Code analysis and transformation

Loop transformations

Simone Campanoni
simone.campanoni@northwestern.edu

Outline

- Simple loop transformations
- Loop invariants based transformations
- Induction variables based transformations
- Complex loop transformations

Simple loop transformations

Simple loop transformations are used to

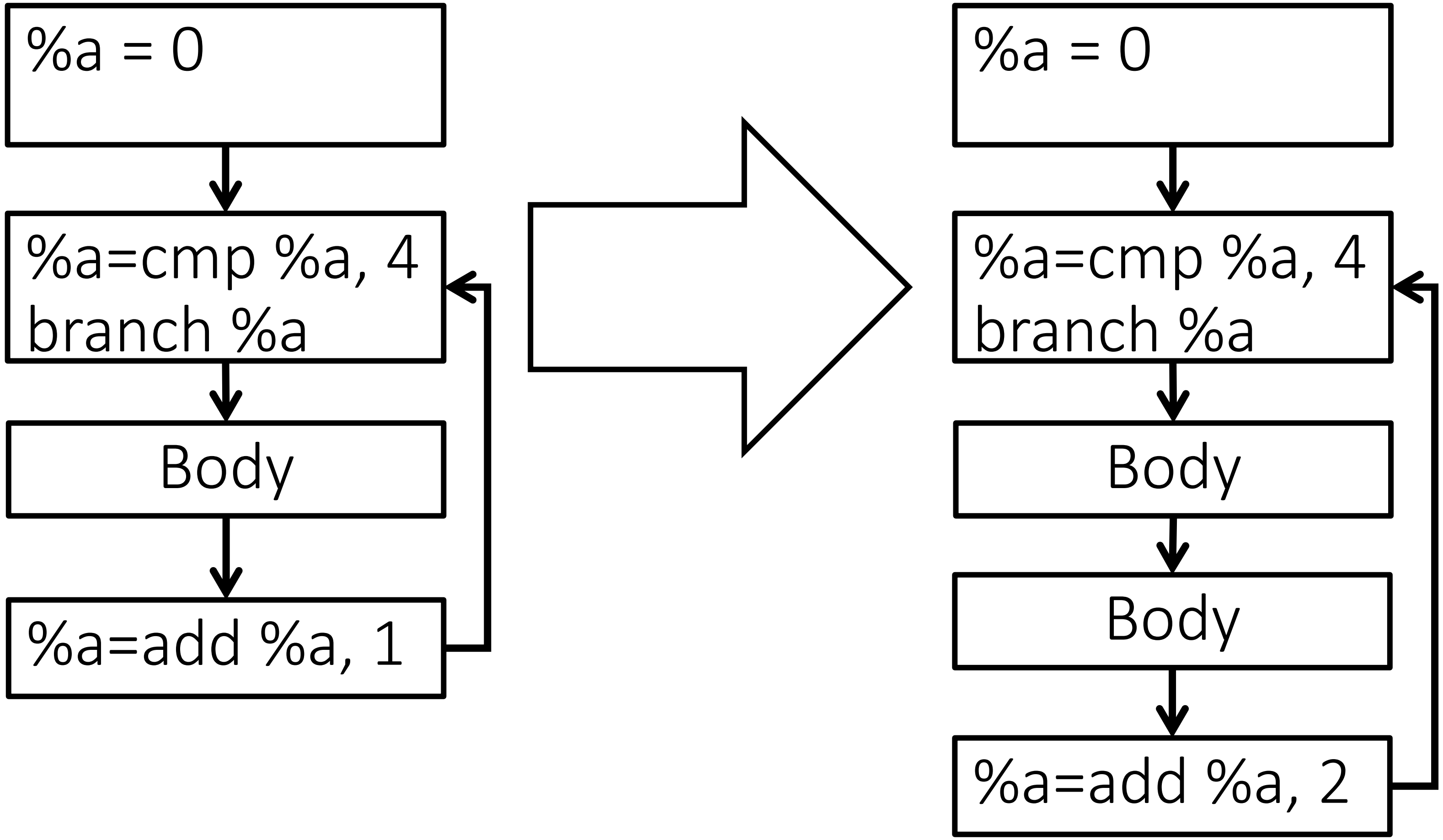
- Increase performance/energy savings

  and/or

- Unblock other transformations
  - E.g., increase the number of constant propagations
  - E.g., Extract thread-level parallelism from sequential code
  - E.g., Generate vector instructions

Loop unrolling

```
for (a=0; a < 4; a++){
   … // Body
}
```



Loop unrolling in LLVM: requirements

- The loop you want to unroll must be in LCSSA form

Loop unrolling in LLVM: dependences

```
void getAnalysisUsage(AnalysisUsage &AU) const override {
  AU.addRequired<AssumptionCacheTracker>();
  AU.addRequired<DominatorTreeWrapperPass>();
  AU.addRequired<LoopInfoWrapperPass>();
  AU.addRequired<ScalarEvolutionWrapperPass>();
  AU.addRequired<TargetTransformInfoWrapperPass>();
}
```

Loop unrolling in LLVM: headers

```
#include "llvm/Analysis/OptimizationRemarkEmitter.h"
#include "llvm/IR/Dominators.h"
#include "llvm/Transforms/Utils/LoopUtils.h"
#include "llvm/Transforms/Utils/UnrollLoop.h"
#include "llvm/Analysis/AssumptionCache.h"
#include "llvm/Analysis/ScalarEvolution.h"
#include "llvm/Analysis/ScalarEvolutionExpressions.h"
#include "llvm/Analysis/TargetTransformInfo.h"
```

Loop unrolling in LLVM

Get the results of the required analyses

```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
const auto &TTI = getAnalysis<TargetTransformInfoWrapperPass>().getTTI(F);
```

Fetch a loop

```
for (auto i : LI){
   auto loop = &*i;
   ...
}
```

```
void getAnalysisUsage(AnalysisUsage &AU) const override {
  AU.addRequired<AssumptionCacheTracker>();
  AU.addRequired<DominatorTreeWrapperPass>();
  AU.addRequired<LoopInfoWrapperPass>();
  AU.addRequired<ScalarEvolutionWrapperPass>();

  return ;
}
```

```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
const auto &TTI = getAnalysis<TargetTransformInfoWrapperPass>().getTTI(F);
```

Loop unrolling in LLVM: API

Unrolling factor

```
UnrollLoopOptions ULO;
ULO.Count = 2;
ULO.Force = false;
ULO.Runtime = false;
ULO.AllowExpensiveTripCount = true;
ULO.UnrollRemainder = false;
ULO.ForgetAllSCEV = true;
```

```
auto tripCount = SE.getSmallConstantTripCount(loop);
```

It is 0, or the number of iterations per invocation

```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
const auto &TTI = getAnalysis<TargetTransformInfoWrapperPass>().getTTI(F);
```

```
void getAnalysisUsage(AnalysisUsage &AU) const override {
  AU.addRequired<AssumptionCacheTracker>();
  AU.addRequired<DominatorTreeWrapperPass>();
  AU.addRequired<LoopInfoWrapperPass>();
  AU.addRequired<ScalarEvolutionWrapperPass>();
  AU.addRequired<TargetTransformInfoWrapperPass>();
}
```

Loop to unroll

```
auto unrolled = UnrollLoop(
  loop, ULO,
  &LI, &SE, &DT, &AC, &TTI, &ORE, true
  );
```

