

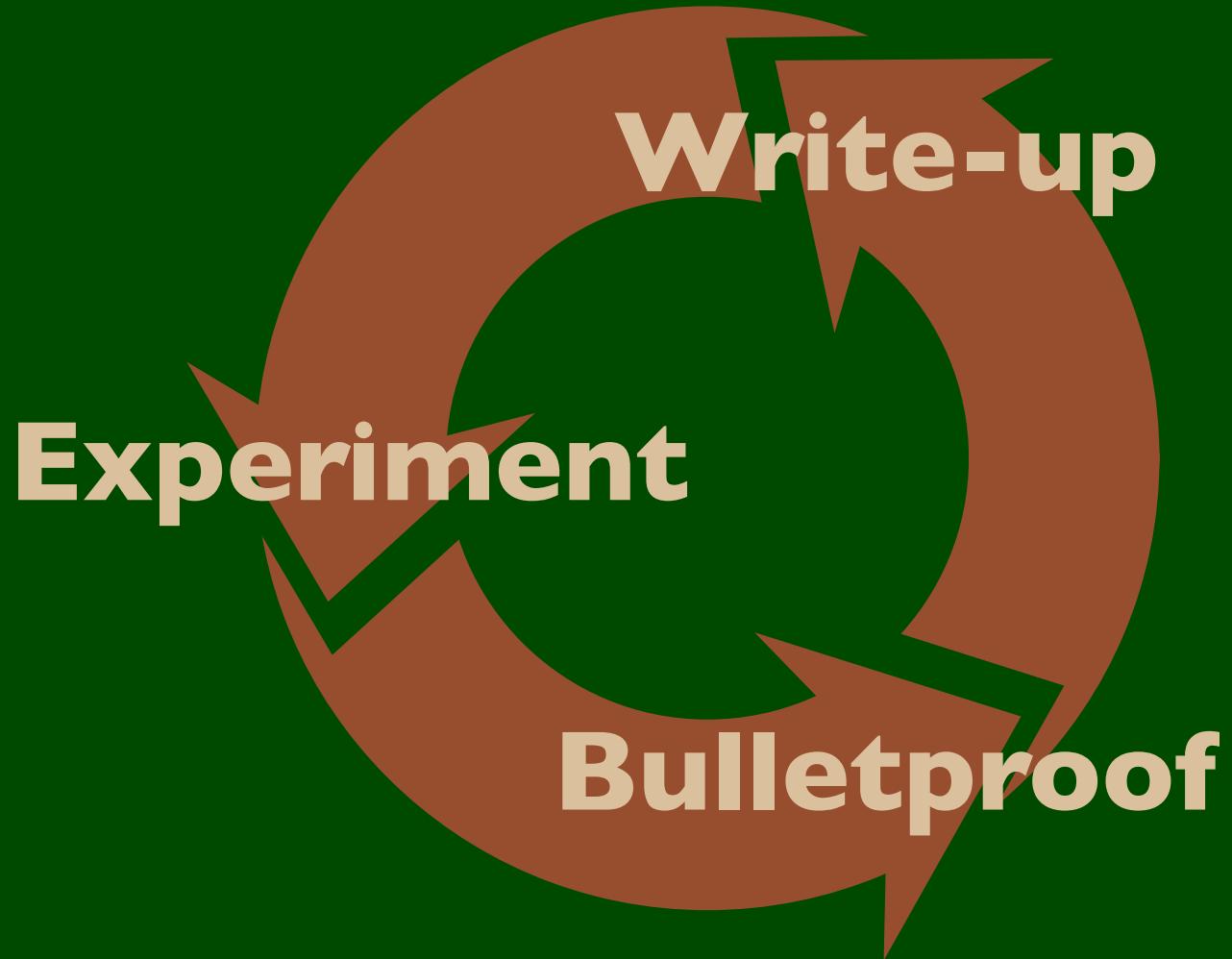
Semantics Engineering: more than just Theorem Proving

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Semantics Lifecycle



Outline

- **An overview of Redex**
- **An in-class assignment**

Outline

- **An overview of Redex**

- Operational semantics
- Redex's DSL & tools
- First-order class model

- **An in-class assignment**

- Develop a higher-order variant of model
- How good is random testing?
- Plug for Racket's class system

$\mathcal{E}val$: program \rightarrow answer

$\mathcal{E}val(p) = a$ iff $\cdots p \cdots a \cdots$

$\mathcal{E}val$: program \rightarrow answer

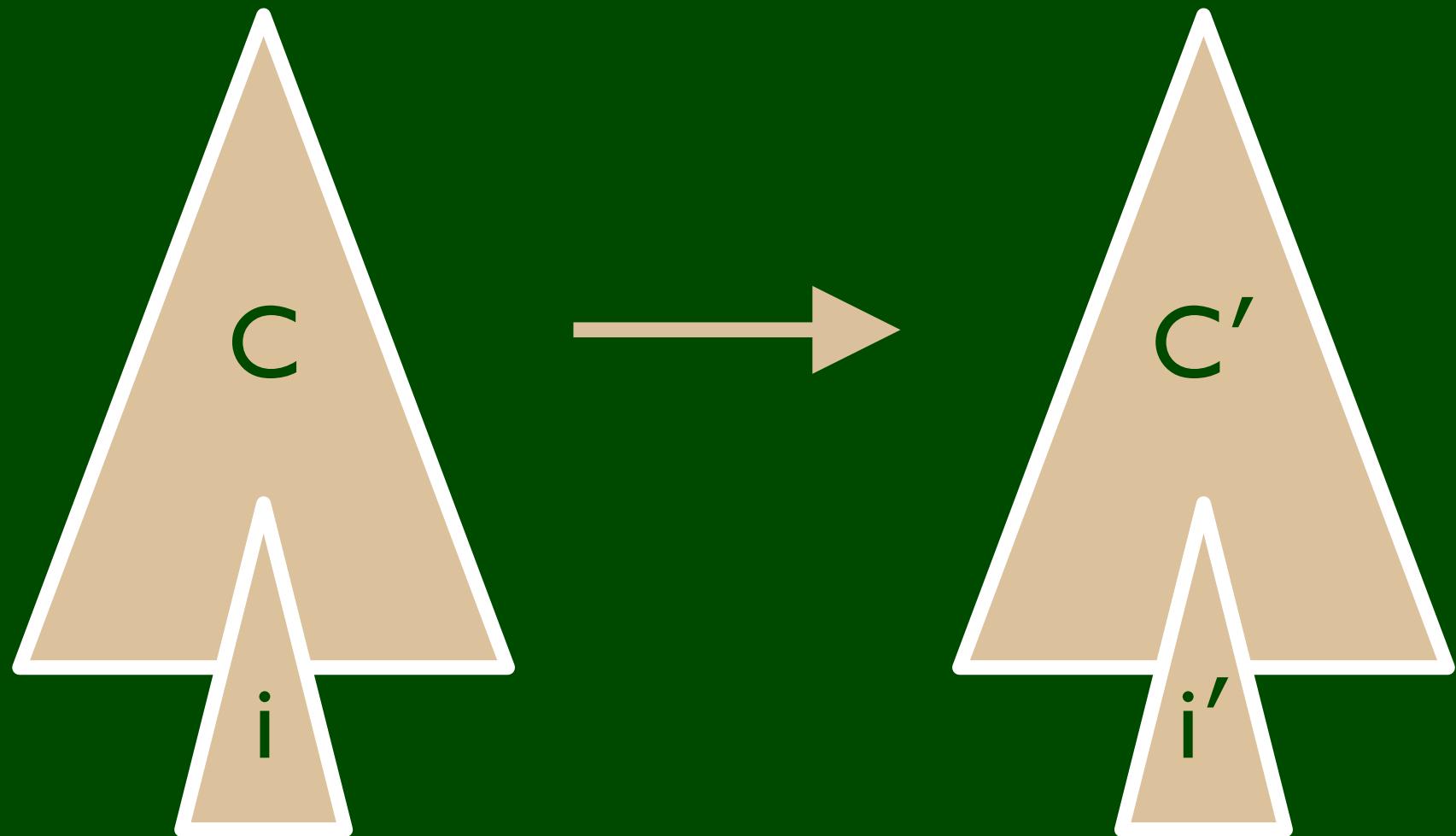
$\mathcal{E}val(p) = a$ iff $p \longrightarrow^* a$

where

answer \subset program

$\longrightarrow \subset$ program \times program

Reduction



Semantics Recap

- Specify programs & answers (grammar)
- Specify evaluation contexts (grammar)
- Specify a reduction relation

```
p ::= (begin d . . . e)
d ::= (define z c)
c ::= (class object%
       (init-field i)
       m . . .
       (super-make-object) )
m ::= (define/public (x y . . .)
        e)
e ::= (make-object x e)
     | (send e x e . . .)
     | x
     | this
     | number
     | (if0 e e e)
     | (+ e . . .)
```

```

p ::= (begin d ... e)
d ::= (define z c)
c ::= (class object%  

       (init-field i)  

       m ...  

       (super-make-object) )
m ::= (define/public (x y ...)  

        e)
e ::= (make-object x e)  

  | (send e x e ...)  

  | x  

  | this  

  | number  

  | (if0 e e e)  

  | (+ e ...)

```

```

(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%  

       (init-field i)  

       m ...  

       (super-make-object)))
  (m (define/public (x y ...)  

        e))
  (e (make-object x e)  

      (send e x e ...)  

      x  

      this  

      number  

      (if0 e e e)  

      (+ e ...)))

```

```

p ::= (begin d . . . e)
d ::= (define z c)
c ::= (class object%  

       (init-field i)  

       m . . .  

       (super-make-object) )
m ::= (define/public (x y . . .)  

        e)
e ::= (make-object x e)  

  | (send e x e . . .)  

  | x  

  | this  

  | number  

  | (if0 e e e)  

  | (+ e . . .)

