (e.g., qualitative orientations of each surface) tended to reduce retrieval accuracy. When MAC/FAC returned multiple examples, the one mentioning the sought quantity with the highest structural evaluation score was chosen. Candidate inferences from the mapping are examined for conjectures about structural abstractions and causal models. All such conjectures are assumed to be true. The causal model from the example is augmented by applying the QM domain theory to the situation. Note that this critically relies on the structural abstraction information imported by analogy from the example: Without it, there would be no model fragments. This completes the model formulation step.

Once the model is formulated, solving an outcome problem involves standard qualitative reasoning. For instance, when asked which way a gear is rotating, the QM domain theory model fragments are used to find the answer. For comparative analysis questions, we do model formulation for each of the systems being compared. (While some problems involve three systems, the principles are the same so we only discuss two-system examples.) Thus for the example of Figure 2, a causal model is built up for each ladder independently. (This may seem like extra work, but the two systems being compared might be quite different in structure.) Next, we compare the situations, using SME. This mapping provides the correspondences between the two systems needed for differential qualitative analysis (DQA). This is an important departure from [21]: We discovered that analogy provides a general mechanism for providing the frame of reference needed for comparative analyses. This allows a wider class of systems to be analyzed, since the correspondences between systems are computed dynamically. For example, some problems require DQA between different parts of the same system, e.g., which wheel of a railcar presses harder on the rail, when the load is nearer to the back wheel? This does not fit the standard "perturbed system" format of DQA. But SME creates an appropriate reference frame by mapping the railcar to itself, constraining the rear wheel to correspond to the front wheel.

Once the reference frame is set up, differential qualitative analysis proceeds by chaining backward from the goal quantity through the causal model. Conceptual quantities that are not causally constrained by other parameters and are not known to be different are assumed to be the same, i.e., a value of `UnchangedDQ`. DQ values for visual quantities are measured from the sketch. For example, in Figure 2, the annotations defining the base widths, combined with measurements on the sketch, indicate that the width of the base decreases from the left ladder to the right ladder, which in turn implies that the stability of the ladder decreases from the left situation to the right situation.

6 An Experiment

We conducted an experiment to see how well this model performs. We selected 13 problems from the BMC, focusing on problems involving net force, revolution rate, stability, and smoothness of ride. We developed a list of 18 example situations, 15 of which were intended to be good analogs for specific test questions. These examples were described via a single sentence of text. We recruited three graduate students, with varying degrees of familiarity with sKEA, to serve as knowledge enterers (KEs), drawing each of the situations on the list. They were told to sketch each situation, breaking it down into glyphs just far enough to explain the principle(s) that they thought were operating in the situation. While drawing, they used sKEA's visual/conceptual relationships interface to fill in their intended relationships, selecting from the candidates offered. KEs were also given a list of non-QM conceptual parameters that were relevant in this subset of the test (smoothness of ride, revolution rate, and stability), and asked to add these to the sketch when appropriate, along with their causal model and how they are constrained by any visual quantities. The visual quantities were defined via annotations, and the other aspects of the causal model were entered via a concept map interface linked to sKEA. After they considered their sketch complete, the QM system was run to derive model fragment instances. If they were not satisfied with the model fragments found, they were encouraged to modify their sketch (typically by adding structural abstraction information, through sKEA's standard conceptual labeling mechanism, or a visual/conceptual relationship) until they were. For example, in a sketch depicting two meshed gears and one of the gears is rotating (as indicated by an annotation), then there is something wrong if there is no mention of torque transfer in the active model fragment list. Once finished, each example sketch was stored in a case library for that particular KE.

Example Library	Number Correct	%
KE1	4	31%
KE2	8	62%
KE3	2	15%

Table 1: Problem Solving Results

The problems were drawn by a fourth graduate student, and given to the system, using each KE's case library in turn. The 13 problems included 10 basic differential qualitative analysis problems, 2 qualitative mechanics problems, and 1 differential qualitative analysis problem that required comparisons between three objects. The results are summarized in Table 1. KE2 had the most amount of experience with sKEA, and KE3 had the least. The result for KE2's cases is statistically significant ($P < 0.04$), but the other two are not. Analyzing the causes of failures is illuminating. There are two broad kinds of failures the system can exhibit. The first is retrieving a poor example, an example so far off that its causal model could never yield an appropriate answer for this problem. The second is not being able to use the example it retrieved to solve the problem. That is, the causal model is potentially applicable, but the system could not apply it correctly or could not solve it once it was applied. Scrutinizing the retrieval results is illuminating. Let us consider a retrieval to be correct if the example retrieved was intended to be one of the analogs for that test problem. The results are summarized in Table 2.