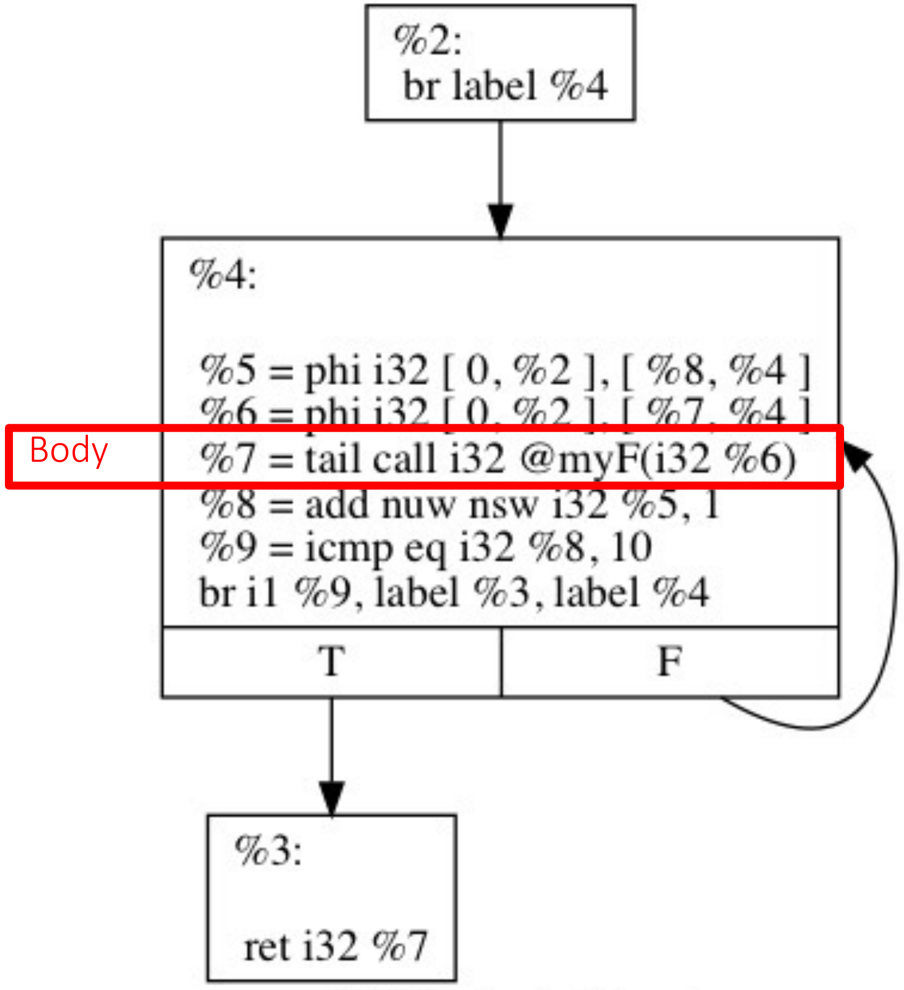
Unrolling options

Loop unrolling in LLVM: result

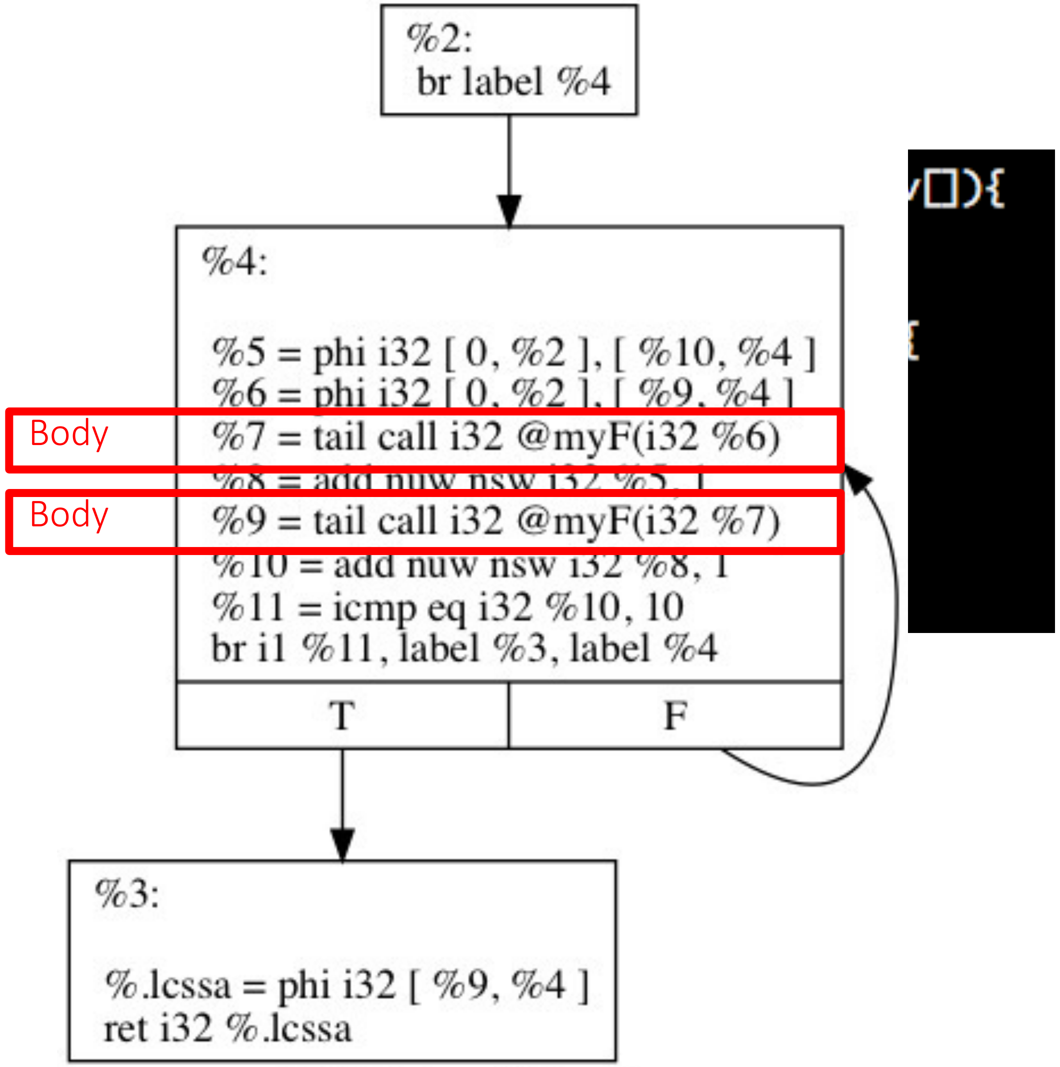
```
switch (unrolled){
  case LoopUnrollResult::FullyUnrolled :
    errs() << "   Fully unrolled\n";
    return true ;

  case LoopUnrollResult::PartiallyUnrolled :
    errs() << "   Partially unrolled\n";
    return true ;

  case LoopUnrollResult::Unmodified :
    errs() << "   Not unrolled\n";
    break ;

  default:
    abort();
}
```

```
auto unrolled = UnrollLoop(
  loop, ULO,
  &LI, &SE, &DT, &AC, &TTI, &ORE, true
  );
```

Loop unrolling in LLVM: example



CFG for 'main' function

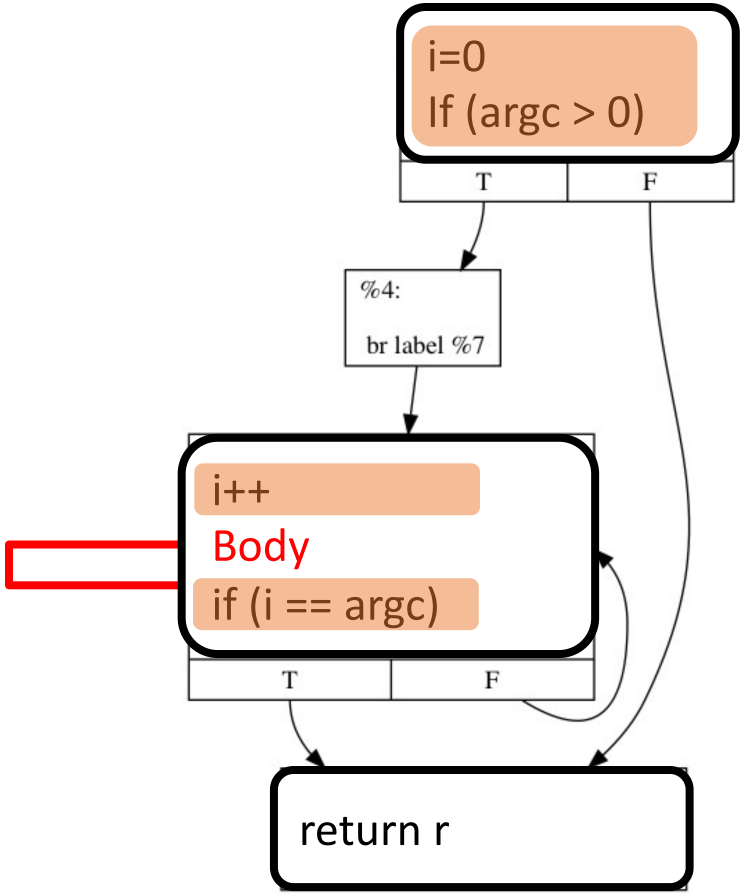


CFG for 'main' function

Loop unrolling in LLVM: Demo

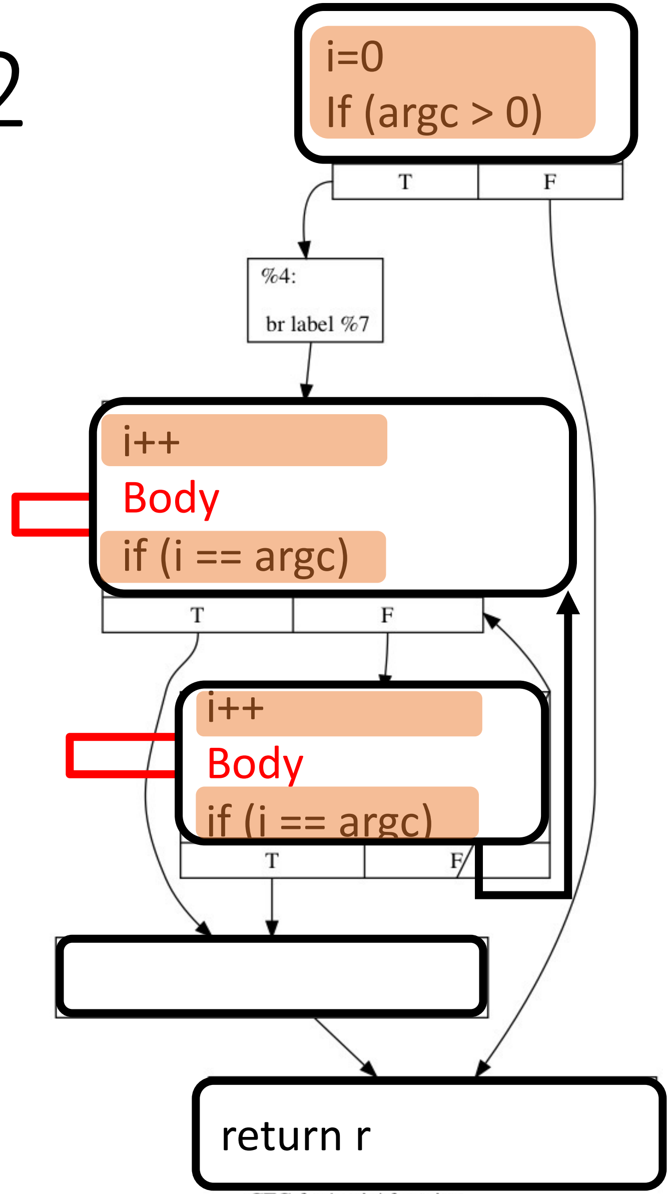
- Detail: Loops/README
- Pass: Loops/llvm/7
- C program: Loops/code/12
- C program: Loops/code/0

Loop unrolling in LLVM: example 2



CFG for 'main' function

```
int main (int argc, char *argv[]){
  auto r = 0;

  for (auto i=0; i < argc; i++){
    r = myF(r);
  }

  return r;
}
```



CFG for 'main' function

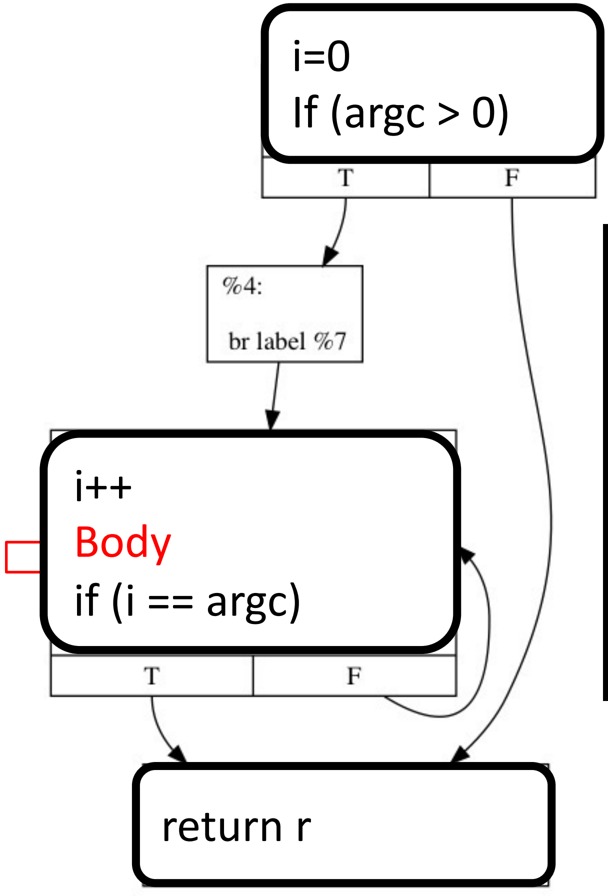
There is still the same amount of loop overhead!