```

```

(define-language Roo
  (p (begin d . . . e))
  (d (define z c)))
  (c (class object%  

       (init-field i)  

       m . . .  

       (super-make-object)))
  (m (define/public (x y . . .)  

       e))
  (e (make-object x e)  

      (send e x e . . .))
  x
  this
  number
  (if0 e e e)
  (+ e . . .))
  ((i x y z)
   variable-not-otherwise-mentioned))

```

A grammar associates
non-terminals with patterns;
parens are significant,
indicating tree structure (aka
“regular tree grammars”)

```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%
    (init-field i)
    m ...
    (super-make-object)))
  (m (define/public (x y ...)
    e))
  (e (make-object x e)
    (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
  variable-not-otherwise-mentioned))
```

Some patterns, e.g. **number**,
are like built-in non-terminals;
in this case matching all
Racket numbers

```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%
    (init-field i)
    m ...
    (super-make-object)))
  (m (define/public (x y ...)
    e))
  (e (make-object x e)
    (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
  variable-not-otherwise-mentioned))
```

The ellipsis is a Kleene star; a post-fix operator that allows zero or more repetitions of the pattern it follows

```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%
    (init-field i)
    m ...
    (super-make-object)))
  (m (define/public (x y ...)
    e))
  (e (make-object x e)
    (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
  variable-not-otherwise-mentioned))
```

These are the non-terminals;
the definitions come from the
grammar

```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%
    (init-field i)
    m ...
    (super-make-object)))
  (m (define/public (x y ...)
    e))
  (e (make-object x e)
    (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
  variable-not-otherwise-mentioned))
```

Which leaves the literals; this is a catch-all category and they act like keywords for the language

```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%
    (init-field i)
    m ...
    (super-make-object)))
  (m (define/public (x y ...)
    e))
  (e (make-object x e)
    (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
  variable-not-otherwise-mentioned))
```

This pattern matches any identifier except literals (making it sensitive to the language where it appears)

```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%
    (init-field i)
    m ...
    (super-make-object)))
  (m (define/public (x y ...)
    e))
  (e (make-object x e)
    (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
  variable-not-otherwise-mentioned))
```

Enough of Redex—now for
the language; a program
consists of definitions & an
expression

```
(define-language Roo
  (p (begin d ... e))
  (d (define z c))
  (c (class object%
            (init-field i)
            m ...
            (super-make-object)))
  (m (define/public (x y ...)
        e))
  (e (make-object x e)
      (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
   variable-not-otherwise-mentioned))
```

Definitions pair variables with
classes

```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%
    (init-field i)
    m ...
    (super-make-object)))
  (m (define/public (x y ...)
    e))
  (e (make-object x e)
    (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
  variable-not-otherwise-mentioned))
```

Each class has a single initialization argument **i**, a bunch of methods, and a call to **super-make-object**; much of this is to mimic the syntactic structure of Racket's class system

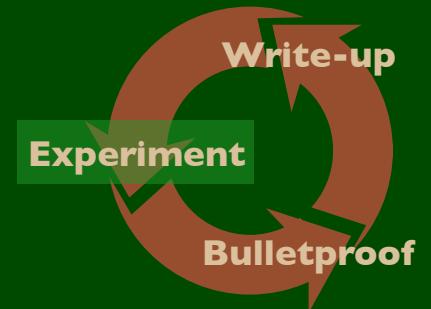
```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
(c (class object%
  (init-field i)
  m ...
  (super-make-object)))
(m (define/public (x y ...)
  e))
(e (make-object x e)
  (send e x e ...))
x
this
number
(if0 e e e)
(+ e ...))
((i x y z)
variable-not-otherwise-mentioned))
```

Each method has a name **x**,
multiple arguments **y**, and a
body **e**

```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%
    (init-field i)
    m ...
    (super-make-object)))
  (m (define/public (x y ...)
    e))
  (e (make-object x e)
    (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
  variable-not-otherwise-mentioned))
```

Expressions either are object creation, method invocation, variables, the `this` keyword, numbers, conditionals, or addition expressions

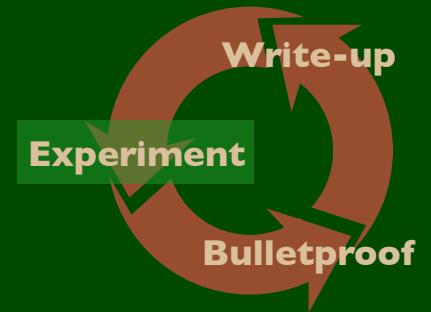
```
(define-language Roo
  (p (begin d ... e))
  (d (define z c)))
  (c (class object%
    (init-field i)
    m ...
    (super-make-object)))
  (m (define/public (x y ...)
    e))
  (e (make-object x e)
    (send e x e ...))
  x
  this
  number
  (if0 e e e)
  (+ e ...))
  ((i x y z)
  variable-not-otherwise-mentioned))
```



```
> (redex-match Roo
      (+ e_1 e_2)
      (term (+ (if0 1 2 3)
                4) )))

(list
 (match
 (list
 (bind 'e_1 '(if0 1 2 3))
 (bind 'e_2 4))))
```

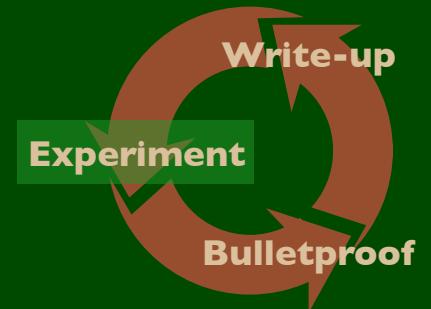
A **redex-match** expression accepts a language, a pattern, and a term; it tests the pattern against the expression and returns bindings for the pattern variables



```
> (redex-match Roo
  (+ e_1 e_2)
  (term (+ (if0 1 2 3)
            4) )))

(list
 (match
 (list
 (bind 'e_1 '(if0 1 2 3))
 (bind 'e_2 4))))
```

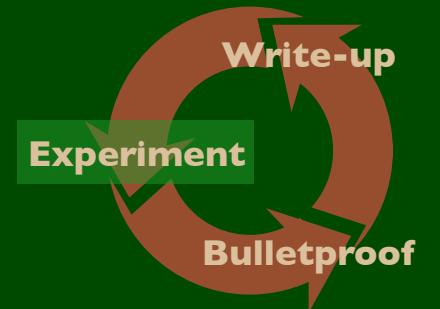
A **redex-match** expression accepts a **language**, a pattern, and a term; it tests the pattern against the expression and returns bindings for the pattern variables



```
> (redex-match Roo
      (+ e_1 e_2)
      (term (+ (if0 1 2 3)
                4) )))

(list
 (match
 (list
 (bind 'e_1 '(if0 1 2 3))
 (bind 'e_2 4))))
```

A **redex-match** expression accepts a language, a pattern, and a term; it tests the pattern against the expression and returns bindings for the pattern variables

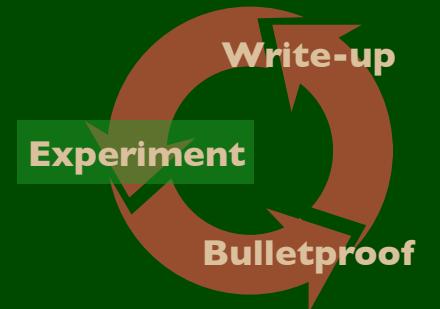


```
> (redex-match Roo
      (+ e_1 e_2)
      (term (+ (if0 1 2 3)
                 4) ))
```



```
(list
  (match
    (list
      (bind 'e_1 ' (if0 1 2 3))
      (bind 'e_2 4))))
```

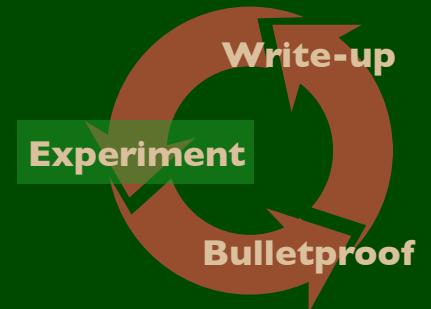
A **redex-match** expression accepts a language, a pattern, and a term; it tests the pattern against the expression and returns bindings for the pattern variables



```
> (redex-match Roo
      (+ e_1 e_2)
      (term (+ (if0 1 2 3)
                 4) ))
```

```
(list
  (match
    (list
      (bind 'e_1 ' (if0 1 2 3))
      (bind 'e_2 4) )) )
```

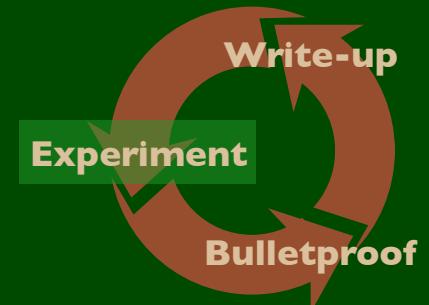
A **redex-match** expression accepts a language, a pattern, and a term; it tests the pattern against the expression and returns **bindings for the pattern variables**



```
> (redex-match Roo
      (+ e . . .)
      (term (+ (if0 1 2 3)
                4) )))

(list
 (match
 (list
 (bind 'e ' ((if0 1 2 3) 4))))))
```

When a pattern variable is behind an ellipsis, it is bound to a list whose elements match pieces of the term



```
> (redex-match
  Roo
  (+ e_1 ... e_2 e_3 ...)
  (term (+ 1 2 3)))
```

Doubled ellipses are ambiguous; so here we get three possible matches, with `e_2` taking on either 1, 2, or 3, and `e_1` and `e_3` absorbing the remaining numbers

```
(list
  (match
    (list
      (bind 'e_1 ' (1 2))
      (bind 'e_2 3)
      (bind 'e_3 ' ()))))

  (match
    (list
      (bind 'e_1 ' (1))
      (bind 'e_2 2)
      (bind 'e_3 ' (3)))))

  (match
    (list
      (bind 'e_1 ' ())
      (bind 'e_2 1)
      (bind 'e_3 ' (2 3)))))
```

Evaluation contexts, answers, and values

$a ::= (\text{begin } d \dots v)$	(a (begin d ... v))
$v ::= (\text{make-object } x v)$ $number$	(v (make-object x v) number)
$P ::= (\text{begin } d \dots E)$	(P (begin d ... E))
$E ::= (\text{make-object } x E)$	(E (make-object x E))
$(\text{send } E x e \dots)$	(send E x e ...)
$(\text{send } v x v \dots E e \dots)$	(send v x v ... E e ...)
$(\text{if0 } E e e)$	(if0 E e e)
$(+ v \dots E e \dots)$	(+ v ... E e ...)
[]	hole)

Evaluation contexts, answers, and values

The only new pattern here, **hole**, collaborates with **in-hole** to decompose terms into contexts and expressions at the hole

```
(a (begin d ... v) )  
(v (make-object x v)  
    number)  
(P (begin d ... E) )  
(E (make-object x E)  
    (send E x e ...))  
    (send v x v ... E e ...))  
    (if0 E e e)  
    (+ v ... E e ...))  
hole)
```

Evaluation contexts, answers, and values

An answer is the final result from a program, definitions plus a value

```
(a (begin d ... v) )
(v (make-object x v)
  number)
(P (begin d ... E) )
(E (make-object x E)
  (send E x e ...))
  (send v x v ... E e ...))
(if0 E e e)
(+ v ... E e ...)
hole)
```

Evaluation contexts, answers, and values

Values are objects and numbers

```
(a (begin d ... v) )
(v (make-object x v)
  number)
(P (begin d ... E) )
(E (make-object x E)
  (send E x e ...))
  (send v x v ... E e ...))
(if0 E e e)
(+ v ... E e ...)
hole)
```

