

Register Allocation, i

Overview & spilling

L1

```
p ::= (label f ...)  
f ::= (label nat nat i ...)  
i ::= (w <- s)  
    | (w <- (mem x n8))  
    | ((mem x n8) <- s)  
    | (w aop= t)  
    | (w sop= sx)  
    | (w sop= num)  
    | (w <- t cmp t)  
    | label  
    | (goto label)  
    | (cjump t cmp t label label)  
    | (call u nat)  
    | (call print 1)  
    | (call allocate 2)  
    | (call array-error 2)  
    | (tail-call u nat0-6)  
    | (return)
```

```
aop ::= += | -= | *= | &=
```

```
sop ::= <<= | >>=
```

```
cmp ::= < | <= | =
```

```
u ::= x | label
```

```
t ::= x | num
```

```
s ::= x | num | label
```

```
x ::= w | rsp
```

```
w ::= a | rax | rbx | rbp | r10 | r11 | r12 | r13 | r14 | r15
```

```
a ::= rdi | rsi | rdx | sx | r8 | r9
```

```
sx ::= rcx
```

```
label ::= sequence of chars matching #rx"^[a-zA-Z_][a-zA-Z_0-9]*$"
```

L2

```
p ::= (label f ...)  
f ::= (label nat nat i ...)  
i ::= (w <- s)  
    | (w <- (mem x n8))  
    | ((mem x n8) <- s)  
    | (w aop= t)  
    | (w sop= sx)  
    | (w sop= num)  
    | (w <- t cmp t)  
    | label  
    | (goto label)  
    | (cjump t cmp t label label)  
    | (call u nat)  
    | (call print 1)  
    | (call allocate 2)  
    | (call array-error 2)  
    | (tail-call u nat0-6)  
    | (return)  
    | (w <- (stack-arg n8))  
aop ::= += | -= | *= | &=  
sop ::= <<= | >>=  
cmp ::= < | <= | =  
u ::= x | label  
t ::= x | num  
s ::= x | num | label  
x ::= w | rsp  
w ::= a | rax | rbx | rbp | r10 | r11 | r12 | r13 | r14 | r15  
a ::= rdi | rsi | rdx | sx | r8 | r9  
sx ::= rcx | var  
label ::= sequence of chars matching #rx"^[a-zA-Z_][a-zA-Z_0-9]*$"  
var ::= sequence of chars matching #rx"^[a-zA-Z_][a-zA-Z_0-9]*$"
```

L2 semantics: variables

L2 behaves just like L1, except that non-reg variables are function local, e.g.,

```
(define (f x)      ⇒  (:main
  (+ (g x) 1))    (:main
                  (rdi <- 10)
                  (call :f))
(define (g x)      (:f (temp <- 1)
  (+ x 2))        (call :g)
                  (rax += temp)
                  (return))
(f 10)           (:g (temp <- 2)
                  (rax += temp)
                  (return)))
```

The assignment to `temp` in `g` does not break `f`, but if `temp` were a register, it would.

L2 semantics: stack-arg

L2 has a convenience for accessing stack-based arguments: `(stack-arg n8)`. It is equivalent to `(mem rsp ?)` where the `?` is the `n8`, plus enough space for the spills. That is, `(stack-arg 0)` is always the last stack argument, `(stack-arg 8)` is always the second to last argument, etc.

This means that if the second natural number in the function header changes, then the `stack-arg` references don't have to change – they will still be referring to the same arguments

From L2 to L1

Register allocation, in three parts; for each function body we do:

- **Liveness analysis** \Rightarrow interference graph (nodes are variables; edges indicate “cannot be in the same register”)
- **Graph coloring** \Rightarrow register assignments
- **Spilling**: coping with too few registers
- Bonus part, **coalescing** eliminating redundant $(x \leftarrow y)$ instructions