Loop unrolling in LLVM: the runtime checks

```
UnrollLoopOptions ULO;
ULO.Count = 2;
ULO.Force = false;
ULO.Runtime = false;
ULO.AllowExpensiveTripCount = true;
ULO.UnrollRemainder = false;
ULO.ForgetAllSCEV = true;
```

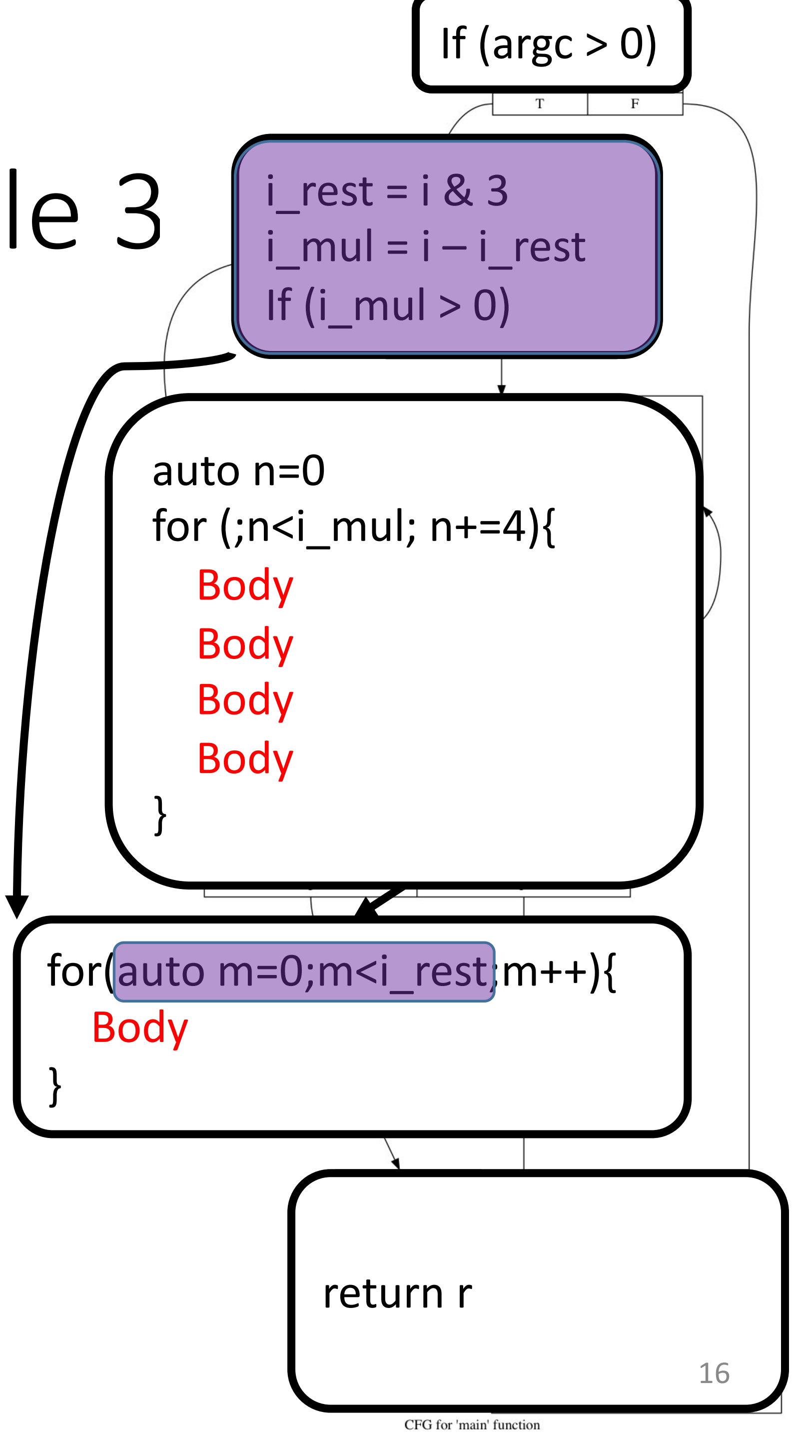
→ true

Loop unrolling in LLVM: example 3



CFG for 'main' function

```
int main (int argc, char *argv[]){
  auto r = 0;

  for (auto i=0; i < argc; i++){
    r = myF(r);
  }

  return r;
}
```



Runtime checks

Loop unrolling in LLVM: API

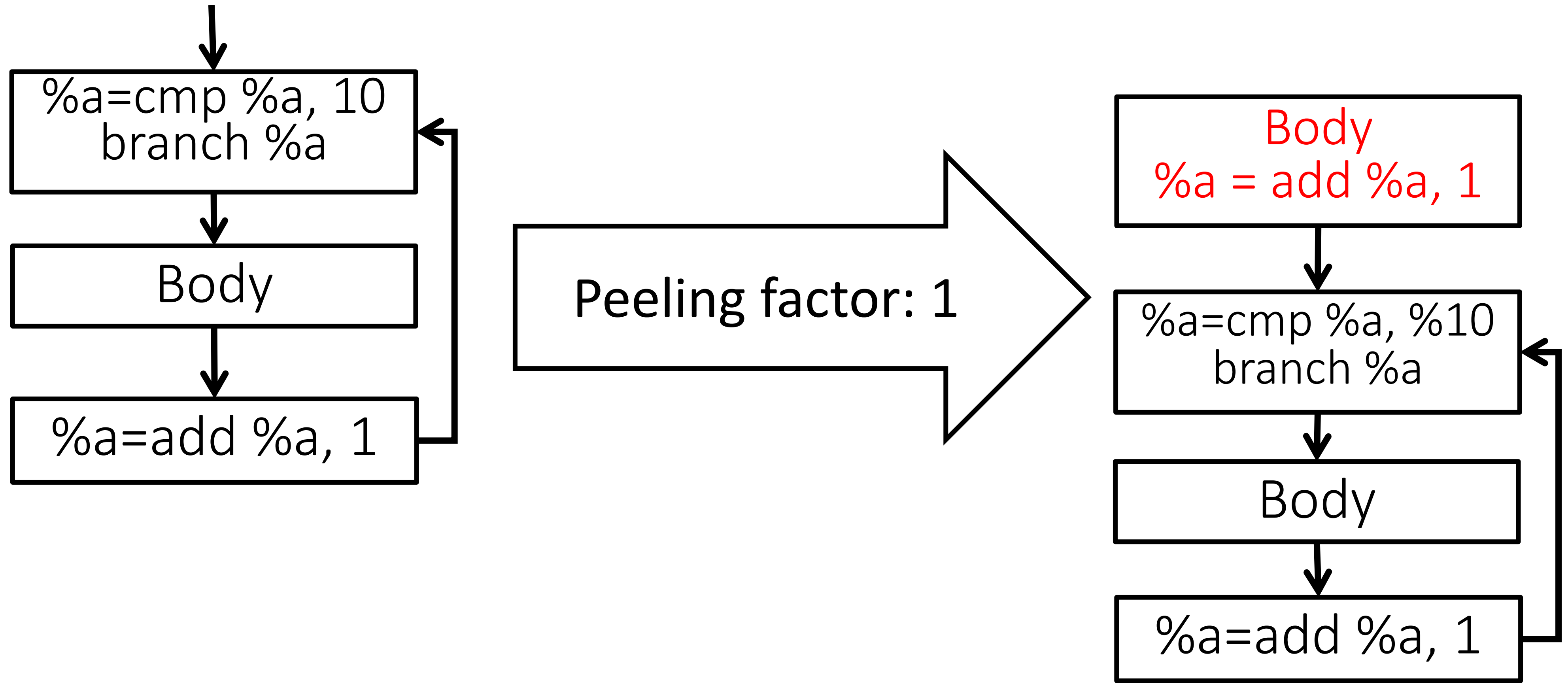
```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
const auto &TTI = getAnalysis<TargetTransformInfoWrapperPass>().getTTI(F);
```

```
auto unrolled = UnrollLoop(
  loop, ULO,
  &LI, &SE, &DT, &AC, &TTI, &ORE, true
  );
```

```
OptimizationRemarkEmitter ORE(&F);
```