Evaluation contexts, answers, and values

P stands for a program
evaluation context;
evaluation only happens in
the main expression

```
(a (begin d ... v) )  
(v (make-object x v)  
    number)  
(P (begin d ... E) )  
(E (make-object x E)  
    (send E x e ...)  
    (send v x v ... E e ...)  
    (if0 E e e)  
    (+ v ... E e ...)  
hole)
```

Evaluation contexts, answers, and values

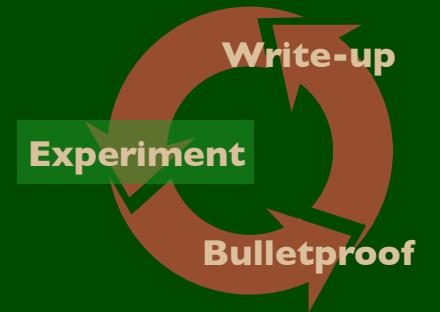
Expression evaluation can happen in the argument to **make-object**, in **send** expressions, **if0** expressions, and **+** expressions

```
(a (begin d ... v) )  
(v (make-object x v)  
    number)  
(P (begin d ... E) )  
(E (make-object x E)  
    (send E x e ...)  
    (send v x v ... E e ...)  
    (if0 E e e)  
    (+ v ... E e ...)  
    hole)
```

Evaluation contexts, answers, and values

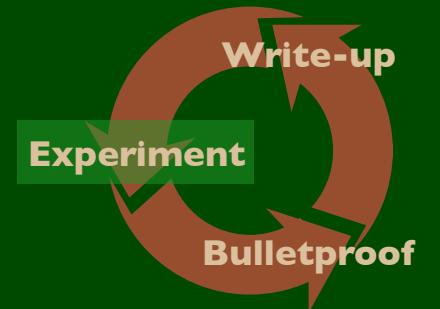
Note that use of doubled ellipses forcing left-to-right order of evaluation

```
(a (begin d ... v) )
(v (make-object x v)
  number)
(P (begin d ... E) )
(E (make-object x E)
  (send E x e ...))
  (send v x v ... E e ...))
(if0 E e e)
(+ v ... E e ...)
hole)
```



```
> (redex-match
  Roo
  (in-hole E (+ number_1 number_2))
  (term (if0 (+ 1 2) 3 4)) )

(list
 (match
 (list
  (bind 'E (list 'if0 (hole) 3 4))
  (bind 'number_1 1)
  (bind 'number_2 2))))
```



```
> (redex-match
  Roo
  (in-hole E e)
  (term (if0 (+ 1 2) 3 4)) )

(list
  (match
    (list
      (bind 'e' '(+ 1 2))
      (bind 'E' (list 'if0 (hole) 3 4)))))

(match
  (list
    (bind 'e' 2)
    (bind 'E' (list 'if0 (list '+ 1 (hole)) 3 4)))))

(match
  (list
    (bind 'e' '(if0 (+ 1 2) 3 4)))
    (bind 'E' (hole)))))

(match
  (list
    (bind 'e' 1)
    (bind 'E' (list 'if0 (list '+ (hole) 2) 3 4)))))
```

$P[(+ \ number \ \dots)] \longrightarrow P[\Sigma[[number \ \dots]]] \ [+]$

$P[(\text{if0} \ 0 \ e_1 \ e_2)] \longrightarrow P[e_1] \quad [\text{if0t}]$

$P[(\text{if0} \ number \ e_1 \ e_2)] \longrightarrow P[e_2] \quad [\text{if0f}]$
where $number \neq 0$

```

(define num-rules
  (reduction-relation
    Roo
    (--> (in-hole P (+ number ...))
          (in-hole P ( $\Sigma$  number ...)))
    "+")
    (--> (in-hole P (if0 0 e_1 e_2))
          (in-hole P e_1)
          "if0t")
    (--> (in-hole P (if0 number e_1 e_2))
          (in-hole P e_2)
          (side-condition (term (nonzero number))))
          "if0f"))
  (define-metafunction Roo
     $\Sigma$  : number ... -> number
    [ ( $\Sigma$  number ...) , (apply + (term (number ...))) ] )

```

```

(define num-rules
  (reduction-relation
    Roo
    (--> (in-hole P (+ number ...))
          (in-hole P ( $\Sigma$  number ...)))
    "+")
    (--> (in-hole P (if0 0 e_1 e_2))
          (in-hole P e_1)
          "if0t")
    (--> (in-hole P (if0 number e_1 e_2))
          (in-hole P e_2)
          (side-condition (term (nonzero number))))
          "if0f"))
  (define-metaprocedure Roo
     $\Sigma$  : number ...  $\rightarrow$  number
    [ ( $\Sigma$  number ...) , (apply + (term (number ...))) ] )

```

Three reduction rules; the prefix operator --> introduces each one

```

(define num-rules
  (reduction-relation
    Roo
    (--> (in-hole P (+ number ...))
          (in-hole P ( $\Sigma$  number ...)))
    "+")
    (--> (in-hole P (if0 0 e_1 e_2))
          (in-hole P e_1)
          "if0t")
    (--> (in-hole P (if0 number e_1 e_2))
          (in-hole P e_2)
          (side-condition (term (nonzero number)))
          "if0f")))
(define-metafunction Roo
   $\Sigma$  : number ... -> number
  [ ( $\Sigma$  number ...) , (apply + (term (number ...))) ] )

```

+ reduces via the metafunction Σ
 (redex supports unicode so Σ is just a regular identifier)

```

(define num-rules
  (reduction-relation
    Roo
    (--> (in-hole P (+ number ...))
          (in-hole P ( $\Sigma$  number ...)))
    "+")
    (--> (in-hole P (if0 0 e_1 e_2))
          (in-hole P e_1)
          "if0t")
    (--> (in-hole P (if0 number e_1 e_2))
          (in-hole P e_2)
          (side-condition (term (nonzero number))))
          "if0f"))
  (define-metafunction Roo
     $\Sigma$  : number ... -> number
    [ ( $\Sigma$  number ...) , (apply + (term (number ...)))] )

```

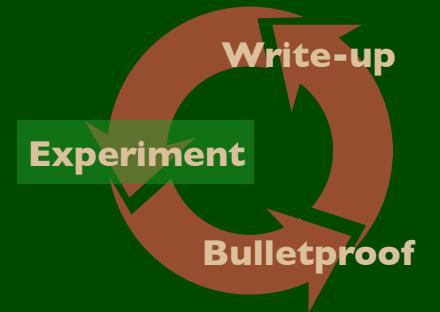
The comma means unquote, so we are just exploiting Racket's + to implement addition in the model

```

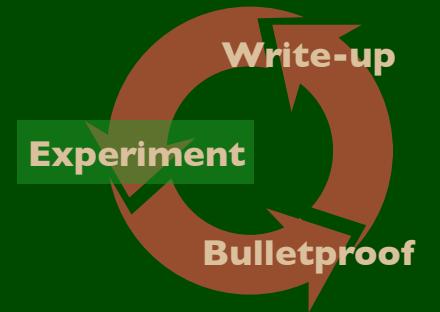
(define num-rules
  (reduction-relation
    Roo
    (--> (in-hole P (+ number ...))
          (in-hole P ( $\Sigma$  number ...)))
    "+")
    (--> (in-hole P (if0 0 e_1 e_2))
          (in-hole P e_1)
          "if0t")
    (--> (in-hole P (if0 number e_1 e_2))
          (in-hole P e_2)
          (side-condition (term (nonzero number)))
          "if0f")))
(define-metafunction Roo
   $\Sigma$  : number ... -> number
  [ ( $\Sigma$  number ...) , (apply + (term (number ...))) ] )

```

The first if0 rule uses the literal 0 but the second has to use a trivial metafunction (not shown) to test non-zeroness

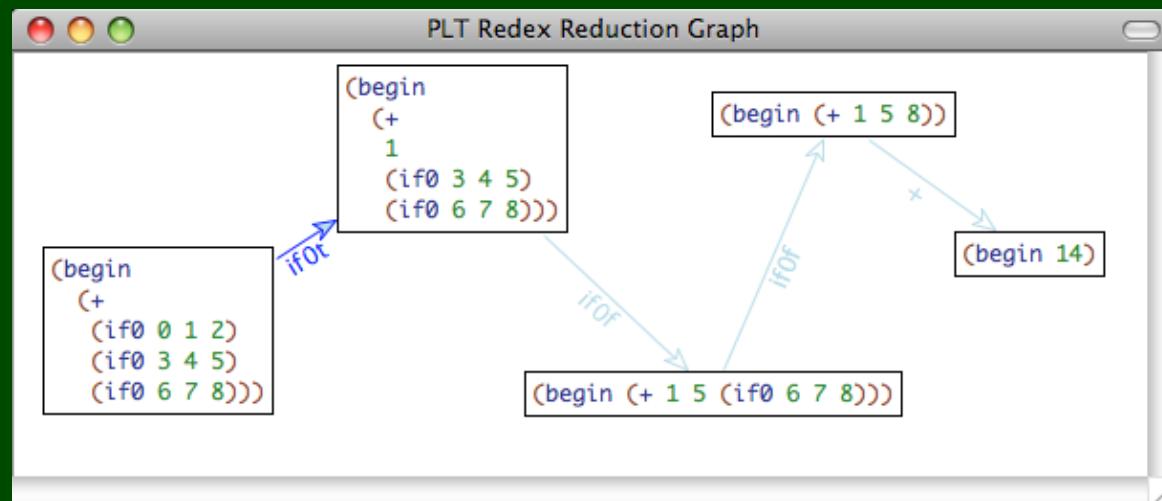
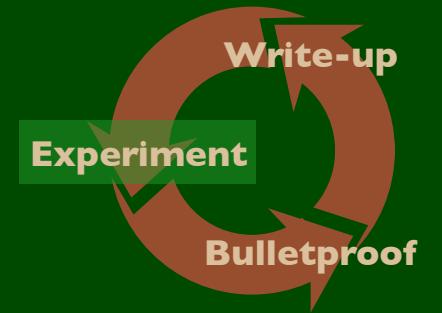


```
> (apply-reduction-relation
  num-rules
  (term (begin (if0 (+ 1 2) 3 4)) ))
' ((begin (if0 3 3 4)))
```



```
> (traces
  num-rules
  (term (begin (+ (if0 0 1 2)
                  (if0 3 4 5)
                  (if0 6 7 8))))
```

Run



<pre>(begin d₁ ... (define z (class object% (init-field i) m₁ ... (define/public (x y ...) e) m₂ ... (super-make-object))) d₂ ... E[(send (make-object z v_i) x v_y . . .)])</pre>		<pre>(begin d₁ ... (define z (class object% (init-field i) m₁ ... (define/public (x y ...) e) m₂ ... (super-make-object))) d₂ ... E[e{y := v_y . . . , i := v_i, this := (make-object z v_i) }])] [send]</pre>
--	--	---

```
(begin
  d_1 ...
  (define z
    (class object%
      (init-field i)
      m_1 ...
      (define/public (x y ...) e)
      m_2 ...
      (super-make-object) )))
  d_2 ...
  (in-hole E (send (make-object z v_i)
                    x
                    v_y ...))))
```

Begin reading at the bottom, in the main expression; this is a method invocation, with arguments $v_y \dots$ to an object of class z with init field v_i

```

(begin
  d_1 ...
(define z
  (class object%
    (init-field i)
    m_1 ...
    (define/public (x y ...) e)
    m_2 ...
    (super-make-object)))
d_2 ...
(in-hole E (send (make-object z v_i)
                  x
                  v_y ...)))