These values are all statistically significant ($P < 10^{-5}$), which is good news: MAC/FAC is returning examples that should be reasonable. Let us now consider the second problem, of not being able to apply the example. Even if the example is a correct retrieval, there is a lot of room for how to construe each situation, given that each KE started with a single sentence describing the situation. Using a sentence instead of a diagram was our attempt to introduce variability into our system, and not give too much away by giving them a drawing. That it did. Consider the instruction to draw "A person carrying a bundle with a pole." The problem sketch includes glyphs for the person, their hand, their body, the stick, and the bag. The KE sketches for this problem were more elaborate, and the system thus tended to retrieve other examples containing the same entities, because they were constructed more simply.

Example Library	Correct Retrievals	%
KE1	10	77%
KE2	10	77%
KE3	7	54%

Table 2: Retrieval Results

Eleven out of the 13 problem solving failures were due to problems mapping the retrieved causal model, and the other two were mapping failures in setting up the appropriate reference frame for DQA. When sketches decompose systems in different ways, leading to different numbers and types of glyphs, matches tend to fail because of the 1:1 constraint.

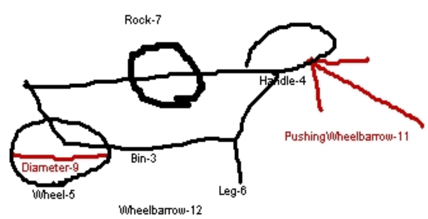

Figure 3a. Example Wheelbarrow

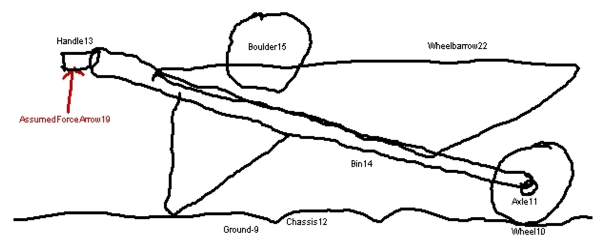

Figure 3b. Problem Wheelbarrow

Figure 3 illustrates an error in mapping with the correct retrieval. There are different numbers of glyphs in these sketches, and the entities that the glyphs depict are of different types. The problem sketch contains glyphs for the ground and the axle, but the example does not. The chassis in the problem sketch is conceptually labeled as a leg in the example. Fundamental differences like these lead to incorrect mappings, e.g., the bin in the example to the chassis in the problem. This leads to incorrect candidate inferences, which in turn forces the system to attempt to reason about surfaces or glyphs that do not exist. Currently the system fails when it detects such problems.

People seem to have several methods for dealing with such problems. First, they use *rerepresentation* [23] to bring the base and target into closer alignment. Knowledge about depiction seems crucial: If two sketches are misaligned, simplifying the more complex one, or postulating new glyphs in the simplified one, seems to be a promising strategy. Second, people try other examples, going back to memory to find an example that is more congenial. We plan to explore both techniques.

7 Related Work

Previous efforts in qualitative mechanics have created systems that can reason about clocks [17], mechanisms [13, 19], and internal combustion engines [14]. Some researchers use high-resolution inputs or CAD systems to produce structural descriptions containing metric information which can also be exploited by the system. Aside from the annotations described here, we do not exploit metric information at all. SketchIt [19] used sketched input, but allowed only a handful of abstract types to be drawn.

The use of experience in model formulation was proposed by [4], in the context of improving compositional modeling by choosing appropriate perspectives and levels of detail. Unlike this effort, he assumed a complete and correct domain theory as a starting point.

Similarity-based qualitative simulation [22] also uses analogy for qualitative reasoning, but focuses on multi-state reasoning. Here the problems all involve single-state reasoning.

8 Discussion

This work provides evidence that qualitative reasoning combined with analogical reasoning is a promising explanation for how common sense reasoning works. The mapping from structural descriptions to structural abstractions, we hypothesize, is learned incrementally from examples. As shown here, examples can be used by analogy to aid in formulating models for new situations. Similarly, sketch annotations provide a means for defining visual quantities in examples which can be used by analogy to define visual quantities for new situations. Analogy also plays an important role in qualitative analysis once the model has been formulated, e.g. our use of mappings as frames of reference for differential qualitative analysis.

As noted earlier, there are several paths to explore in future work. Variations on the baseline model of analogical processing used here constitute one line of investigation. This includes more aggressive retrieval strategies (e.g., trying different examples, using multiple examples), rerepresentation to improve mappings, and using our SEQL model [15] to learn generalizations from examples. We are also designing an interactive interface so that we can expand the system's capabilities by teaching it. For instance, given an incorrect mapping or retrieval, providing feedback about what a better answer would be gives the system evidence on how to reorganize its methods for encoding situations, which should lead to improved performance. Our goal is to expand its capabilities via instruction to include the full range of phenomena covered in the Bennett Mechanical Comprehension test. Being able to perform at an expert level on such an exam would be a landmark in qualitative reasoning.

Acknowledgements

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