Example Function

```
int f(int x) = 2x2 + 3x + 4
```

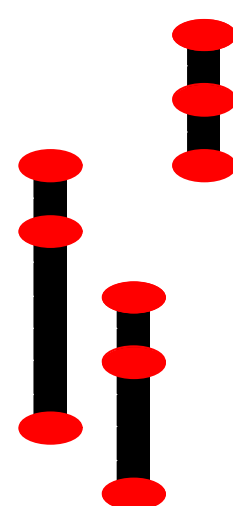
```
(:f  
1 0  
(x2 <- rdi)  
(x2 *= x2)  
(dx2 <- x2)  
(dx2 *= 2)  
(tx <- rdi)  
(tx *= 3)  
(rax <- dx2)  
(rax += tx)  
(rax += 4)  
(return))
```

Example Function: live ranges

```
int f(int x) = 2x2 + 3x + 4
```

dx2 tx x2

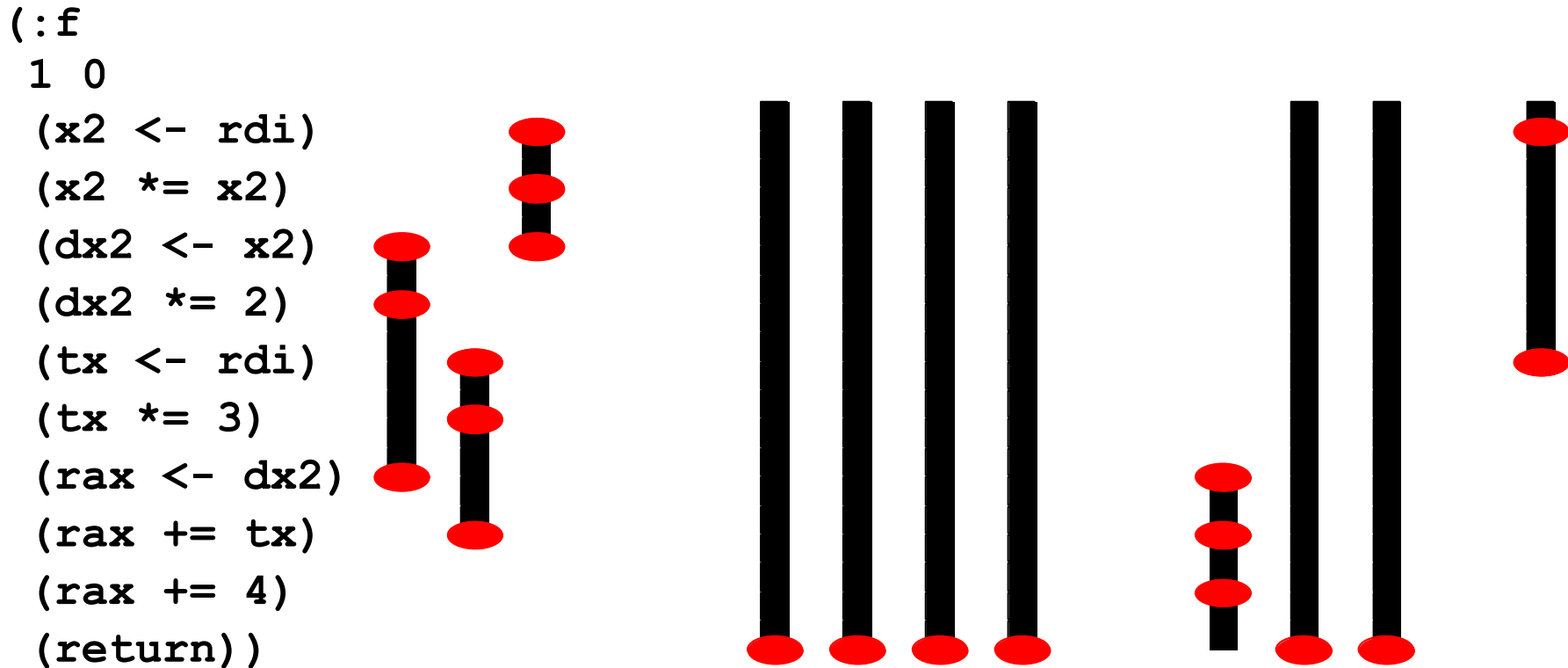
```
(:f  
1 0  
(x2 <- rdi)  
(x2 *= x2)  
(dx2 <- x2)  
(dx2 *= 2)  
(tx <- rdi)  
(tx *= 3)  
(rax <- dx2)  
(rax += tx)  
(rax += 4)  
(return))
```