```

Since the `z` appears twice, it must be the same identifier; this forces the `d_1` and `d_2` to “absorb” all of the irrelevant class definitions

```

(begin
  d_1 ...
(define z
  (class object%
    (init-field i)
    m_1 ...
    (define/public (x y ...) e)
    m_2 ...
    (super-make-object) )))
d_2 ...
(in-hole E (send (make-object z v_i)
  x
  v_y ...)))

```

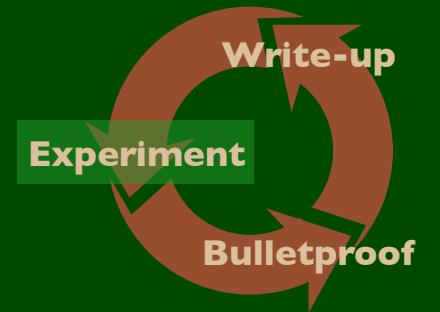
Ditto for `m_1` and `m_2` absorbing all of the irrelevant methods

```

(begin
  d_1 ...
(define z
  (class object%
    (init-field i)
    m_1 ...
    (define/public (x y ...) e)
    m_2 ...
    (super-make-object)))
d_2 ...
(in-hole E (substs-e e
  (y v_y) ...
  (i v_i)
  (this
    (make-object z v_i))))))

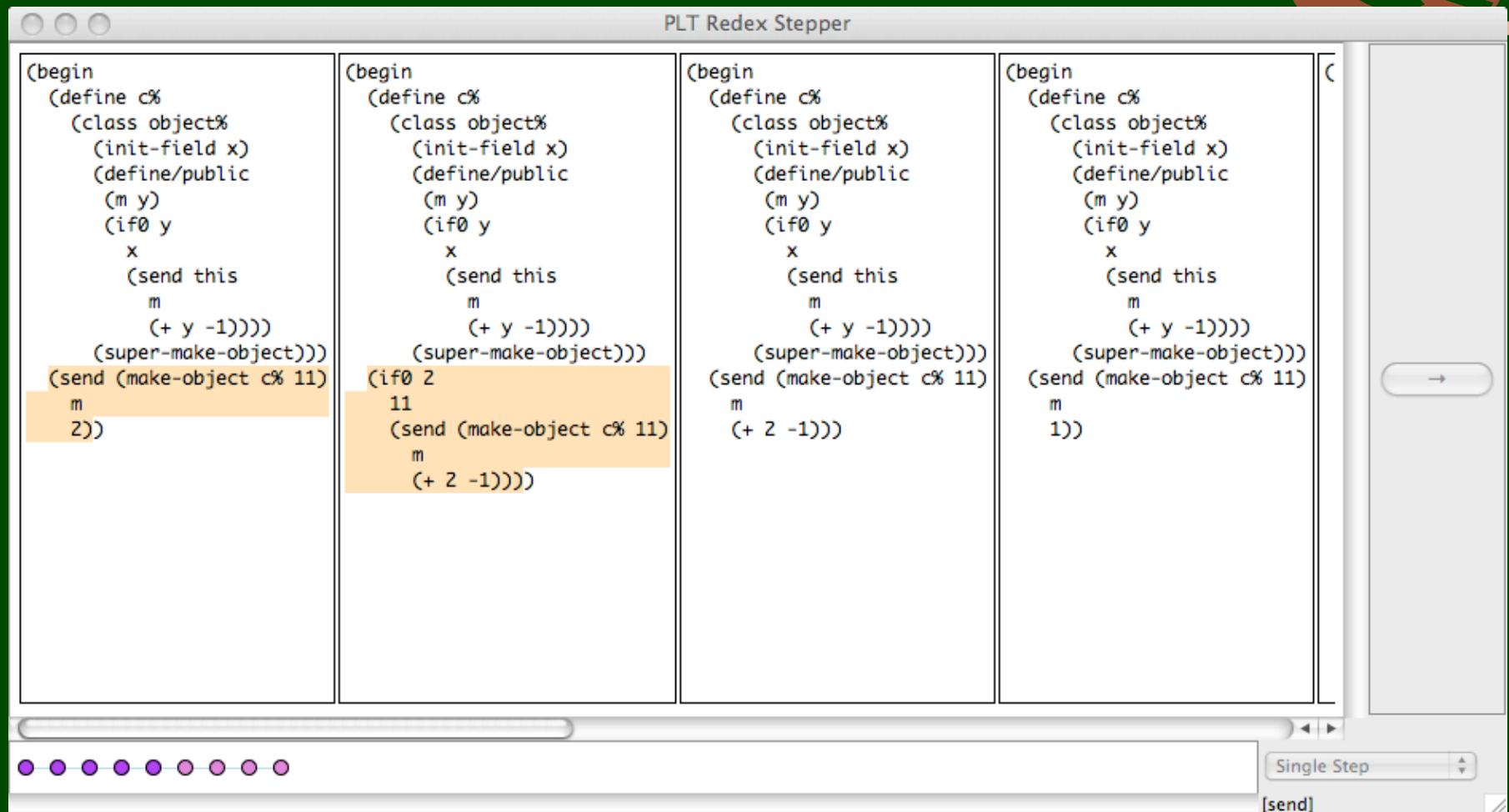
```

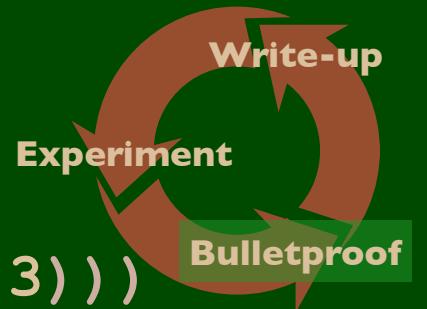
This, the right-hand side of the rule, is just like the left, except we replace method invocation with the body of the method, modulo appropriate substitutions



```
> (stepper
  red
  (term (begin
    (define c%
      (class object%
        (init-field x)
        (define/public (m y)
          (if0 y
            x
            (send this m (+ y -1)))))
      (super-make-object)))
    (send (make-object c% 11) m 2))))
```

Run



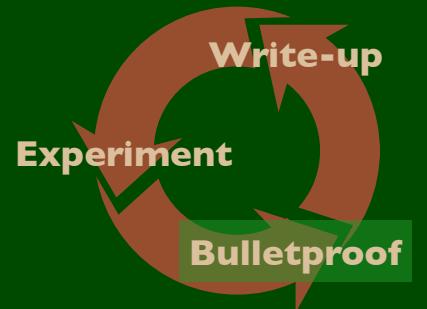


```
> (test-->> red
  (term (begin (if0 1 2 3) ))
  (term (begin 3) ))

> (test-->> red
  (term (begin (if0 0 1 2) ))
  (term (begin 2) ))

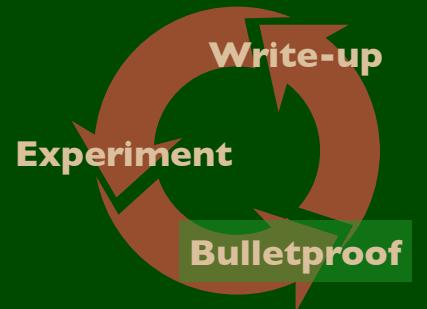
FAILED slides.rkt:124.11
expected: '(begin 2)
actual: '(begin 1)
> (test-->> red
  (term (begin (+ 1 2 3 4) ))
  (term (begin 10) ))

> (test-results)
1 test failed (out of 3 total).
```



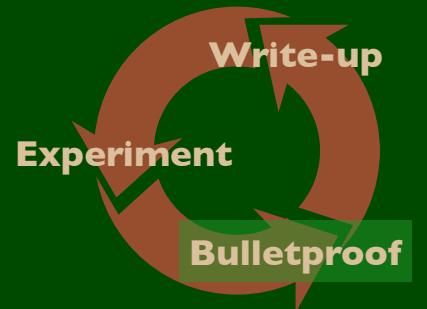
```
> (redex-check
  Roo
  p
  (equal? (eval-expr (term p))
          (reduce-expr (term p))))
```

redex-check accepts a language, a non-terminal, and a Racket expression, and makes up random examples of the pattern to test the Racket expression, looking for #f



```
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  Roo
  p
  (equal? (eval-expr (term p))
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redex-check accepts a language, a non-terminal, and a Racket expression, and makes up random examples of the pattern to test the Racket expression, looking for #f



```
> (redex-check
```

```
Roo
```

```
p
```

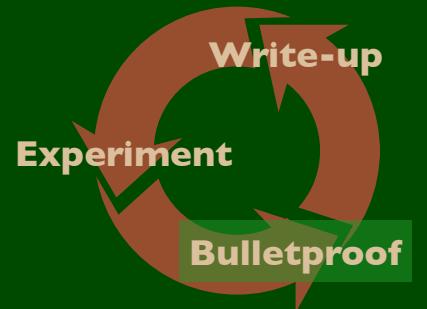
```
(equal? (eval-expr (term p))  
       (reduce-expr (term p))))
```

redex-check accepts a language, a non-terminal, and a Racket expression, and makes up random examples of the pattern to test the Racket expression, looking for #f



```
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  p
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```

redex-check accepts a language, a non-terminal, and a Racket expression, and makes up random examples of the pattern to test the Racket expression, looking for #f