Normalize the generated loop to LCSSA

Loop peeling



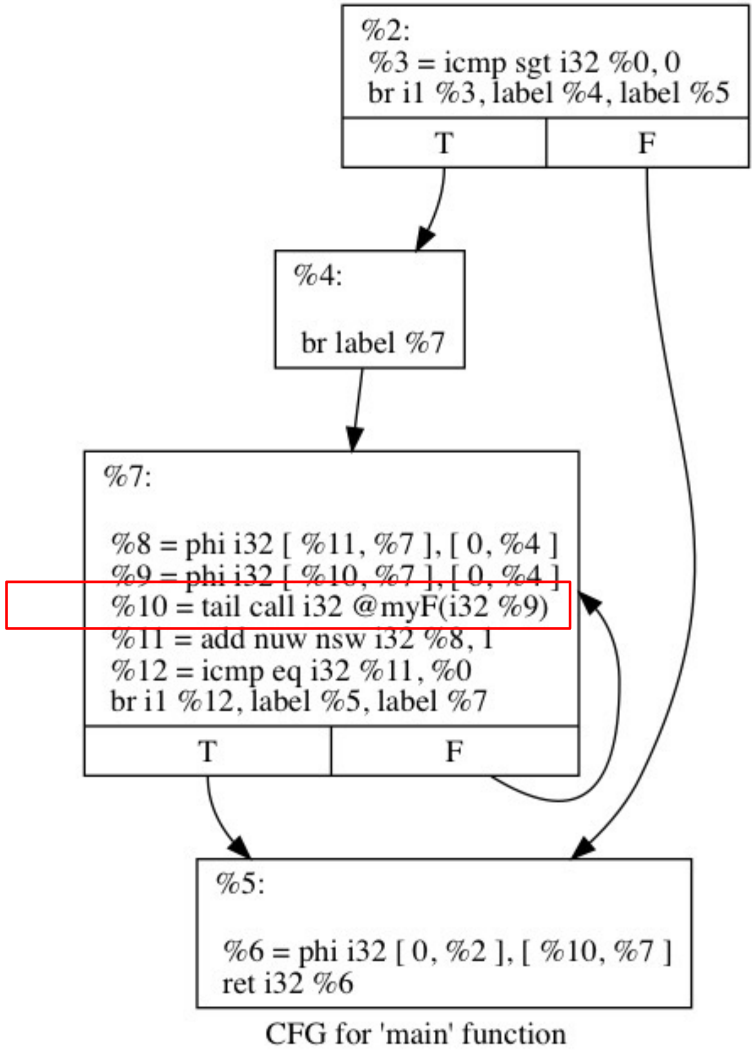
Loop peeling in LLVM

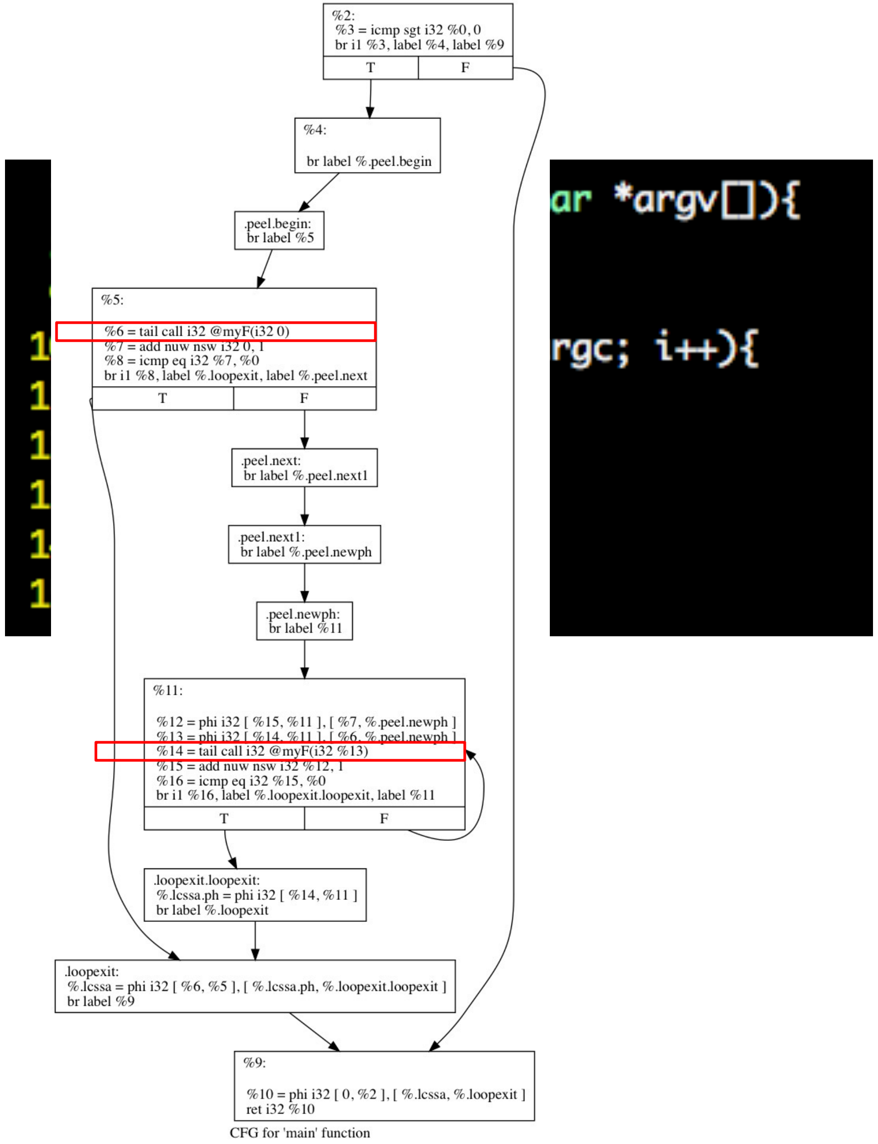
- API
```
#include "llvm/Transforms/Utils/LoopPeel.h"
```

```
auto peeled = peelLoop(
  loop, peelingCount,
  &LI, &SE, &DT, &AC,
  true);
```

  - No trip count
  - No flags
  - (almost) always possible
  - To check if you can peel, invoke the following API: bool canPeel(Loop *loop)

Loop peeling in LLVM: example





Fetching analyses outputs from a module pass

- From a function pass

```
auto& LI = getAnalysis<LoopInfoWrapperPass>().getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>().getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>().getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
```

- From a module pass

```
auto& LI = getAnalysis<LoopInfoWrapperPass>(F).getLoopInfo();
auto& DT = getAnalysis<DominatorTreeWrapperPass>(F).getDomTree();
auto& SE = getAnalysis<ScalarEvolutionWrapperPass>(F).getSE();
auto& AC = getAnalysis<AssumptionCacheTracker>().getAssumptionCache(F);
```

Outline

- Simple loop transformations
- Loop invariants based transformations
- Induction variables based transformations
- Complex loop transformations

Optimizations in small, hot loops

- Most programs: 90% of time is spent in few, small, hot loops

```
while (){
  statement 1
  statement 2
  statement 3
}
```

- Deleting a single statement from a small, hot loop
  might have a big impact
  (100 seconds -> 70 seconds)

Loop example

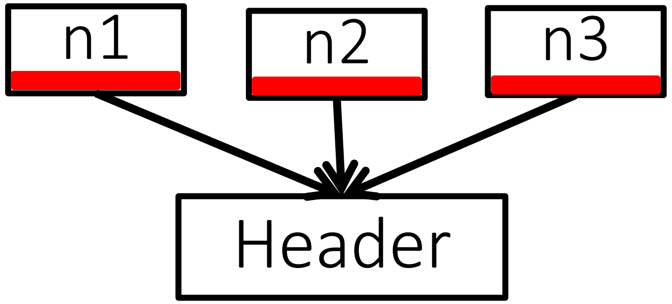
```
if (N>5){ k = 1; z = 4;}
else {k = 2; z = 3;}
  do {
    a = 1;
    y = x + N;
    b = k + z;
    c = a * 3;
    if (N < 0){
      m = 5;
      break;
    }
    x++;
} while (x < N);
```

- **Observation**: each statement in that loop will contribute to the program execution time
- **Idea**: what about moving statements from inside a loop to outside it?
- Which statements can be moved outside our loop?
- How to identify them automatically? (code analysis)
- How to move them? (code transformation)

Hoisting code

- In order to “hoist” a loop-invariant computation out of a loop, we need a place to put it
- We could copy it to all immediate predecessors of the loop header...

```
for (auto pBB : predecessors(H)){
  p = pBB->getTerminator();
  inv->moveBefore(p);
}
```



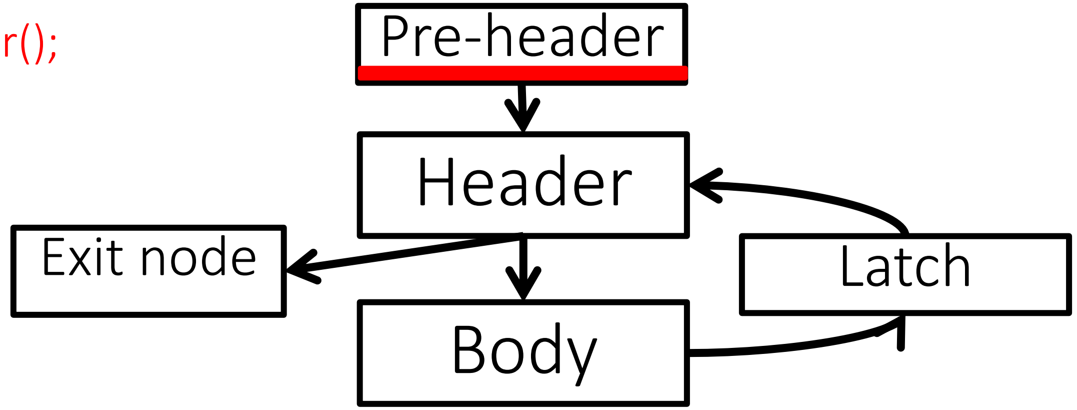
Is it correct?

- ...But we can avoid code duplication (and bugs) by taking advantage of loop normalization that guarantees the existence of the pre-header

Hoisting code

- In order to "hoist" a loop-invariant computation out of a loop, we need a place to put it
- We could copy it to all immediate predecessors of the loop header...