Example Function: live ranges

```
int f(int x) = 2x2 + 3x + 4
```

dx2 tx x2 r10 r11 r12 r13 r14 r15 r8 r9 rax rbp rbx rcx rdi rdx rsi



Example Function 2

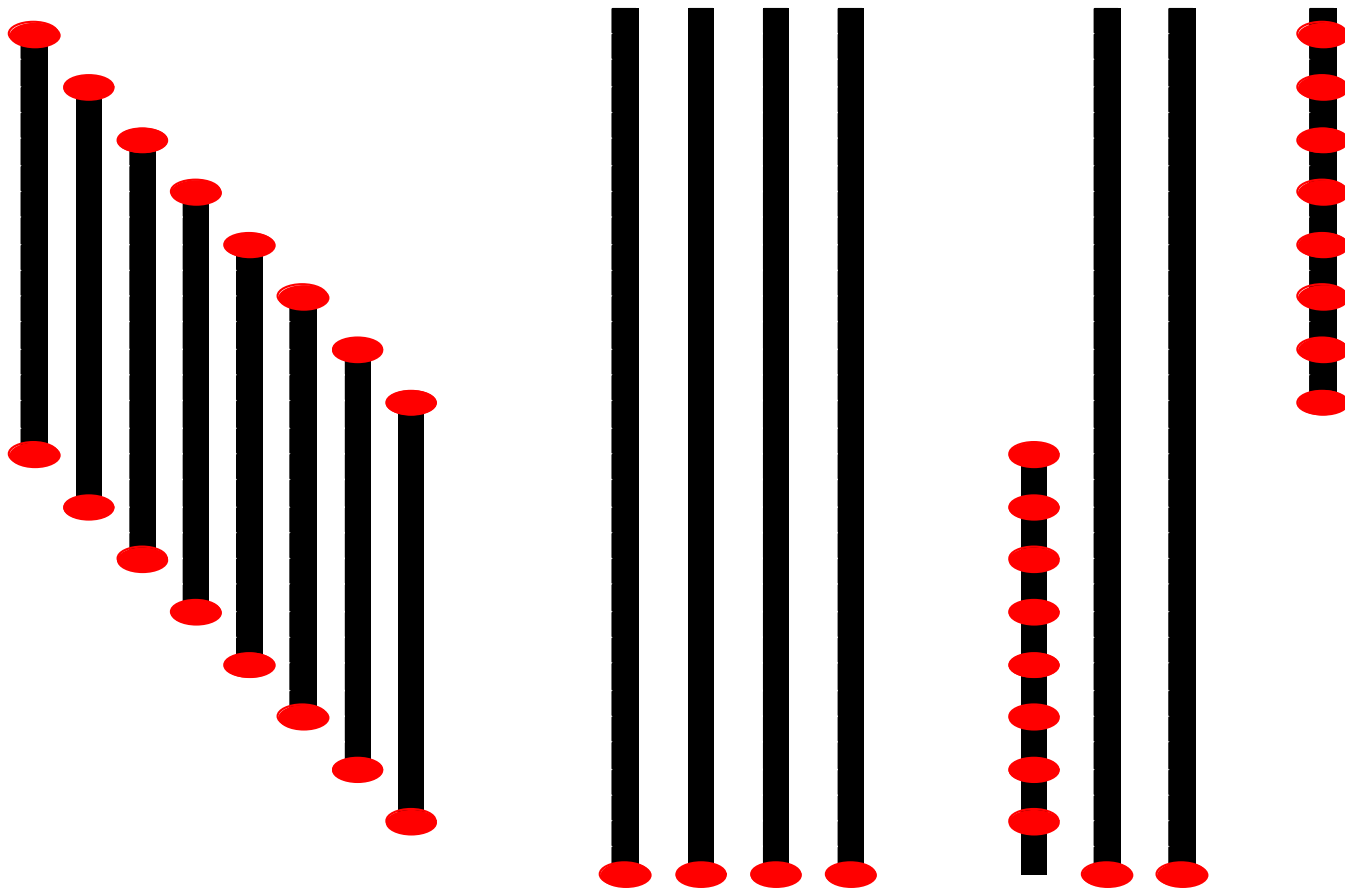
`int f(int x) = x+x+x+x+x+x+x+x` (in a stupid compiler)