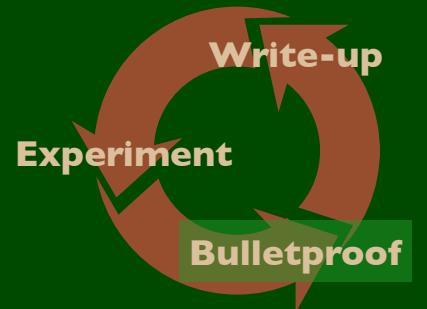


```

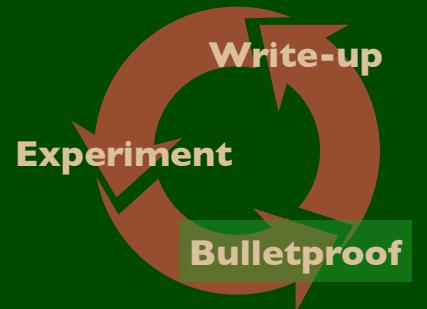
; reduce-expr : any -> (or/c 'error
;                                'object
;                                number? )
(define (reduce-expr e)
  (define results (apply-reduction-relation* red e))
  (define result (car results))
  (define value (last result))
  (if (redex-match R0 a result)
      (if (number? value)
          value
          'object)
      'error))

; eval-expr : any -> (or/c 'error 'object number? )
(define (eval-expr e)
  ; evaluate the expression using Racket
  )

```

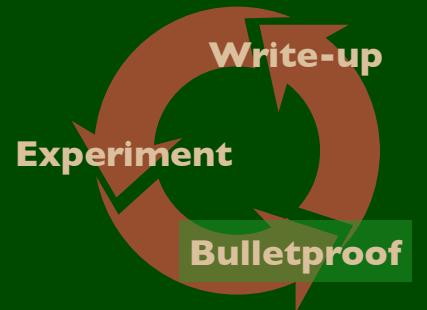


```
> (redex-check
  Roo
  p
  (equal? (eval-expr (term p))
          (reduce-expr (term p))))
```



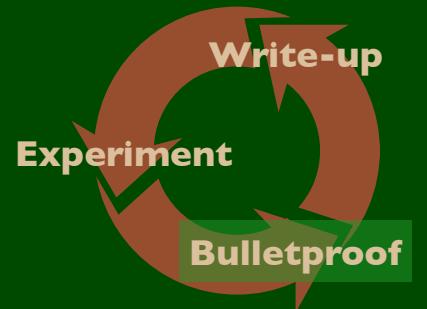
```
> (redex-check
  Roo
  p
  (equal? (eval-expr (term p))
           (reduce-expr (term p))))
```

redex-check: checking (begin (define U (class object%
 (init-field Jjp) (super-make-object))) (define FR (class object%
 (init-field x) (define/public (o) this) (define/public (k) this)
 (define/public (x) 0) (define/public (f) 6) (define/public (Q) 2)
 (super-make-object))) (make-object c (+ (make-object X 3)
 (send this hn) 2 (send 2 U) 0 this 0 -2 (make-object C -l) (if0
 this 5 6) (+)))) raises an exception:
class: duplicate declared identifier in: x



```
> (redex-check
  Roo
  p
  (equal? (eval-expr (term p))
           (reduce-expr (term p))))
```

redex-check: checking
(begin (define FR (class object%
 (init-field x)
 (define/public (x) 0)
 (super-make-object)))
 0)
raises an exception:
class: duplicate declared identifier in: x



```
> (redex-check
  Roo
  p
  (with-handlers ((exn:fail:syntax?
                  (λ (x) #t)))
    (equal? (eval-expr (term p))
            (reduce-expr (term p)))))
```

redex-check: no counterexamples in 1000 attempts

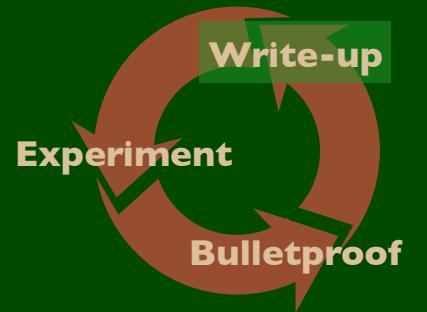


> (**render-reduction-relation** num-rules)

$$P[(+ \text{ } number \dots)] \quad [+] \\ \longrightarrow P[\Sigma[[number \dots]]]$$

$$P[(\text{if0} \text{ } 0 \text{ } e_1 \text{ } e_2)] \quad [\text{if0t}] \\ \longrightarrow P[e_1]$$

$$P[(\text{if0} \text{ } number \text{ } e_1 \text{ } e_2)] \text{ } [\text{if0f}] \\ \longrightarrow P[e_2] \\ \text{where nonzero}[[number]]$$

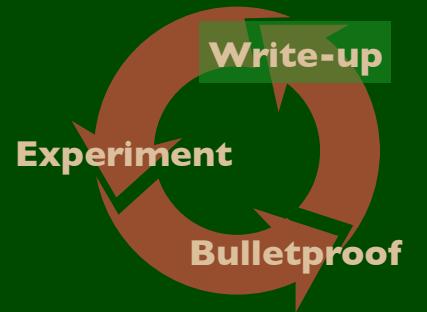


```
> (parameterize ([rule-pict-style 'horizontal])
  (render-reduction-relation num-rules))
```

$P[(+ \text{ number} \dots)] \longrightarrow P[\Sigma[[\text{number} \dots]]] \text{ [+]}$

$P[(\mathbf{if0} \ 0 \ e_1 \ e_2)] \longrightarrow P[e_1] \quad [\text{if0t}]$

$P[(\mathbf{if0} \ \text{number} \ e_1 \ e_2)] \longrightarrow P[e_2] \quad [\text{if0f}]$
where $\text{nonzero}[[\text{number}]]$



```
> (parameterize ([rule-pict-style 'horizontal])
  (with-rewriters
    (render-reduction-relation num-rules) ))
```

$$P[(+ \text{ } number \dots)] \longrightarrow P[\Sigma[[number \dots]]] \text{ [+]}$$

$$P[(\mathbf{if0} \text{ } 0 \text{ } e_1 \text{ } e_2)] \longrightarrow P[e_1] \quad [\text{if0t}]$$

$$P[(\mathbf{if0} \text{ } number \text{ } e_1 \text{ } e_2)] \longrightarrow P[e_2] \quad [\text{if0f}]$$

where $number \neq 0$

Assignment: Generalize the model to first-class
classes (i.e., classes as values)

The next few slides show a model that we went through together in the live talk. It starts with the complete version of the first-order calculus and then adjusts definitions so that any expression can appear on the right-hand side, and then makes classes be expressions. It then exploits Redex's testing facilities to find bugs in the model.

To try this yourself, download:

www.eecs.northwestern.edu/~robby/talks/fool2010-hw.rkt

It contains the complete code from the model but with two changes:

- **e** now contains **c** and
- definitions are of the form (**define** **x** **e**).

Run it to see the first counter example.

0.rkt

$p ::= (\text{begin } d \dots e)$
 $d ::= (\text{define } z c)$
 $c ::= (\text{class object} \%$
 $\quad (\text{init-field } i))$
 $m \dots$
 $\quad (\text{super-make-object}))$
 $m ::= (\text{define/public } (x y \dots)$
 $\quad e)$
 $e ::= (\text{make-object } x e)$
 $\quad | (\text{send } e x e \dots)$
 $\quad | x$
 $\quad | \text{this}$
 $\quad | \text{number}$
 $\quad | (\text{if0 } e e e)$
 $\quad | (+ e \dots)$
 $a ::= (\text{begin } d \dots v)$
 $v ::= (\text{make-object } x v)$
 $\quad | \text{number}$
 $P ::= (\text{begin } d \dots E)$
 $E ::= (\text{make-object } x E)$
 $\quad | (\text{send } E x e \dots)$
 $\quad | (\text{send } v x v \dots E e \dots)$
 $\quad | (\text{if0 } E e e)$
 $\quad | (+ v \dots E e \dots)$
 $\quad | []$

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

$(\text{begin } d_1 \dots (\text{define } z (\text{class object} \% (\text{init-field } i) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object}))) d_2 \dots E[e\{y := v_y \dots, i := v_i, \text{this} := (\text{make-object } z v_i)\}]) \rightarrow (\text{begin } d_1 \dots (\text{define } z (\text{class object} \% (\text{init-field } i) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object}))) d_2 \dots E[e\{y := v_y \dots, i := v_i, \text{this} := (\text{make-object } z v_i)\}])$

$P[(+ \text{number} \dots)] \quad [+]$
 $\rightarrow P[\Sigma[\text{number} \dots]]$
 $P[(\text{if0 } 0 e_1 e_2)] \quad [\text{if0t}]$
 $\rightarrow P[e_1]$
 $P[(\text{if0 } \text{number } e_1 e_2)] \quad [\text{if0f}]$
 $\rightarrow P[e_2]$
 $\quad \text{where number } \neq 0$

.rkt

$p ::= (\text{begin } d \dots e)$
 $d ::= (\text{define } z e)$
 $c ::= (\text{class object}^{\%})$
 $\quad (\text{init-field } i)$
 $m \dots$
 $\quad (\text{super-make-object})$
 $m ::= (\text{define/public } (x y \dots))$
 $\quad e)$
 $e ::= c$
 $\quad | (\text{make-object } x e)$
 $\quad | (\text{send } e x e \dots)$
 $\quad | x$
 $\quad | \text{this}$
 $\quad | \text{number}$
 $\quad | (\text{if0 } e e e)$
 $\quad | (+ e \dots)$
 $a ::= (\text{begin } d \dots v)$
 $v ::= (\text{make-object } x v)$
 $\quad | \text{number}$
 $P ::= (\text{begin } d \dots E)$
 $E ::= (\text{make-object } x E)$
 $\quad | (\text{send } E x e \dots)$
 $\quad | (\text{send } v x v \dots E e \dots)$
 $\quad | (\text{if0 } E e e)$
 $\quad | (+ v \dots E e \dots)$
 $\quad | []$

$(\text{begin}$ $\quad d_1 \dots$ $\quad (\text{define } z$ $\quad \quad (\text{class object}^{\%})$ $\quad \quad (\text{init-field } i)$ $\quad \quad m_1 \dots$ $\quad \quad (\text{define/public } (x y \dots))$ $\quad \quad e)$ $\quad m_2 \dots$ $\quad (\text{super-make-object}))$ $\quad d_2 \dots$ $\quad E[(\text{send } (\text{make-object } z v_i)$ $\quad \quad x$ $\quad \quad v_y \dots)])]$	$\longrightarrow (\text{begin}$ $\quad d_1 \dots$ $\quad (\text{define } z$ $\quad \quad (\text{class object}^{\%})$ $\quad \quad (\text{init-field } i)$ $\quad \quad m_1 \dots$ $\quad \quad (\text{define/public } (x y \dots))$ $\quad \quad e)$ $\quad m_2 \dots$ $\quad (\text{super-make-object}))$ $\quad d_2 \dots$ $\quad E[e\{y := v_y \dots,$ $\quad \quad i := v_i,$ $\quad \quad \text{this} := (\text{make-object } z v_i)\}]])$
$P[(+ \text{ number} \dots)]$ $\longrightarrow P[\sum [\text{number} \dots]]]$	$[+]$
$P[(\text{if0 } 0 e_1 e_2)]$ $\longrightarrow P[e_1]$	$[\text{if0t}]$
$P[(\text{if0 } \text{number } e_1 e_2)]$ $\longrightarrow P[e_2]$	$[\text{if0f}]$ $\quad \text{where number} \neq 0$