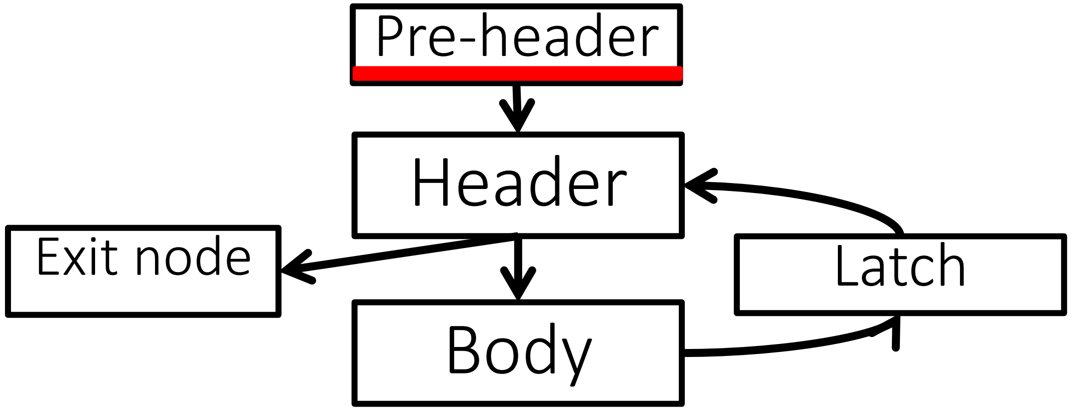
```
pBB = loop->getLoopPreheader();
p = pBB->getTerminator();
inv->moveBefore(p);
```



- ...but we can avoid code duplication (and bugs) by taking advantage of loop normalization that guarantees the existence of the pre-header

Can we hoist
all invariant instructions of a loop L
in the pre-header of L?

```
for (inv : invariants(loop)){
  pBB = loop->getLoopPreheader();
  p = pBB->getTerminator();
  inv->moveBefore(p);
}
```



Hoisting conditions

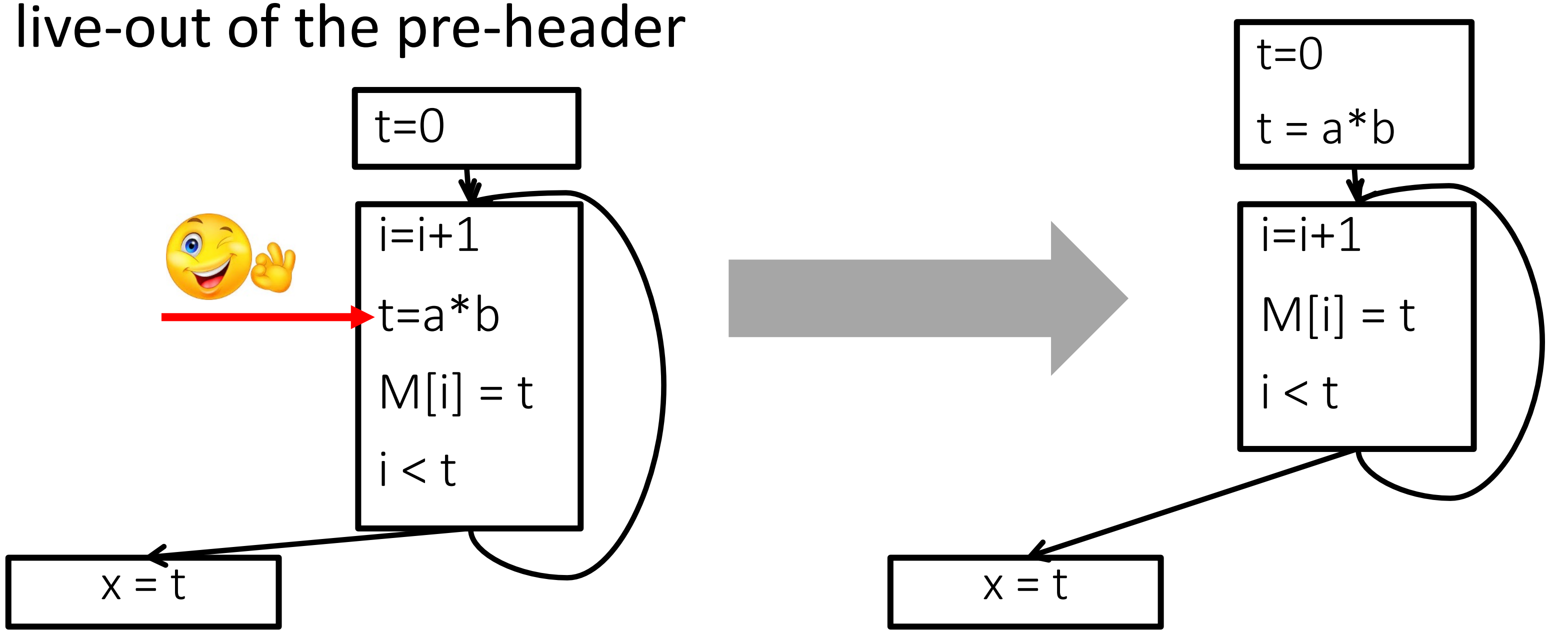
Loop invariant code motion

- For a loop-invariant definition

(d) t = x op y

- Assuming no SSA, we can hoist d into the loop's pre-header if
    1. d dominates all loop exits at which t is live-out, and
    2. there is only one definition of t in the loop, and
    3. t is not live-out of the pre-header

??



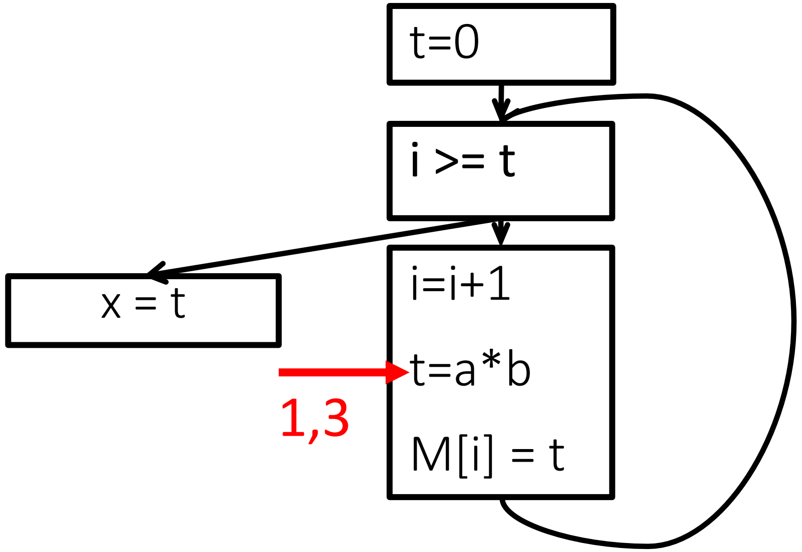
Hoisting conditions

Loop invariant code motion

- For a loop-invariant definition

(d) t = x op y

- Assuming no SSA, we can hoist d into the loop's pre-header if
  1. d dominates all loop exits at which t is live-out, and
  2. there is only one definition of t in the loop, and
  3. t is not live-out of the pre-header



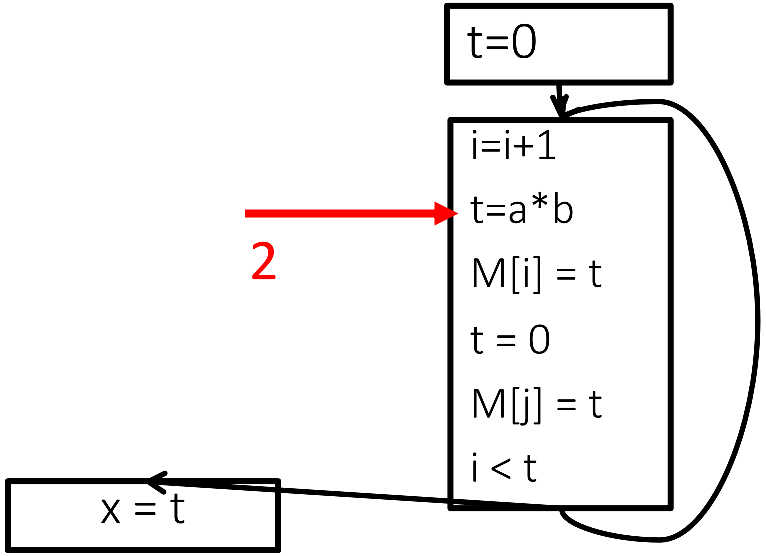
Hoisting conditions

Loop invariant code motion

- For a loop-invariant definition

(d) t = x op y

- Assuming no SSA, we can hoist d into the loop's pre-header if
    1. d dominates all loop exits at which t is live-out, and
    2. there is only one definition of t in the loop, and
    3. t is not live-out of the pre-header



Hoisting conditions

Loop invariant code motion

- For a loop-invariant definition

(d) t = x op y

- Assuming SSA, we can hoist d into the loop's pre-header if
  1. ~~d dominates all loop exits at which t is live-out, and~~
  2. ~~there is only one definition of t in the loop, and~~
  3. t is not live-out of the pre-header

Hoisting conditions

Loop invariant code motion

- For a loop-invariant definition

(d) t = x op y

- Assuming SSA, we can hoist d into the loop's pre-header if t is not live-out of the pre-header

Hoisting conditions

Loop invariant code motion

- For a loop-invariant definition

(d) t = load X

- Assuming SSA, we can hoist d into the loop's pre-header if

??

Outline

- Simple loop transformations
- Loop invariants based transformations
- Induction variables based transformations
- Complex loop transformations

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;
11:} while (x < N);
```

Assuming a,b,c,m are used after our code

Do we have to execute 4 for every iteration?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;
11:} while (x < N);
```

y=N

Do we have to execute 4 for every iteration?

Compute manually values of x and y for every iteration
What do you see?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;y++;
11:} while (x < N);
```

y=N

Do we have to execute 4 for every iteration?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++; y++;
11:} while (y < (2*N));
```

y=N

Do we have to execute 4 for every iteration?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  y++;
11:} while (y < (2*N));
```

y=N

Do we have to execute 4 for every iteration?

Do we have to execute 10 for every iteration?

Loop example

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  y++;
11:} while (y < tmp);
```

y=N;tmp=2*N;

Do we have to execute 4 for every iteration?

x, y are induction variables

Do we have to execute 10 for every iteration?

Is the code transformation worth it?