a b c d e f g h r10 r11 r12 r13 r14 r15 r8 r9 rax rbp rbx rcx rdi rdx rsi

(:f

1 0

```
(a <- rdi)
(b <- rdi)
(c <- rdi)
(d <- rdi)
(e <- rdi)
(f <- rdi)
(g <- rdi)
(h <- rdi)
(rax <- a)
(rax += b)
(rax += c)
(rax += d)
(rax += e)
(rax += f)
(rax += g)
(rax += h)
(return)
```



No way to get all of **a**, **b**, **c**, **d**, **e**, **f**, **g**, and **h** into their own registers; so we need to *spill* one of them.

Spilling

Spilling is a program rewrite to make it easier to allocate registers

- Pick a variable
- Make a new location on the stack (increment the second `nat` in the function definition)
- Replace all writes to the variable with writes to the new stack location
- Replace all reads from the variable with reads from the new stack location

Sometimes that means introducing new temporaries

Spilling Example

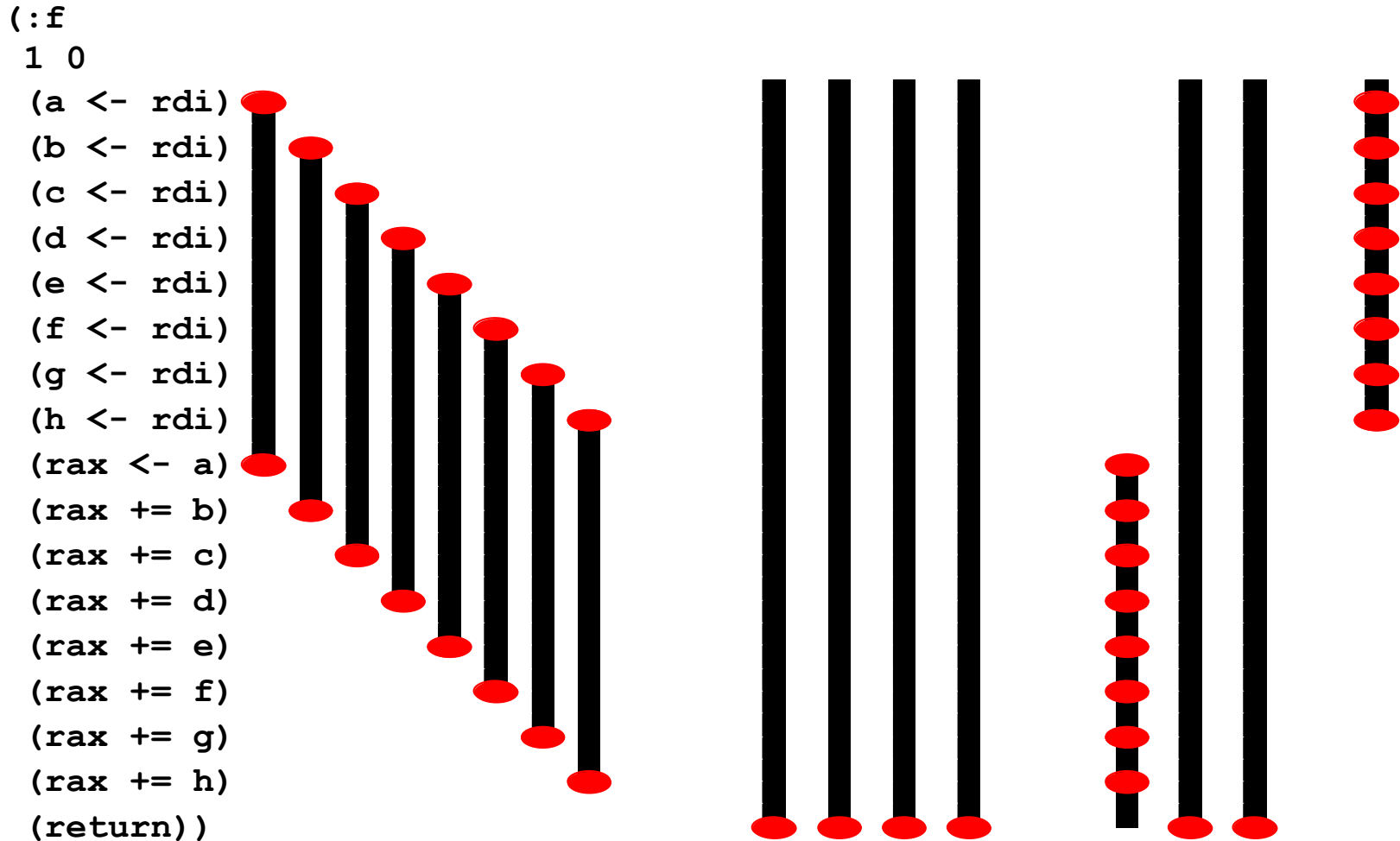
Say we want to spill **a** to first location on the stack,
`(mem rsp 0)`; two easy cases:

$$(a \leftarrow 1) \Rightarrow ((\text{mem } \text{rsp } 0) \leftarrow 1)$$
$$(x \leftarrow a) \Rightarrow (x \leftarrow (\text{mem } \text{rsp } 0))$$

Example Function 2, need to spill

`int f(int x) = x+x+x+x+x+x+x+x` (in a stupid compiler)

a b c d e f g h r10 r11 r12 r13 r14 r15 r8 r9 rax rbp rbx rcx rdi rdx rsi

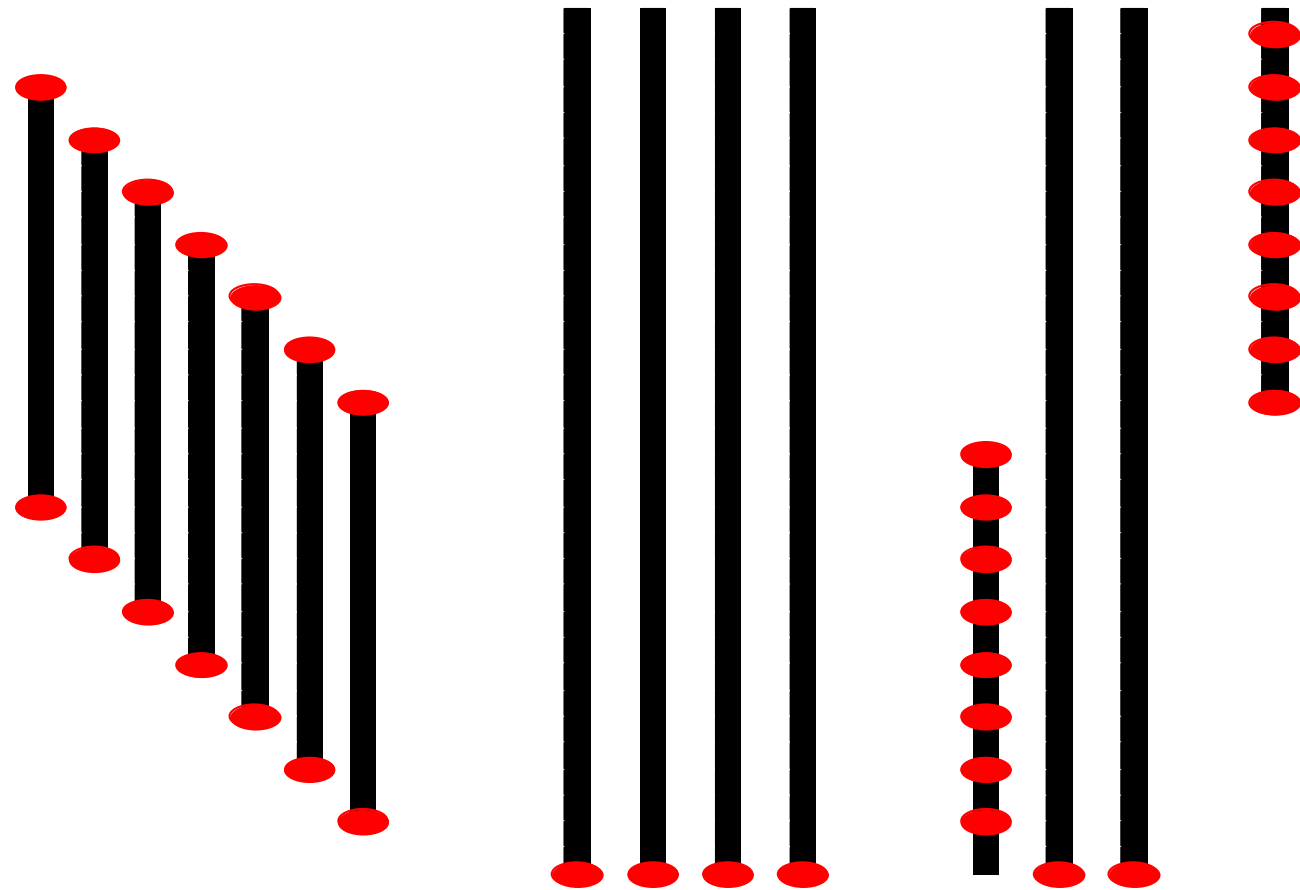


Example Function 2, spilling a

`int f(int x) = x+x+x+x+x+x+x+x` (in a stupid compiler)

b c d e f g h r10 r11 r12 r13 r14 r15 r8 r9 rax rbp rbx rcx rdi rdx rsi

```
(:f
1 1
(mem rsp 0) <- rdi)
(b <- rdi)
(c <- rdi)
(d <- rdi)
(e <- rdi)
(f <- rdi)
(g <- rdi)
(h <- rdi)
(rax <- (mem rsp 0))
(rax += b)
(rax += c)
(rax += d)
(rax += e)
(rax += f)
(rax += g)
(rax += h)
(return))
```



Spilling Example

A trickier case:

```
(a *= a) ⇒ (a_new <- (mem rsp 0))  
           (a_new *= a_new)  
           ((mem rsp 0) <- a_new)
```

In general, make up a new temporary for each instruction that uses the variable to be spilled