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

I.rkt Error

```


$$\begin{array}{l}
p ::= (\textbf{begin } d \dots e) \\
d ::= (\textbf{define } z e) \\
c ::= (\textbf{class } object \% \\
       (\textbf{init-field } i) \\
       m \dots \\
       (\textbf{super-make-object})) \\
m ::= (\textbf{define/public } (x y \dots) \\
       e) \\
e ::= c \\
| (\textbf{make-object } x e) \\
| (\textbf{send } e x e \dots) \\
| x \\
| \textbf{this} \\
| number \\
| (\textbf{if0 } e e e) \\
| (+ e \dots) \\
a ::= (\textbf{begin } d \dots v) \\
v ::= (\textbf{make-object } x v) \\
| number \\
P ::= (\textbf{begin } d \dots E) \\
E ::= (\textbf{make-object } x E) \\
| (\textbf{send } E x e \dots) \\
| (\textbf{send } v x v \dots E e \dots) \\
| (\textbf{if0 } E e e) \\
| (+ v \dots E e \dots)
\end{array}$$


```

x, y, z ::= variable-not-otherwise-mentioned

2.rkt

$p ::= (\text{begin } d \dots e)$
 $d ::= (\text{define } z e)$
 $c ::= (\text{class object} \% (\text{init-field } i))$
 $m ::= (\text{super-make-object})$
 $m ::= (\text{define/public } (x y \dots) e)$
 $e ::= c$
 | $(\text{make-object } x e)$
 | $(\text{send } e x e \dots)$
 | x
 | this
 | $number$
 | $(\text{if0 } e e e)$
 | $(+ e \dots)$
 $a ::= (\text{begin } d \dots v)$
 $v ::= (\text{make-object } x v)$
 | $number$
 | c
 $P ::= (\text{begin } d \dots E)$
 $E ::= (\text{make-object } x E)$
 | $(\text{send } E x e \dots)$
 | $(\text{send } v x v \dots E e \dots)$
 | $(\text{if0 } E e e)$
 | $(+ v \dots E e \dots)$
 | $[]$

$(\text{begin } d_1 \dots (\text{define } z (\text{class object} \% (\text{init-field } i)) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object})) d_2 \dots E[e\{y := v_y \dots, i := v_i, \text{this} := (\text{make-object } z v_i)\}]) \rightarrow (\text{begin } d_1 \dots (\text{define } z (\text{class object} \% (\text{init-field } i)) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object})) d_2 \dots E[e\{y := v_y \dots, i := v_i, \text{this} := (\text{make-object } z v_i)\}])$
 $P[(+ number \dots)] \rightarrow P[\sum [number \dots]]$
 $P[(\text{if0 } 0 e_1 e_2)] \rightarrow P[e_1]$
 $P[(\text{if0 } number e_1 e_2)] \rightarrow P[e_2]$
 where $number \neq 0$
 $[+]$
 $[\text{if0f}]$

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

2.rkt Error

$p ::= (\text{begin } d \dots e)$
 $d ::= (\text{define } z e)$
 $c ::= (\text{class object} \%$
 (init-field i)
 $m \dots$
 (super-make-object))
 $m ::= (\text{define/public } (x y \dots)$
 $e)$
 $e ::= c$
 | (make-object $x e$)
 | (send $x e \dots$)
 | x
 | this
 | number
 | (if0 $e_1 e_2$)
 | (+ $e \dots$)
 $a ::= (\text{begin } d \dots v)$
 $v ::= (\text{make-object } x v)$
 | number
 | c
 $P ::= (\text{begin } d \dots E)$
 $E ::= (\text{make-object } x E)$
 | (send $E x e \dots$)
 | (send $v x v \dots E e \dots$)
 | (if0 $E e e$)
 | (+ $v \dots E e \dots$)
 | []

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

$(\text{begin}$ $d_1 \dots$ $(\text{define } z$ (class object% (init-field i) $m_1 \dots$ (define/public $(x y \dots)$ $e)$ $m_2 \dots$ (super-make-object))) $d_2 \dots$ $E[\text{send } (\text{make-object } z v_i)$ x $v_{i+1} \dots])])$	$\longrightarrow (\text{begin}$ $d_1 \dots$ $(\text{define } z$ (class object% (init-field i) $m_1 \dots$ (define/public $(x y \dots)$ $e)$ $m_2 \dots$ (super-make-object))) $d_2 \dots$ $E[e\{y := v_y \dots,$ $i := v_i,$ this := (make-object $z v_i\})])$
--	--

(begin
 $\longrightarrow (\text{define } c \ (\text{send } 0 \ G))$
 $\longrightarrow 3)_{\text{if0 } 0 \ e_1 \ e_2}] \ [\text{if0}]$
 $\longrightarrow P[e_1]$

$P[\ (\text{if0 } \text{number } e_1 \ e_2) \] \ [\text{if0}]$
 $\longrightarrow P[e_2]$
 where $\text{number} \neq 0$

3.rkt

$p ::= (\text{begin } d \dots e)$
 $d ::= (\text{define } z e)$
 $c ::= (\text{class object} \% (\text{init-field } i))$
 $m ::= (\text{super-make-object})$
 $m ::= (\text{define/public } (x y \dots) e)$
 $e ::= c$
 | $(\text{make-object } x e)$
 | $(\text{send } e x e \dots)$
 | x
 | this
 | $number$
 | $(\text{if0 } e e e)$
 | $(+ e \dots)$
 $a ::= (\text{begin } (\text{define } x v) \dots v)$
 $v ::= (\text{make-object } x v)$
 | $number$
 | c
 $P ::= (\text{begin } d \dots E)$
 $E ::= (\text{make-object } x E)$
 | $(\text{send } E x e \dots)$
 | $(\text{send } v x v \dots E e \dots)$
 | $(\text{if0 } E e e)$
 | $(+ v \dots E e \dots)$
 | $[]$

$(\text{begin } d_1 \dots (\text{define } z (\text{class object} \% (\text{init-field } i)) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object})) d_2 \dots E[e\{y := v_y \dots, i := v_i, \text{this} := (\text{make-object } z v_i)\}]]) \rightarrow (\text{begin } d_1 \dots (\text{define } z (\text{class object} \% (\text{init-field } i)) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object})) d_2 \dots E[e\{y := v_y \dots, i := v_i, \text{this} := (\text{make-object } z v_i)\}])$
 $P[(+ number \dots)] \rightarrow P[\sum [number \dots]]]$
 $P[(\text{if0 } 0 e_1 e_2)] \rightarrow P[e_1]$
 $P[(\text{if0 } number e_1 e_2)] \rightarrow P[e_2]$
 where $number \neq 0$

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

3.rkt Error

$p ::= (\text{begin } d \dots e)$	$(\text{begin} \quad \quad \quad \longrightarrow (\text{begin}$	
$d ::= (\text{define } z e)$	$d_1 \dots \quad \quad \quad d_1 \dots$	
$c ::= (\text{class object}%$	$(\text{define } z \quad \quad \quad (\text{define } z$	
$\quad (\text{init-field } i)$	$(\text{class object}% \quad \quad \quad (\text{class object}%$	
$m \dots$	$\quad (\text{init-field } i) \quad \quad \quad \quad (\text{init-field } i)$	
$\quad (\text{super-make-object}))$	$m_1 \dots \quad \quad \quad m_1 \dots$	
$m ::= (\text{define/public } (x y \dots)$	$(\text{define/public } (x y \dots) \quad \quad \quad (\text{define/public } (x y \dots)$	
$e)$	$ \quad \quad \quad e) \quad \quad \quad e)$	
$e ::= c$	$m_2 \dots \quad \quad \quad m_2 \dots$	
$\quad (\text{make-object } x e)$	$(\text{super-make-object})) \quad \quad \quad (\text{super-make-object}))$	
$\quad (\text{send } e e \dots)$	$d_2 \dots \quad \quad \quad d_2 \dots$	
$\quad x$	$E[(\text{send } (\text{make-object } z v_i) \quad \quad \quad E[e\{y := v_y \dots,$	
$\quad \text{this}$	$x \quad \quad \quad i := v_i,$	
$\quad \text{number}$	$v_y \dots)]]) \quad \quad \quad \text{this} := (\text{make-object } z v_i) \}]])$	
$\quad (\text{if0 } e e e)$		
$\quad (+ e \dots)$		
$a ::= (\text{begin } (\text{define } x v) \dots$		
$v ::= (\text{make-object } x v)$		
$\quad \text{number}$		
$\quad c$		
$P ::= (\text{begin } d \dots E)$	$P[(\text{if0 } 0 e_1 e_2)] \quad \quad \quad [\text{if0t}]$	
$E ::= (\text{make-object } x E)$	$\longrightarrow P[e_1]$	
$\quad (\text{send } E x e \dots)$		
$\quad (\text{send } v x v \dots E e \dots)$	$P[(\text{if0 } \text{number } e_1 e_2)] \quad [\text{if0f}]$	
$\quad (\text{if0 } E e e)$	$\longrightarrow P[e_2]$	
$\quad (+ v \dots E e \dots)$	$\quad \quad \quad \text{where number } \neq 0$	
$\quad []$		
$x, y, z ::= \text{variable-not-otherwise-mentioned}$		

4.rkt

$p ::= (\text{begin } d \dots e)$
 $d ::= (\text{define } z e)$
 $c ::= (\text{class object}^{\%})$
 $\quad (\text{init-field } i)$
 $m \dots$
 $\quad (\text{super-make-object})$
 $m ::= (\text{define/public } (x y \dots))$
 $\quad e)$
 $e ::= c$
 $\quad | (\text{make-object } x e)$
 $\quad | (\text{send } e e \dots)$
 $\quad | x$
 $\quad | \text{this}$
 $\quad | \text{number}$
 $\quad | (\text{if0 } e e e)$
 $\quad | (+ e \dots)$
 $a ::= (\text{begin } (\text{define } x v) \dots v)$
 $v ::= (\text{make-object } x v)$
 $\quad | \text{number}$
 $\quad | c$
 $P ::= (\text{begin } (\text{define } x v) \dots$
 $\quad (\text{define } x E)$
 $\quad (\text{define } x e) \dots$
 $\quad e)$
 $\quad | (\text{begin } (\text{define } x v) \dots$
 $\quad E)$
 $E ::= (\text{make-object } x E)$
 $\quad | (\text{send } E x e \dots)$
 $\quad | (\text{send } v x v \dots E e \dots)$
 $\quad | (\text{if0 } E e e)$
 $\quad | (+ v \dots E e \dots)$
 $\quad | []$