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
A :y=N;tmp=2*N;
   do {
3:   a = 1;

5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  y++;
11:} while (y < tmp);
```

Induction variable elimination

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;
11:} while (x < N);
```

… and after Loop Invariant Code Motion …

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}
A :y=N;tmp=2*N;
3 :a=1;
5 :b=k+z;
6: c=a*3;

    do{
7:   if (N < 0){
8:      m = 5;
9:      break;
     }
10: y++;
11:} while (y < tmp);
```

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}

    do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:      m = 5;
9:      break;
     }
10: x++;
11:} while (x < N);
```

… and with a better Loop Invariant Code Motion …

```
 1: if (N>5){ k = 1; z = 4;}
 2: else {k = 2; z = 3;}
A :y=N;tmp=2*N;
3 :a=1;
5 :b=k+z;
6: c=a*3;
7: if (N < 0){
8:    m=5;
    }

    do{
10:      y++;
11:} while (y < tmp);
```

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}

    do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;
11:} while (x < N);
```

… and after dead code elimination …

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}

3 :a=1;
5 :b=k+z;
6: c=a*3;
7: if (N < 0){
8:    m=5;
    }
```

Assuming a,b,c,m are used after our code

```
1: if (N>5){ k = 1; z = 4;}
2: else {k = 2; z = 3;}

   do {
3:   a = 1;
4:   y = x + N;
5:   b = k + z;
6:   c = a * 3;
7:   if (N < 0){
8:     m = 5;
9:     break;
     }
10:  x++;
11:} while (x < N);
```

Induction variable elimination

- Suppose we have a loop variable
  - i initially set to i_0; each iteration $i = i + 1$
- and a variable that linearly depends on it
  - $x = i * c_1 + c_2$

Loop invariants

- We can
  - Initialize $x = i_o * c_1 + c_2$
  - Increment x by c_1 each iteration

Is it faster?

1: $i = i_o$
2: do {
3: $i = i + 1;$
...
A: $x = i * c_1 + c_2$
B:} while (i < maxI);

1: $i = i_o$
N1: $x = i_o * c_1 + c_2$
2: do {
3: $i = i + 1;$
...
A: $x = x + c_1$
B:} while (i < maxI);

On some hardware, adds are faster than multiplies

- Strength reduction

Induction variable elimination: step 1

Run induction variable identification

① Iterate over IVs

$k = j * c1 + c2$

- where the IV $j = (i, a, b)$, and
- this is the only def of k in the loop, and
- there is no def of i between the def of j and the def of k

```
i = ...
...
j = i ...
...
k = j ...
```

② Record as $k = (i, a*c1, b*c1+c2)$

Induction variable elimination: step 2

For an induction variable k = (i, c1, c2)

① Initialize k = i * c1 + c2 in the pre-header

② Replace k's def in the loop by k = k + c1

- Make sure to do this after i's definition

Outline

- Simple loop transformations
- Loop invariants based transformations
- Induction variables based transformations
- Complex loop transformations

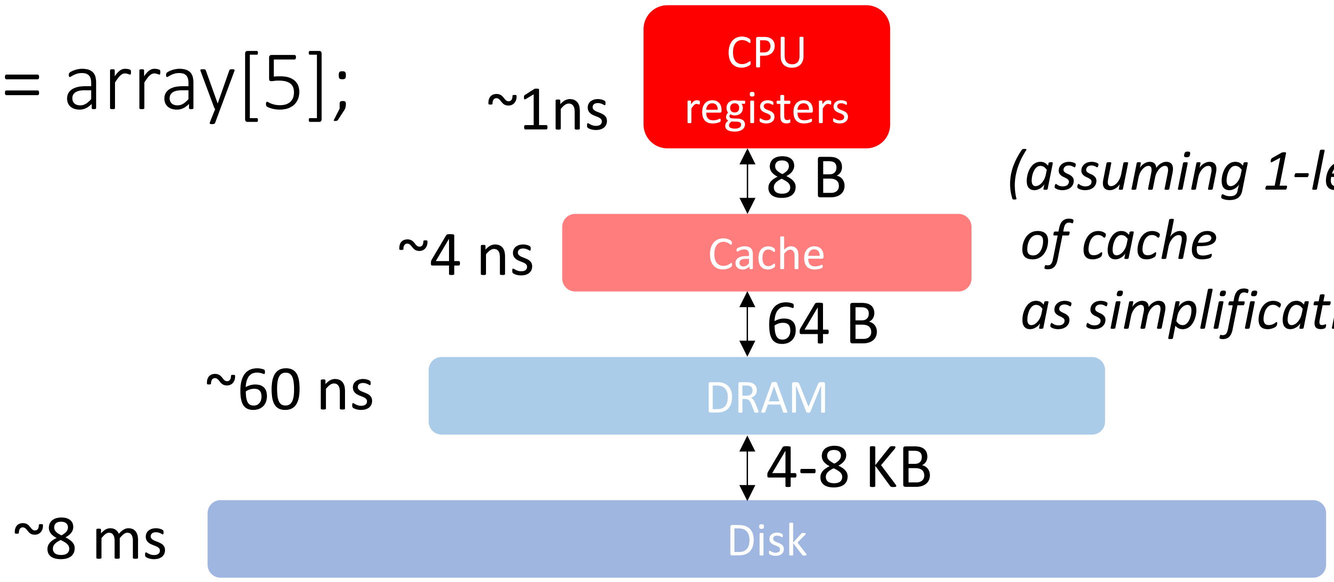
Loop transformations

- Restructure a loop to expose more optimization opportunities and/or transform the “loop overhead”
  - Loop unrolling, loop peeling, …
- Reorganize a loop to improve memory utilization
  - Cache blocking, skewing, loop reversal
- Distribute a loop over cores/processors
  - DOACROSS, DOALL, DSWP, HELIX

Loop transformations for memory optimizations

- How many clock cycles will it take?

```
...
varX = array[5];
...
```



(assuming 1-level of cache as simplification)

Goal: improve cache performance

- **Temporal locality**

  A resource that has just been referenced
  will more likely be referenced again in the near future

- **Spatial locality**

  The likelihood of referencing a resource is higher
  if a resource near it was just referenced

- Ideally, a compiler generates code
  with high temporal and spatial locality
  for the target architecture
  - What to minimize: bad replacement decisions

What a compiler can do

- Time:
  - When is an object accessed?
- Space:
  - Where does an object exist in the address space?
  - What is the data layout of an object in memory?
- These are the two “knobs” a compiler can manipulate

First understand cache behavior ...

- When do cache misses occur?
  - Use locality analysis
- Can we change the visitation order to produce better behavior?
  - Evaluate costs
- Does the new visitation order still produce correct results?
  - Use dependence analysis

… and then rely on loop transformations

- loop interchange
- cache blocking
- loop fusion
- loop reversal
- …

Code example

```
double A[N][N], B[N][N];
...
for i = 0 to N-1{
  for j = 0 to N-1{
    ... = A[i][j] ...
  }
}
```

How can we represent the different memory accesses of between all loop iterations?

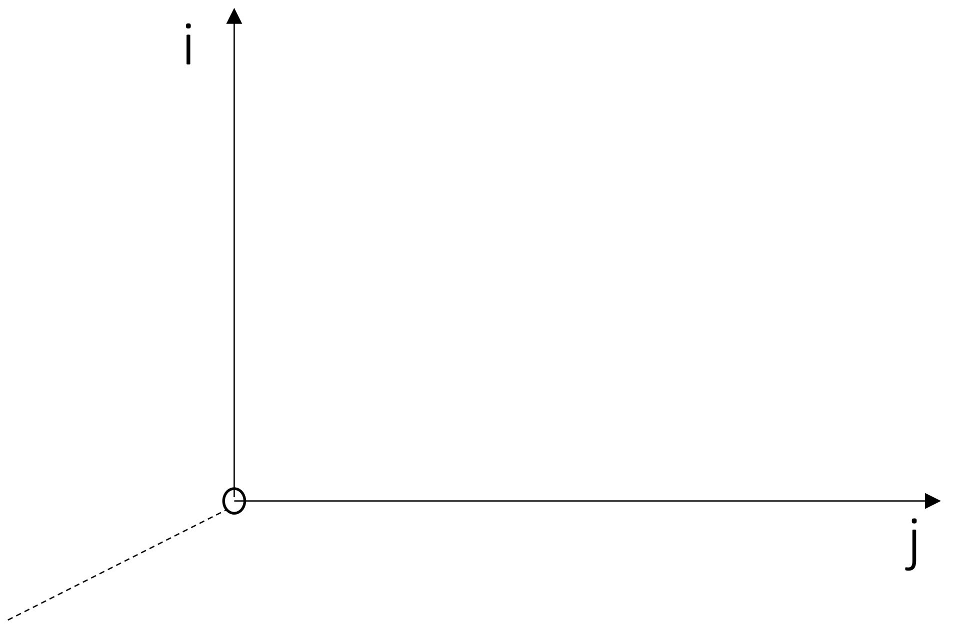
Iteration space for A

A[0][0]
A[0][1]
...
A[1][0]
A[1][1]
...

Code example

```
double A[N][N], B[N][N];
…
for i = 0 to N-1{
  for j = 0 to N-1{
     … = A[i][j] …
  }
}
```

Iteration space for A

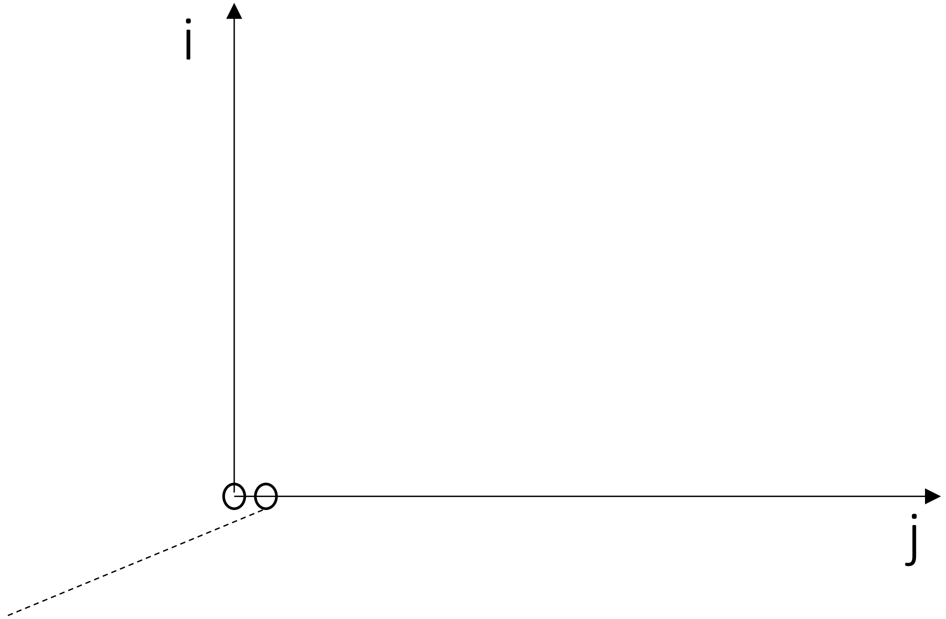


Memory access performed at the iteration i=0 *and* j=0

Code example

```
double A[N][N], B[N][N];
…
for i = 0 to N-1{
  for j = 0 to N-1{
     … = A[i][j] …
  }
}
```