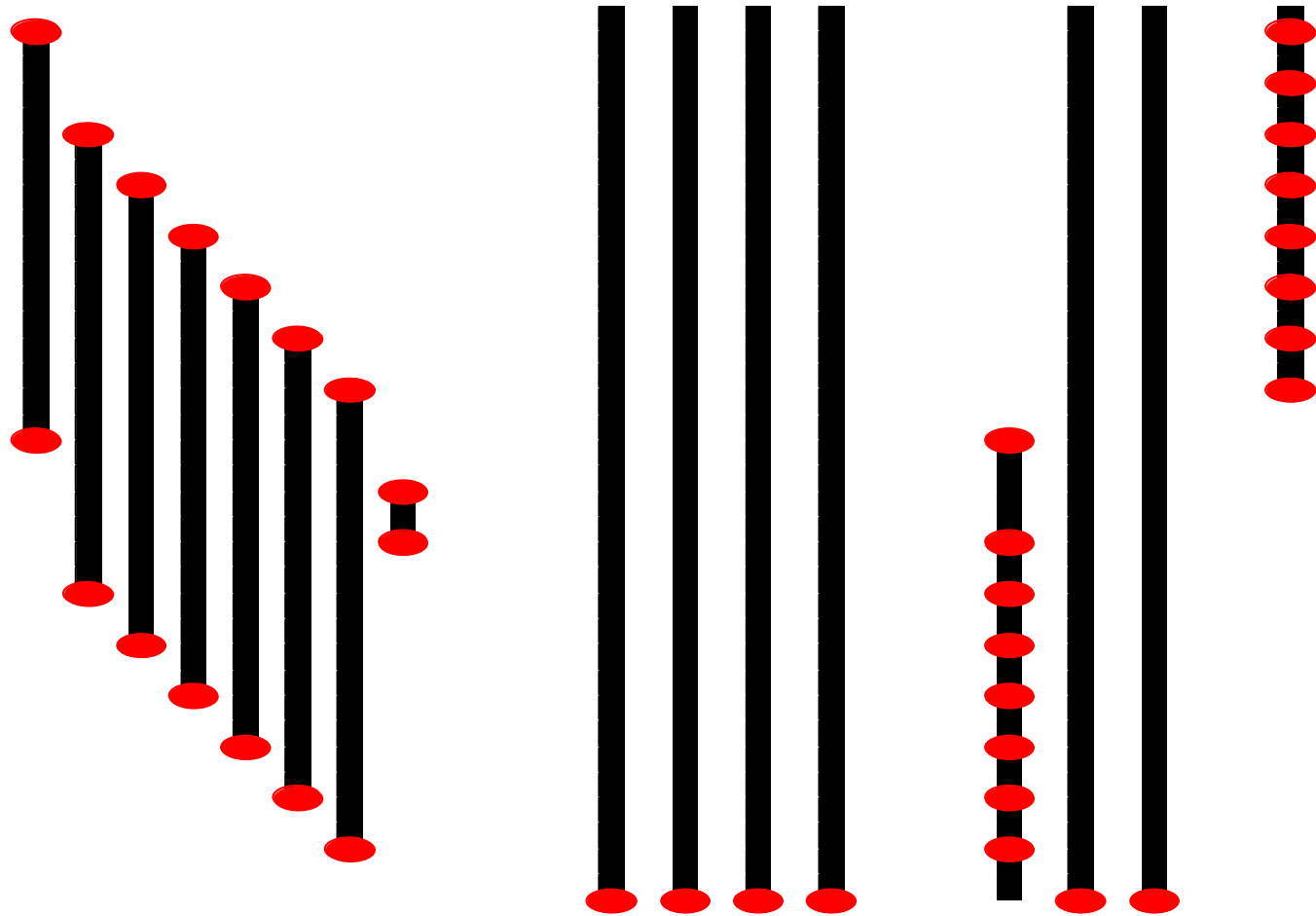
This makes for very short live ranges.

Example Function 2, spilling b

`int f(int x) = x+x+x+x+x+x+x+x` (in a stupid compiler)

a c d e f g h s0 r10 r11 r12 r13 r14 r15 r8 r9 rax rbp rbx rcx rdi rdx rsi

```
(:f
1 1
(a <- rdi)
(mem rsp 0) <- rdi)
(c <- rdi)
(d <- rdi)
(e <- rdi)
(f <- rdi)
(g <- rdi)
(h <- rdi)
(rax <- a)
(s0 <- (mem rsp 0))
(rax += s0)
(rax += c)
(rax += d)
(rax += e)
(rax += f)
(rax += g)
(rax += h)
(return))
```



Example Function 2, spilling b

Even though we still have eight temporaries, we can still allocate them to our seven unused registers because the live ranges of `s0` and `a` don't overlap and so they can go into the same register.

Your job

Implement:

```
spill : (label nat nat i ...) ;; original func
var      ;; to spill
var      ;; prefix for temporaries
-> (label nat nat i ...) ;; spilled func
```

Here's how two example spilled functions from the earlier slides would look like as calls to spill:

```
(spill «the original program»  
  'a  
  's)
```

```
(spill «the original program»  
  'b  
  's)
```

See the assignment handout for more details on the precise spec for test cases and your spill function's interface