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

$(\text{begin } d_1 \dots (\text{define } z (\text{class object}^{\%}) (\text{init-field } i) m_1 \dots (\text{define/public } (x y \dots)) e) m_2 \dots (\text{super-make-object})) \rightarrow (\text{begin } d_1 \dots (\text{define } z (\text{class object}^{\%}) (\text{init-field } i) m_1 \dots (\text{define/public } (x y \dots)) e) m_2 \dots (\text{super-make-object}))$
 $E[e\{y := v_y \dots, i := v_i, \text{this} := (\text{make-object } z v_i)\}]$
 $P[(+ \text{number} \dots)] \quad [+]$
 $\rightarrow P[\sum [\text{number} \dots]]$
 $P[(\text{if0 } 0 e_1 e_2)] \quad [\text{if0t}]$
 $\rightarrow P[e_1]$
 $P[(\text{if0 } \text{number } e_1 e_2)] \quad [\text{if0f}]$
 $\rightarrow P[e_2]$
 $\quad \text{where } \text{number} \neq 0$

4.rkt Error

$ \begin{aligned} p ::= & (\text{begin } d \dots e) \\ d ::= & (\text{define } z e) \\ c ::= & (\text{class object} \% \\ & \quad (\text{init-field } i) \\ & \quad m \dots \\ & \quad (\text{super-make-object})) \\ m ::= & (\text{define/public } (x y \dots) \\ & \quad e) \\ e ::= & c \\ & \mid (\text{make-object } x e) \\ & \mid (\text{send } e x e \dots) \\ & \mid x \\ & \mid \text{this} \\ & \mid \text{number} \\ & \mid (\text{if0 } e e e) \\ & \mid (+ e \dots) \\ a ::= & (\text{begin } (\text{define } x v) \dots v) \\ v ::= & (\text{make-object } x v) \\ & \mid \text{number} \\ & \mid c \\ P ::= & (\text{begin } (\text{define } x v) \dots \\ & \quad (\text{define } x E) \\ & \quad (\text{define } x e) \dots \\ & \quad e) \\ & \mid (\text{begin } (\text{define } x v) \dots \\ & \quad E) \\ E ::= & (\text{make-object } x E) \\ & \mid (\text{send } E x e \dots) \\ & \mid (\text{send } v x v \dots E e \dots) \\ & \mid (\text{if0 } E e e) \\ & \mid (+ v \dots E e \dots) \\ & \mid [] \end{aligned} $	$ \begin{aligned} & (\text{begin } \\ & \quad d_1 \dots \\ & \quad (\text{define } z \\ & \quad \quad (\text{class object} \% \\ & \quad \quad \quad (\text{init-field } i) \\ & \quad \quad \quad m_1 \dots \\ & \quad \quad (\text{define/public } (x y \dots) \\ & \quad \quad \quad e) \\ & \quad \quad m_2 \dots \\ & \quad \quad (\text{super-make-object}))) \\ & \quad (\text{begin } \\ & \quad \quad d_2 \dots \\ & \quad \quad E[e\{y := v_y \dots, \\ & \quad \quad \quad i := v_i, \\ & \quad \quad \quad \text{this} := (\text{make-object } z v)\}]) \\ & \quad (\text{class } \\ & \quad \quad P[\text{(+ number } \dots)] \quad [+] \\ & \quad \quad P[\Sigma] \quad [\Sigma] \\ & \quad \quad P[(\text{if0 } 0 e_1 e_2)] \quad [\text{if0}] \\ & \quad \quad P[(\text{if0 } \text{number } e_1 e_2)] \quad [\text{if0}] \\ & \quad \quad P[e_1] \quad (\text{init-field } h) \\ & \quad \quad P[e_2] \quad (\text{super-make-object})) \\ & \quad \quad \quad \text{where } \text{number} \neq 0 \end{aligned} $	$ \begin{aligned} & \rightarrow (\text{begin } \\ & \quad d_1 \dots \\ & \quad (\text{define } z \\ & \quad \quad (\text{class object} \% \\ & \quad \quad \quad (\text{init-field } i) \\ & \quad \quad \quad m_1 \dots \\ & \quad \quad (\text{define/public } (x y \dots) \\ & \quad \quad \quad e) \\ & \quad \quad m_2 \dots \\ & \quad \quad (\text{super-make-object}))) \end{aligned} $	
<p style="text-align: center;">7)</p>			

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

5.rkt

$p ::= (\text{begin } d \dots e)$	$(\text{begin} \quad d_1 \dots \quad (\text{define } z \quad (\text{class object}\% \quad (\text{init-field } i) \quad m_1 \dots \quad (\text{super-make-object})) \quad m_2 \dots \quad (\text{super-make-object}))) \quad d_2 \dots \quad E[\text{(send (make-object } z \quad v_i) \quad x \quad v_y \dots)]))$	$\rightarrow (\text{begin} \quad d_1 \dots \quad (\text{define } z \quad (\text{class object}\% \quad (\text{init-field } i) \quad m_1 \dots \quad (\text{define/public } (x \quad y \dots) \quad e) \quad m_2 \dots \quad (\text{super-make-object}))) \quad d_2 \dots \quad E[e\{y := v_y \quad \dots, \quad i := v_i, \quad \text{this} := (\text{make-object } z \quad v_i)\}])$	[send]
$d ::= (\text{define } z \quad e)$			
$c ::= (\text{class object}\% \quad (\text{init-field } i) \quad m_1 \dots \quad (\text{super-make-object}))$			
$m ::= (\text{define/public } (x \quad y \dots) \quad e)$			
$e ::= c$			
$ \quad (\text{make-object } x \quad e)$			
$ \quad (\text{send } e \quad x \quad e \dots)$			
$ \quad x$			
$ \quad \text{this}$			
$ \quad \text{number}$			
$ \quad (\text{if0 } e \quad e \quad e)$			
$ \quad (+ \quad e \quad \dots)$			
$a ::= (\text{begin } (\text{define } x \quad v) \quad \dots \quad v)$	$P[(+ \quad \text{number} \quad \dots)] \quad [+]$		
$v ::= (\text{make-object } c \quad v)$	$\rightarrow P[\sum[[\text{number} \quad \dots]]]$		
$ \quad \text{number}$			
$ \quad c$			
$P ::= (\text{begin } (\text{define } x \quad v) \quad \dots \quad (\text{define } x \quad E) \quad (\text{define } x \quad e) \quad \dots \quad e)$	$P[(\text{if0 } 0 \quad e_1 \quad e_2)] \quad [\text{if0t}]$		
$ \quad (\text{begin } (\text{define } x \quad v) \quad \dots \quad E)$	$\rightarrow P[e_1]$		
$E ::= (\text{make-object } x \quad E)$	$P[(\text{if0 } \text{number} \quad e_1 \quad e_2)] \quad [\text{if0f}]$		
$ \quad (\text{send } E \quad x \quad e \quad \dots)$	$\rightarrow P[e_2]$		
$ \quad (\text{send } v \quad x \quad v \quad \dots \quad E \quad e \quad \dots)$	where $\text{number} \neq 0$		
$ \quad (\text{if0 } E \quad e \quad e)$			
$ \quad (+ \quad v \quad \dots \quad E \quad e \quad \dots)$			
$ \quad []$			
$x, y, z ::= \text{variable-not-otherwise-mentioned}$			

5.rkt Error

$p ::= (\text{begin } d \dots e)$	$(\text{begin} \quad \quad \quad \longrightarrow (\text{begin}$	
$d ::= (\text{define } z e)$	$d_1 \dots \quad \quad \quad d_1 \dots$	
$c ::= (\text{class object}%$	$(\text{define } z \quad \quad \quad (\text{define } z$	
$\quad (\text{init-field } i)$	$(\text{class object}% \quad \quad \quad (\text{class object}%$	
$m \dots$	$\quad (\text{init-field } i) \quad \quad \quad \quad (\text{init-field } i)$	
$\quad (\text{super-make-object}))$	$m_1 \dots \quad \quad \quad m_1 \dots$	
$m ::= (\text{define/public } (x y \dots)$	$(\text{define/public } (x y \dots) \quad \quad \quad (\text{define/public } (x y \dots)$	
$e)$	$ \quad \quad \quad e) \quad \quad \quad e)$	
$e ::= c$	$m_2 \dots \quad \quad \quad m_2 \dots$	
$\quad (\text{make-object } x e)$	$(\text{super-make-object})) \quad \quad \quad (\text{super-make-object}))$	
$\quad (\text{send } e e \dots)$	$d_2 \dots \quad \quad \quad d_2 \dots$	
$\quad x$	$E[(\text{send } (\text{make-object } z v_i) \quad \quad \quad E[e\{y := v_y \dots,$	
$\quad \text{this}$	$x \quad \quad \quad i := v_i,$	
$\quad \text{number}$	$v_y \dots)])) \quad \quad \quad \text{this} := (\text{make-object } z v_i) \}]$	
$\quad (\text{if0 } e e e)$		
$\quad (+ e \dots)$		
$a ::= (\text{begin } (\text{define } x v) \dots v)$		
$v ::= (\text{make-object } c v)$		
$\quad \text{number}$		
$\quad c$		
$P ::= (\text{begin } (\text{define } x v) \dots$	$\longrightarrow P[e_1]$	
$\quad (\text{define } x E)$	$P[(\text{if0 } number e_1 e_2)] \quad [\text{if0f}]$	
$\quad (\text{define } x e) \dots$		
$\quad e)$	$\longrightarrow P[e_2]$	
$\quad (\text{begin } (\text{define } x v) \dots$		
$\quad \quad E)$	$ \quad \quad \quad \text{where } number \neq 0$	
$E ::= (\text{make-object } x E)$		
$\quad (\text{send } E x e \dots)$		
$\quad (\text{send } v x v \dots E e \dots)$		
$\quad (\text{if0 } E e e)$		
$\quad (+ v \dots E e \dots)$		
$\quad []$		
$x, y, z ::= \text{variable-not-otherwise-mentioned}$		

Test case (on line 196)

$$\frac{P[\ (\text{if0 } number \ e_1 \ e_2) \] \quad [\text{if0f}]}{P[(\text{if0 } number \ e_1 \ e_2)] \quad [\text{if0f}]}$$

$$\frac{P[(\text{if0 } number \ e_1 \ e_2)] \quad [\text{if0f}]}{P[e_2]}$$

where $number \neq 0$

6.rkt

$p ::= (\text{begin } d \dots e)$
 $d ::= (\text{define } z e)$
 $c ::= (\text{class object} \% (\text{init-field } i) m \dots (\text{super-make-object}))$
 $m ::= (\text{define/public } (x y \dots) e)$
 $e ::= c$
 | $(\text{make-object } x e)$
 | $(\text{send } e x e \dots)$
 | x
 | this
 | $number$
 | $(\text{if0 } e e e)$
 | $(+ e \dots)$
 $a ::= (\text{begin } (\text{define } x v) \dots v)$
 $v ::= (\text{make-object } c v)$
 | $number$
 | c
 $P ::= (\text{begin } (\text{define } x v) \dots (\text{define } x E) (\text{define } x e) \dots e)$
 | $(\text{begin } (\text{define } x v) \dots E)$
 $E ::= (\text{make-object } v E)$
 | $(\text{make-object } E e)$
 | $(\text{send } E x e \dots)$
 | $(\text{send } v x v \dots E e \dots)$
 | $(\text{if0 } E e e)$
 | $(+ v \dots E e \dots)$
 | $[]$