Iteration space for A

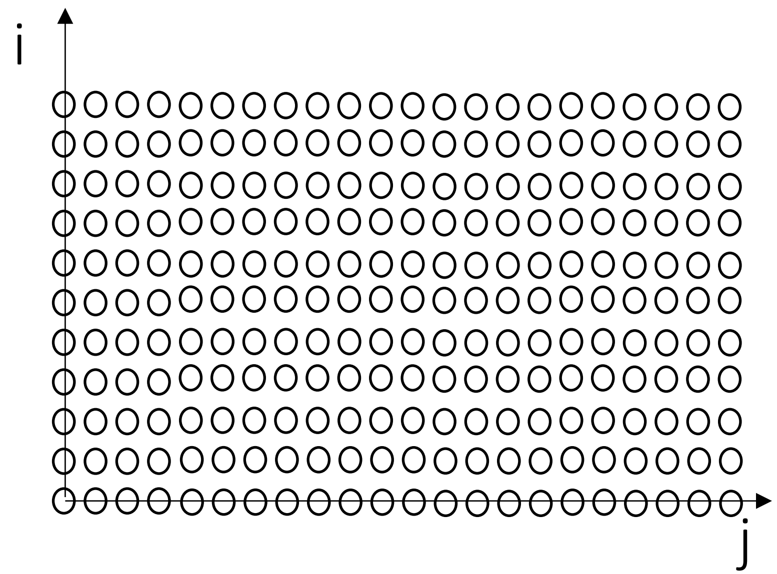


Memory access performed at the iteration i=0 *and* j=1

Code example

```
double A[N][N], B[N][N];
…
for i = 0 to N-1{
 for j = 0 to N-1{
    … = A[i][j] …
 }
}
```

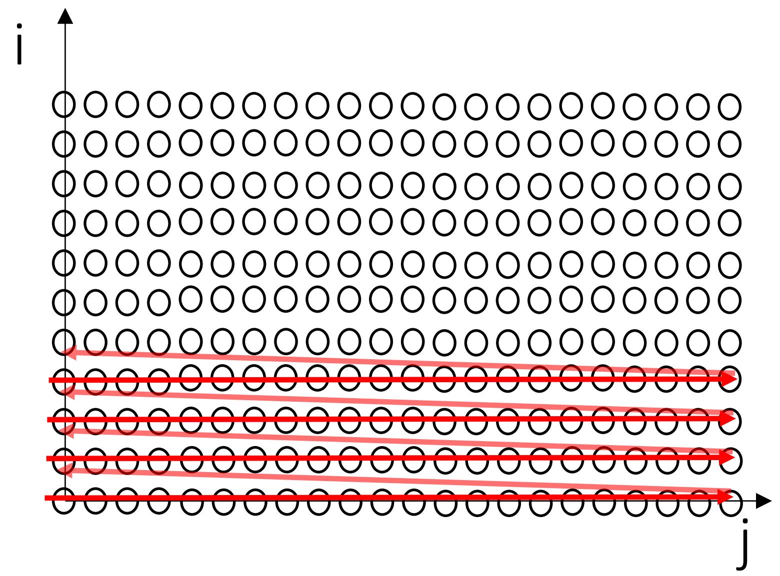
Iteration space for A



Code example

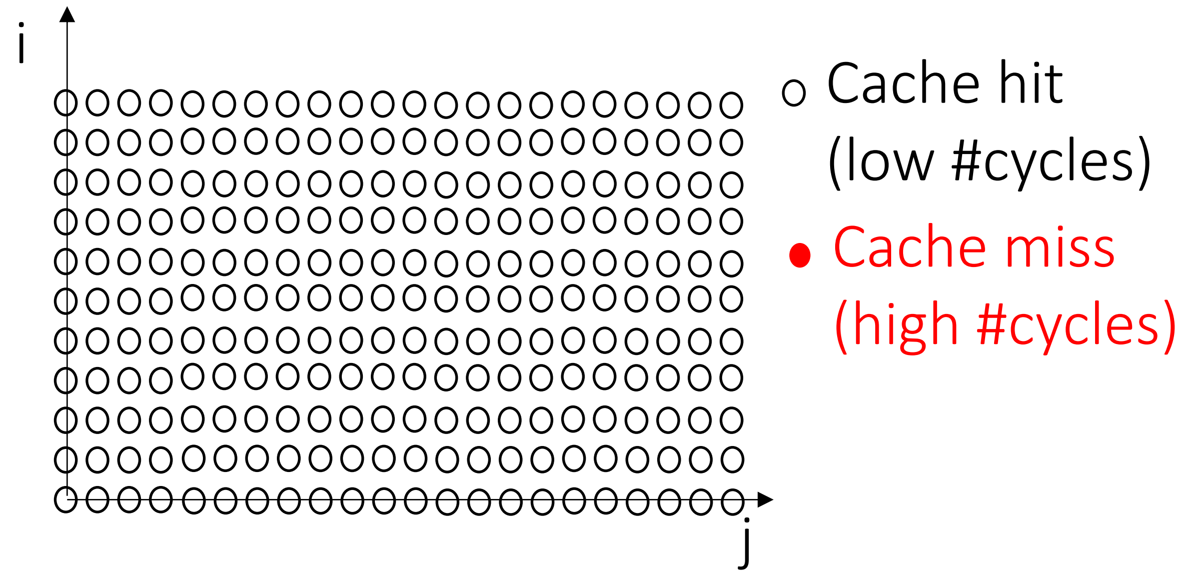
```
double A[N][N], B[N][N];
…
for i = 0 to N-1{
  for j = 0 to N-1{
     … = A[i][j] …
  }
}
```

Iteration space for A



```
for i = 0 to N-1
  for j = 0 to N-1
    ... = A[j][i] ...
```

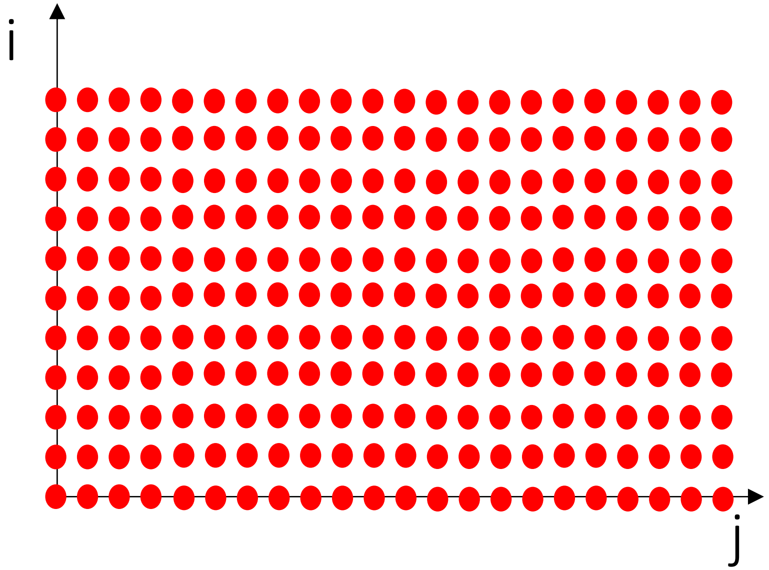
Assumptions: N is large; A is row-major; 8 elements per cache line



```
for i = 0 to N-1
  for j = 0 to N-1
    … = A[j][i] …
```

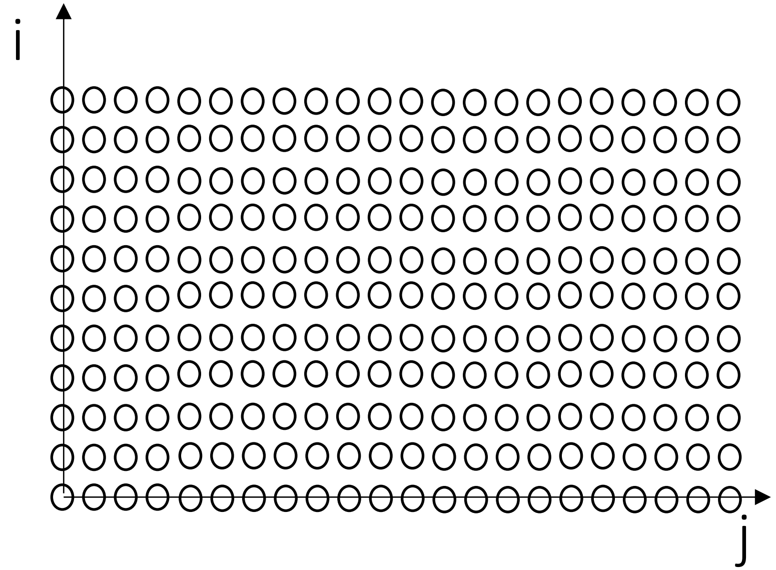
```
For j = 0 to N-1
  for i = 0 to N-1
    … = A[j][i] …
```

Assumptions: N is large; A is row-major; 8 elements per cache line



○ Cache hit (low #cycles)

● Cache miss (high #cycles)



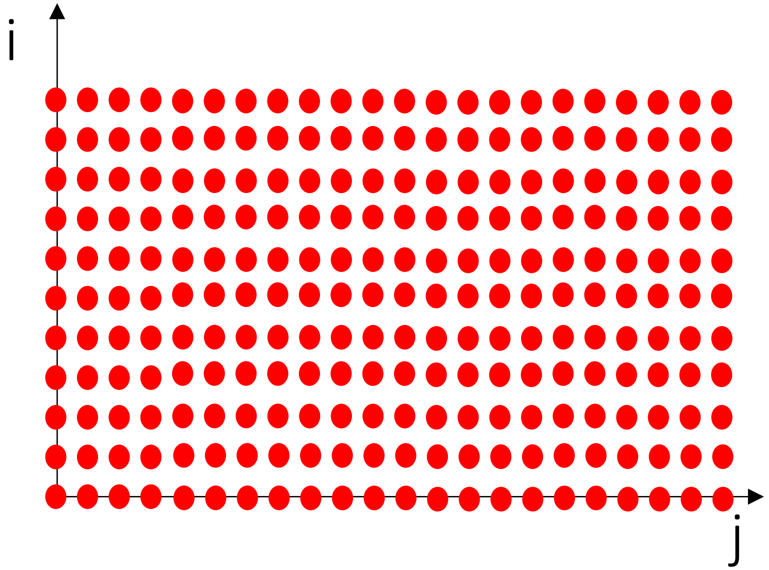
Loop interchange

```
for i = 0 to N-1
  for j = 0 to N-1
    … = A[j][i] …
```

A[][] in C? Java?