$P[(\text{send } (\text{make-object } (\text{class object} \% (\text{init-field } i) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object}))) v_i)]$
 $\longrightarrow P[e \{y := v_y \dots, i := v_i, \text{this} := (\text{make-object } (\text{class object} \% (\text{init-field } i) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object}))) v_i\}]$
 $\qquad [send]$
 $P[(+ number \dots)] \quad [+] \quad (\text{begin } d_1 \dots (\text{define } x v) \dots d_2 \dots E[x]) \longrightarrow (\text{begin } d_1 \dots (\text{define } x v) \dots d_2 \dots E[v])$
 $\longrightarrow P[\sum [number \dots]]$
 $\longrightarrow P[(\text{if0 } 0 e_1 e_2)] \quad [\text{if0t}] \quad E[x])$
 $\longrightarrow P[e_1]$
 $\longrightarrow P[(\text{if0 } number e_1 e_2)] \quad [\text{if0f}]$
 $\longrightarrow P[e_2]$
 $\qquad \text{where } number \neq 0$

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

6.rkt Error

```


$$\begin{array}{l}
p ::= (\text{begin } d \dots e) \\
d ::= (\text{define } z e) \\
c ::= (\text{class object} \% \\
\quad (\text{init-field } i) \\
\quad m \dots \\
\quad (\text{super-make-object})) \\
m ::= (\text{define/public } (x y \dots) \\
\quad e) \\
e ::= c \\
| (\text{make-object } x e) \\
| (\text{send } e x e \dots) \\
| x \\
| \text{this} \\
| \text{number} \\
| (\text{if0 } e e e) \\
| (+ e \dots) \\
a ::= (\text{begin } (\text{define } x v) \dots v) \\
v ::= (\text{make-object } c v) \\
\quad | \text{number} \\
\quad | c \\
P ::= (\text{begin } (\text{define } x v) \dots \\
\quad (\text{define } x E) \\
\quad (\text{define } x e) \dots \\
\quad e) \\
\quad | (\text{begin } (\text{define } x v) \dots \\
\quad \quad E) \\
E ::= (\text{make-object } v E) \\
\quad | (\text{make-object } E e) \\
\quad | (\text{send } E x e \dots) \\
\quad | (\text{send } v x v \dots E e \dots) \\
\quad | (\text{if0 } E e e) \\
\quad | (+ v \dots E e \dots) \\
\quad | []
\end{array}$$


```

substs-e gets a non-“e” as argument in test cases

where $number \neq 0$

$\longrightarrow P[e \{ y := v_y \dots,$
 $i := v_i,$
this := (make-object
 (class object%
 (init-field i)
 $m_1 \dots$
 (define/public ($x y \dots$)
 e)
 $m_2 \dots$
 (super-make-object))
 $\dots \}]$

ment $\longrightarrow (\begin{matrix} d_1 \\ \vdots \\ d_n \end{matrix})$ $\begin{matrix} (begin \\ d_1 \dots \\ (define x v) \\ d_2 \dots \\ E[v]) \end{matrix}$

[send]

substs-e gets a
non-“e” as argument
in test cases

7.rkt

$p ::= (\text{begin } d \dots e)$
 $d ::= (\text{define } z e)$
 $c ::= (\text{class object} \%)$
 $\quad (\text{init-field } i)$
 $m \dots$
 $\quad (\text{super-make-object})$
 $m ::= (\text{define/public } (x y \dots))$
 $\quad e)$
 $e ::= c$
 $\quad | (\text{make-object } e e)$
 $\quad | (\text{send } e x e \dots)$
 $\quad | x$
 $\quad | \text{this}$
 $\quad | \text{number}$
 $\quad | (\text{if0 } e e e)$
 $\quad | (+ e \dots)$
 $a ::= (\text{begin } (\text{define } x v) \dots v)$
 $v ::= (\text{make-object } c v)$
 $\quad | \text{number}$
 $\quad | c$
 $P ::= (\text{begin } (\text{define } x v) \dots$
 $\quad (\text{define } x E)$
 $\quad (\text{define } x e) \dots$
 $\quad e)$
 $\quad | (\text{begin } (\text{define } x v) \dots$
 $\quad E)$
 $E ::= (\text{make-object } v E)$
 $\quad | (\text{make-object } E e)$
 $\quad | (\text{send } E x e \dots)$
 $\quad | (\text{send } v x v \dots E e \dots)$
 $\quad | (\text{if0 } E e e)$
 $\quad | (+ v \dots E e \dots)$
 $\quad | []$

$P[(\text{send } (\text{make-object}$
 $\quad (\text{class object} \%)$
 $\quad (\text{init-field } i))$
 $\quad m_1 \dots$
 $\quad (\text{define/public } (x y \dots))$
 $\quad e)$
 $\quad m_2 \dots$
 $\quad (\text{super-make-object}))$
 $\quad v_i)$
 $\quad x$
 $\quad v_y \dots)]$

$\longrightarrow P[e\{y := v_y \dots,$
 $\quad i := v_i,$
 $\quad \text{this} := (\text{make-object}$
 $\quad (\text{class object} \%)$
 $\quad (\text{init-field } i))$
 $\quad m_1 \dots$
 $\quad (\text{define/public } (x y \dots))$
 $\quad e)$
 $\quad m_2 \dots$
 $\quad (\text{super-make-object}))$
 $\quad v_i)\}]$

$P[(+ \text{number} \dots)] \quad [+] \quad (\text{begin } d_1 \dots$
 $\quad (\text{define } x v) \quad \longrightarrow (\text{begin } d_1 \dots$
 $\quad d_2 \dots \quad (\text{define } x v)$
 $\quad E[x]) \quad d_2 \dots$
 $\longrightarrow P[\sum [\text{number} \dots]]]$
 $P[(\text{if0 } 0 e_1 e_2)] \quad [\text{if0t}] \quad E[v])$
 $\longrightarrow P[e_1]$
 $P[(\text{if0 } \text{number} e_1 e_2)] \quad [\text{if0f}]$
 $\longrightarrow P[e_2]$
 $\quad \text{where number} \neq 0$

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

7.rkt Error

```


$$\begin{array}{l}
p ::= (\textbf{begin } d \dots e) \\
d ::= (\textbf{define } z e) \\
c ::= (\textbf{class object} \% \\
       (\textbf{init-field } i) \\
       m \dots \\
       (\textbf{super-make-object})) \\
m ::= (\textbf{define/public } (x y \dots) \\
       e) \\
e ::= c \\
| (\textbf{make-object } e e) \\
| (\textbf{send } e x e \dots) \\
| x \\
| \textbf{this} \\
| number \\
| (\textbf{if0 } e e e) \\
| (+ e \dots) \\
a ::= (\textbf{begin } (\textbf{define } x v) \dots v) \\
v ::= (\textbf{make-object } c v) \\
| number \\
| c \\
P ::= (\textbf{begin } (\textbf{define } x v) \dots \\
      (\textbf{define } x E) \\
      (\textbf{define } x e) \dots \\
      e) \\
| (\textbf{begin } (\textbf{define } x v) \dots \\
      E)
\end{array}$$


```

x, y, z ::= variable-not-otherwise-mentioned

8.rkt

$p ::= (\text{begin } d \dots e)$
 $d ::= (\text{define } z e)$
 $c ::= (\text{class object} \% (\text{init-field } i) m \dots (\text{super-make-object}))$
 $m ::= (\text{define/public } (x y \dots) e)$
 $e ::= c$
 | $(\text{make-object } e e)$
 | $(\text{send } e x e \dots)$
 | x
 | this
 | $number$
 | $(\text{if0 } e e e)$
 | $(+ e \dots)$
 $a ::= (\text{begin } (\text{define } x v) \dots v)$
 $v ::= (\text{make-object } c v)$
 | $number$
 | c
 $P ::= (\text{begin } (\text{define } x v) \dots (\text{define } x E) (\text{define } x e) \dots e)$
 | $(\text{begin } (\text{define } x v) \dots E)$
 $E ::= (\text{make-object } v E)$
 | $(\text{make-object } E e)$
 | $(\text{send } E x e \dots)$
 | $(\text{send } v x v \dots E e \dots)$
 | $(\text{if0 } E e e)$
 | $(+ v \dots E e \dots)$
 | []

$P[(\text{send } (\text{make-object } (\text{class object} \% (\text{init-field } i) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object}))) v_i)]$
 $\longrightarrow P[e \{y := v_y \dots, i := v_i, \text{this} := (\text{make-object } (\text{class object} \% (\text{init-field } i) m_1 \dots (\text{define/public } (x y \dots) e) m_2 \dots (\text{super-make-object}))) v_i\}]$

$P[(+ number \dots)] \quad [+] \quad (\text{begin } d_1 \dots (\text{define } x v) \dots \rightarrow (\text{begin } d_1 \dots (\text{define } x v))$
 $\longrightarrow P[\sum [number \dots]] \quad d_2 \dots \quad d_2 \dots$
 $P[(\text{if0 } 0 e_1 e_2)] \quad [\text{if0t}] \quad E[x]) \quad E[v])$
 $\longrightarrow P[e_1] \quad (\text{begin } d_1 \dots (\text{define } x v) \dots \rightarrow (\text{begin } d_1 \dots (\text{define } x v))$
 $P[(\text{if0 } number e_1 e_2)] \quad [\text{if0f}] \quad d_2 \dots \quad d_2 \dots$
 $\longrightarrow P[e_2] \quad \text{where } number \neq 0 \quad (\text{define } y E[x]) \quad (\text{define } y E[v])$
 $d_3 \dots \quad d_3 \dots$
 $e) \quad e)$

$x, y, z ::= \text{variable-not-otherwise-mentioned}$

Racket's class system

There's more to the Racket class system, e.g.:

- The superclass position is an expression \Rightarrow mixins
- **define-local-member-name**
(exploiting scope for abstraction)
- inner + super

See Flatt et al [APLAS 2006] or the docs for more

Thank you.

Three parting thoughts:

Semantics Engineering is more than just Theorem Proving;

Random testing gives *no* guarantees, yet is incredibly useful;

Check out Racket's class system:
<http://racket-lang.org/>