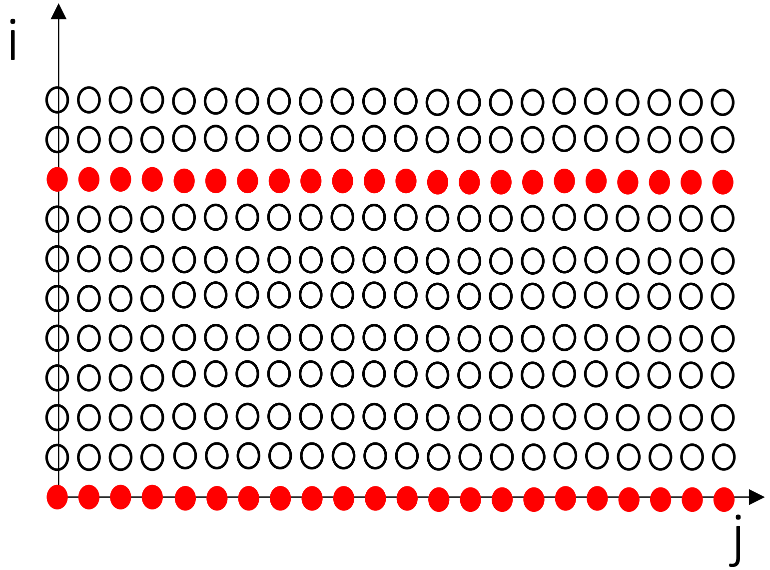
```
For j = 0 to N-1
  for i = 0 to N-1
    … = A[j][i] …
```

Assumptions: N is large; A is row-major; 8 elements per cache line



o Cache hit (low #cycles)

● Cache miss (high #cycles)



Java (similar in C)

To create a matrix:

```
double [][] A = new double[3][3];
```

A is an array of arrays

A is not a 2 dimensional array!

Java (similar in C)

To create a matrix:

```
double [][] A = new double[3][];
A[0] = new double[3];
A[1] = new double[3];
A[2] = new double[3];
```

Java (similar in C)

To create a matrix:

```
double [][] A = new double[3][];
A[0] = new double[10];
A[1] = new double[5];
A[2] = new double[42];
```

A is a jagged array

C#: [][] vs. [,]

```
double [][] A = new double[3][];
A[0] = new double[3];
A[1] = new double[3];
A[2] = new double[3];
```

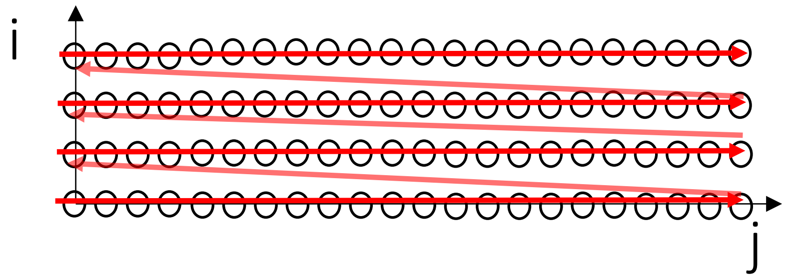
```
double [,] A = new double[3,3];
```

The compiler can easily choose between raw-major vs. column-major

```c
#include <stdio.h>

int main (){
  int a[2][4];

  printf("0x%p\n", &a[0][0]);
  printf("0x%p\n", &a[0][1]);
  printf("  Distance: %d bytes\n", ((unsigned int)(&a[0][1])) - ((unsigned int)(&a[0][0])));

  printf("0x%p\n", &a[0][0]);
  printf("0x%p\n", &a[1][0]);
  printf("  Distance: %d bytes\n", ((unsigned int)(&a[1][0])) - ((unsigned int)(&a[0][0])));

  return 0;
}
```

```
for i = 0 to N-1
  for j = 0 to N-1
    f(A[i], A[j])
```

Assumptions: N is large; 8 elements per cache line

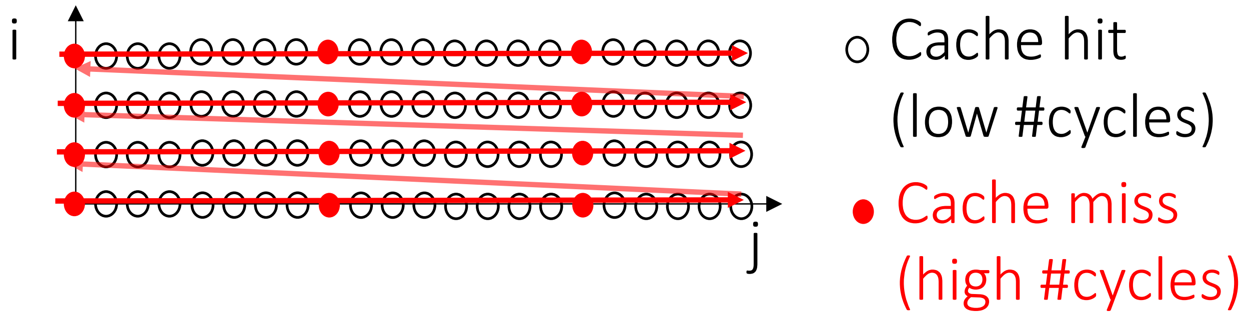


o Cache hit (low #cycles)

● Cache miss (high #cycles)

```
for i = 0 to N-1
  for j = 0 to N-1
    f(A[i], A[j])
```

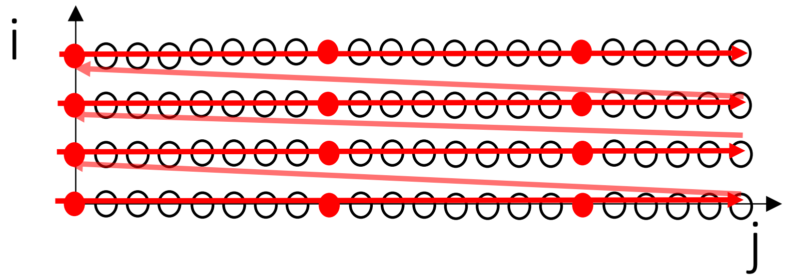
Assumptions: N is large; 8 elements per cache line



```
for i = 0 to N-1
  for j = 0 to N-1
    f(A[i], A[j])
```

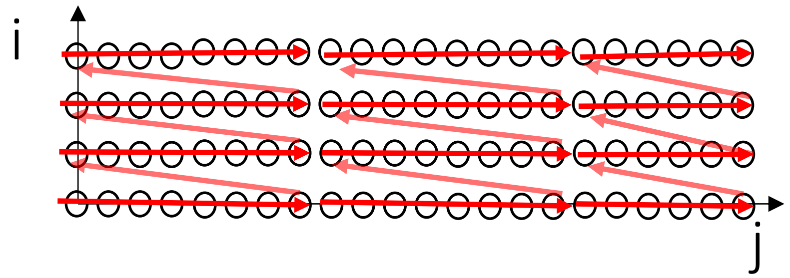
```
for JJ = 0 to N-1 by B
  for i = 0 to N-1
    for j = JJ to min(N-1,JJ+B-1)
      f(A[i], A[j])
```

Assumptions: N is large; 8 elements per cache line



o Cache hit (low #cycles)

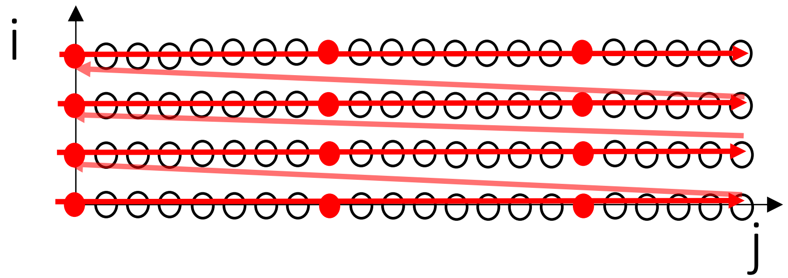
● Cache miss (high #cycles)



```
for i = 0 to N-1
  for j = 0 to N-1
    f(A[i], A[j])
```

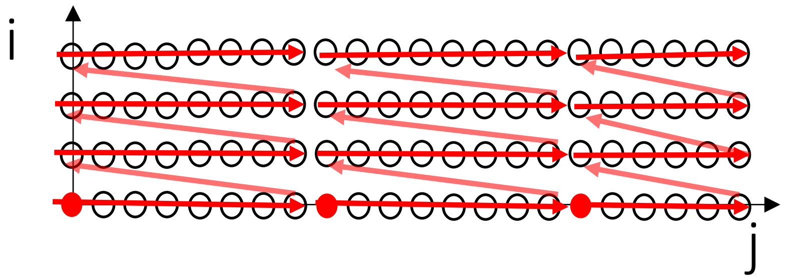
```
for JJ = 0 to N-1 by B
  for i = 0 to N-1
    for j = JJ to min(N-1,JJ+B-1)
      f(A[i], A[j])
```

Assumptions: N is large; 8 elements per cache line



○ Cache hit (low #cycles)

● Cache miss (high #cycles)



Loop fusion

```
for i = 0 to N-1
  C[i] = A[i]*2 + B[i]

for i = 0 to N-1
  D[i] = A[i] * 2
```

```
for i = 0 to N-1
  C[i] = A[i] * 2 + B[i]
  D[i] = A[i] * 2
```

- Reduce loop overhead
- Improve locality by combining loops that reference the same array
- Increase the granularity of work done in a loop

Loop transformations

- They manipulate the order of memory accesses
- They can change both temporal and spatial localities
- They can enable or disable parallelism

Always have faith in your ability

Success will come your way eventually

Best of luck!